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RESEARCH ARTICLE

THE DESIGN OF A LINEAR QUADRATIC OPTIMAL REGULATOR FOR TRANSIENT HEAT FLOW IN CONTINUOUS CASTING OF STEEL

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ABSTRACT

A generalized approach to the modelling of transient heat flow in steel ingots during cooling in the continuous casting mould is presented. The mathematical development starting with the relevant differential equations and their modelling using the Hills' integral profile method gave polynomial functions which characterized the transient heat flow in the steel ingots. An optimal feedback control law $u(\mu)$ was obtained for the system based on the quadratic performance index, and was expressed as a function of x given by $u(\mu) = f(x)$, which assures asymptotic stability $\left(x(\mu) \to 0\right)$ as $\mu \to \infty$. A good functional correspondence was shown to exist between the model and the data reported in the literature.

Notation:

C Specific heat capacity (J/kg^{0}C), h Heat transfer coefficient (W/m^{20}C), k Thermal Conductivity (W/m^{O}C) , Q, q Heat flux (W/m^2)

t Dimension of workpiece (m), V Speed of workpiece (m/s), x, y, z Spatial extent in space (m), α Thermal diffusivity (m^2/s), ε Strain,

θ Temperature (^{O}C), ρ Density (kg/m^3), τ Time (s), ξ Dimensionless time, Δ Small increment.

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INTRODUCTION

Several models of heat flow in hot working processes have been reported in the existing literature. Most of these studies have been confined to laboratory models and relatively simple components, notably finite lengths of steel slabs, have been used to produce solutions to the heat conduction equations. These equations were often defined in one (Wartmann, 1973) and more rarely two (Yu and Sang, 2007) dimensions in space.

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In billet mills, accurate mathematical models of interstand cooling of rolled steel are required for the analysis and control of the shape distortions observed during air-cooling of the final product. In relation to the thermal distortions observed during hot rolling of steel in billet mills (Obinabo, 1991), these one-dimensional models cannot be used with confidence to describe the origins and orientations of these defects. Increasingly, the models reported in the existing literature are aimed at predicting the thermal conditions of the rolls in both hot and cold strip mills while a very scant treatment is, so far, given to the determination of the actual temperature distributions in the workpiece itself.

A variety of methods of solution has been adopted in the literature for the analysis of the unsteady heat conduction problem. In general, they are classified into two broad categories: (a) approximate mathematical solutions using more or less realistic cooling conditions, and (b) exact solutions under conditions rarely achieved in practice. Here the analytical methods are used to find solutions without mathematical approximations. This is made quite difficult by the unsteady heat flow, and solutions obtained generally describe particular ideal cases.

Approximate analytical procedures

The approximate methods of analysis applied to transient heat transfer problems include the electrical and hydraulic analogue techniques (Chapman, 1974), the finite difference methods and the heat balance integrals otherwise know as the integral-profile method (Hills, 1965). A number of works has been reported which makes use of the first two techniques. The finite difference method has the disadvantage of being very lengthy and tedious to apply, and the entire procedure must be repeated each time a parameter is assigned a new value. The integral-profile method, on the other hand, seems to have attracted very little attention although it has been shown (Hills, 1965) to be successful in the in the prediction of heat transfer rates during solidification of steel, for instance, and in a wide range of situations, without involving large amounts of computation time.

The finite element method has also featured in the analysis of heat flow problems in the continuous casting of steel (Zorzi and Mazzantini, 1982). However, the technique has most commonly been limited to stress analysis and related crack formulation problems (Sorimachi and Brimacombe, 1977) which in conjunction with finite difference models, has been used to determine the thermal fields. The work of Soliman and Fakhroo (1972) makes use of a variational formulation and triangular finite elements to describe a two-dimensional problem of heat conduction in steel ingots, accounting for the variations of the specific heat and the thermal conductivity in the model. Other methods of analysis considered in this work include the boundary layer method, the linear temperature profile method and the method of heat balance integral.

The boundary layer method of analysis

Many of the contributions to the development of temperature models in hot processes are based on the principle of fluid flow in a laminar boundary layer, using an approximate method similar to that developed by Von Karman (1934) and Pohlhausen (1921). In this method, the rate of growth of the boundary layer at any point is assumed to be solely determined by its thickness at that point, and by other local properties. Based on this assumption, a first order differential equation was derived (Welty *et al*., 1976) for the growth of the layer. The form of this equation is determined by the use of an auxiliary function for the velocity profile within the boundary layer and by the application of the conservation of momentum. The boundary layer method has also been used (Cooper, 1969) to solve transient heat conduction problems in a semi-infinite solid with temperature-dependent conductivity and constant surface heat flux. The existence and significance of similarity solutions for a semi-infinite solid with temperature-dependent conductivity was established by Peletier (1970) via rigorous mathematical argument. In a related study, Cook (1970) showed that accounting for the variable properties of the material may significantly influence the heat transfer rates deduced from the measurement of a solid metal. When transient heat conduction is confined to a thin layer near the surface of a solid, the solution to the problem was generally approximated to that of a semi-infinite solid (Letcher, 1969). A generalization of this technique has been developed for curved surfaces (Letcher, 1969). A method was also developed for solving the unsteady heat equation in the case in which the bounding surfaces of a solid change with time while maintaining a similar shape (Grinberg, 1969). It has also been reported (Chao and Chen, 1970) that a series solution method in the Laplace transform developed in connection with a convection problem for a fluid sphere is useful in transient heat conduction problems.

Linear temperature profile methods

Several assumptions of linear temperature distributions have been made in the treatment of heat flow problems in the cooling of cast steel billets (Hills, 1963). However, it was observed that this assumption produced some serious errors when the computed values were compared with the measured values. Hills (1963) also noted that with any given surface temperature value, an assumed linear temperature distribution underestimates the thermal energy stored in the solid metal, and that any underestimation of the temperature gradient at the outer surface of the metal is even more serious since the value is needed, where the surface temperature is not specified, to relate the surface temperature to both the dimensions of the solid metal and the heat flux leaving the surface. A simplifying assumption was adopted (Hills, 1963) to avoid the difficulty of specifying the surface temperature.

Mathematical formulations

The problem considered is one of time-and space dependent heat flow in the square steel ingots produced in Delta Steel mill at Ovwian-Aladja, Warri, Nigeria. The approach in the development of the model was based on the assumption that the steel grade considered was homogeneous in terms of its metallurgical constitutions. This enabled the thermal conductivity and the specific heat capacity of the material to be assumed constant across any dimension of the cast ingot. Another important aspect of the model is the provision for temperature independence of these functions. A series of empirical equations has been adopted in the literature (Yang and Lu, 1986) for the different phases in steel over the whole temperature range. It is indicated that above the temperature of 900^{o}C, the thermal conductivity is temperature-dependent. Consequently, two simultaneous ordinary differential equations were derived for these parameters which satisfied the following unsteady heat conduction equation

$$\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)-\left(v_x\frac{\partial\theta}{\partial x}+v_y\frac{\partial\theta}{\partial y}+v_z\frac{\partial\theta}{\partial z}\right)+Q'=\frac{\partial\theta}{\partial\tau}\;\ldots\ldots(1)$$

where $\alpha = \dfrac{k}{\rho c}$, $Q' = \dfrac{Q}{\rho c}$, v_x, v_y, v_z are component of the velocity vector $\vec{v}$. $\dfrac{\partial \theta}{\partial \tau}$ denotes the rate of change of temperature in space and time along the workpiece. The length of the deforming bar was considered infinite since the rolling process in the mill was continuous. Consequently, conduction of heat in that dimension relative to the other dimensions was negligibly small, so that the temperature changes in that direction is a function of time only. Applicability of (1) was based on the assumption that the rolling process in the mill was steady relative to the roll stand, and that the motion of the workpiece was restricted to the direction of rolling only. The heat input, Q, due to the deformation in the roll gap was assumed uniformly distributed in the workpiece. Thus

$$\frac{\partial \theta}{\partial \tau} = 0 \qquad \text{………………………(2)}$$

$$V_x = V_y = 0 \qquad \text{……………………….(3)}$$

At any point *z* along the length of the ingot from a chosen position, the time during which the workpiece was exposed to the cooling effects of the mill was defined (Obinabo, 1991) from a chosen position, this time was defined as:

$$\tau = \frac{z}{v} \qquad \text{……………………….(3)}$$

The origin of z was at the instance the workpiece exits the roll gap, and this is at time $\tau = 0$. Equation (3) applies to the workpiece at any point between two roll stands, that is, at exit from the roll gap of one stand to the point just before entry into the roll gap of the next stand. It also applies to the portion of the mill between the last finishing stand and the cooling bed. In each of these regions the speed of the workpiece is assumed constant. Therefore, to transform the foregoing results (1) in terms of this variable τ, the following operator was derived from (3) as:

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial \tau}\frac{d\tau}{dz} = \frac{1}{V}\frac{\partial}{\partial \tau} \qquad \text{……………………………………(4)}$$

and, on substitution into (1), yields (Yu and Sang, 2007; Kwon and Bang, 2000):

$$\alpha\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{1}{V^2}\frac{\partial^2\theta}{\partial \tau^2}\right) + Q' = \frac{\partial \theta}{\partial \tau} \qquad \text{……........(5)}$$

In the weighted form (Yu and Sang, 2007), equation (5) becomes

$$\int_\Omega W\left[\frac{\partial \theta}{\partial t} - \frac{\partial}{\partial x}\left(\alpha\frac{\partial \theta}{\partial x}\right) - \frac{\partial}{\partial y}\left(\alpha\frac{\partial \theta}{\partial y}\right)\right]d\Omega = 0 \quad \text{…………...(6)}$$

and reduced using Green's theorem (Pepper and Heinrich, 1992) as follows:

$$\iint_\Omega\left[W\frac{\partial \theta}{\partial t} + \alpha\left(\frac{\partial W}{\partial x}\frac{\partial \theta}{\partial x} + \frac{\partial W}{\partial y}\frac{\partial \theta}{\partial y}\right)\right]d\Omega + \int_{\psi_B} W\left(-\alpha\frac{\partial \theta}{\partial n}\right)d\psi = 0 \quad \text{…(7)}$$

where $\alpha = \dfrac{k(\theta)}{\rho c(\theta)}$ and Ω denotes the two-dimensional domain. ψ_B and $d\psi$ represent respectively the boundary and the surface element of ψ_B over which the normal gradients were applied. Also n is the outward normal unit vector at the boundary ψ. To facilitate computation of (5) the dimensionless variables due to Hills (Obinabo, 1991) were employed to transform the equation to the following form

$$\frac{\partial^2\theta^*}{\partial x^{*2}} + \frac{\partial^2\theta^*}{\partial y^{*2}} + \left(\frac{1}{V^*}\right)^2\frac{\partial^2\theta^*}{\partial \xi^2} = \frac{\partial \theta^*}{\partial \xi}$$

……………………(8)

which, on introduction of the integral sign, gave

$$\iint\left(\frac{\partial^2\theta^*}{\partial x^{*2}} + \frac{\partial^2\theta^*}{\partial y^{*2}}\right)dx^*dy^* = \iint\left(\frac{\partial \theta^*}{\partial \xi} - \left(\frac{1}{V^*}\right)^2\frac{\partial^2\theta^*}{\partial \xi^2}\right)dx^*dy^* \quad \text{….(9)}$$

where $V^* = \dfrac{V\rho c\theta_i}{[q_o]_o}$ and which, on further simplification and ignoring the asterisk, gave

$$h(\theta_{t_x} + \theta_{t_y}) - q_{oy}t_x - q_{ox}t_y = \frac{d}{d\xi}\left(\int_0^{t_x}\int_0^{t_y}\theta\, dxdy\right) - \theta_{t_x}\left(\frac{dt_x}{d\xi}\right) - \theta_{t_y}\left(\frac{dt_y}{d\xi}\right)$$
$$-\left(\frac{1}{V}\right)^2\left(\frac{d^2}{d\xi^2}\int_0^{t_x}\int_0^{t_y}\theta\, dxdy\right) - \theta_{t_x}\left(\frac{d^2t_x}{d\xi^2}\right) - \theta_{t_y}\left(\frac{d^2t_y}{d\xi^2}\right)$$

……(10)

The left hand side of (10) was evaluated first by integrating with respect to *y**, then with respect to *x** to yield the following:

$$LHS = t_y \cdot \left.\frac{\partial \theta^*}{\partial x^*}\right|_{t_x} - t_y \cdot \left.\frac{\partial \theta^*}{\partial x^*}\right|_{x=0} + t_x \cdot \left.\frac{\partial \theta^*}{\partial y^*}\right|_{t_y} - t_x \cdot \left.\frac{\partial \theta^*}{\partial y^*}\right|_{y=0} \quad \text{…(11)}$$

where suffix *o* represents the origin of the coordinate axes, x^* and y^* are dimensionless spatial extents in the x and y directions respectively. Writing (11) in terms of the surface heat flux, *q* gives

$$q^*_{t_y} = h^*\Delta\theta^*_{t_y} \qquad \text{………………………………………(12)}$$

then (12) becomes:

$$h^*\left(\theta^*_{t_x}+\theta^*_{t_y}\right)-q^*_{ox}t^*_y-q^*_{oy}t^*_x \quad \text{……….(13)}$$

The first term on the right hand side of equation (10) was evaluated (Obinabo, 1991) to yield the following result:

$$\iint\left(\frac{\partial\theta^*}{\partial\xi}\right)dx^*dy^*=\frac{d}{d\xi}\left(\iint\theta^*dx^*dy^*\right)-\theta^*_{t_x}\left(\frac{dt^*_x}{d\xi}\right)-\theta^*_{t_y}\left(\frac{dt^*_y}{d\xi}\right) \quad \text{…. (14)}$$

Similarly, evaluating the second term on the same right hand side of the equation yields:

$$-\left(\frac{1}{V^*}\right)^2\iint\left(\frac{\partial^2\theta^*}{\partial\xi^2}\right)dx^*dy^*=-\left(\frac{1}{V^*}\right)^2\frac{d^2}{d\xi^2}\left(\iint\theta^*dx^*dy^*\right)-\theta^*_{t_x}\left(\frac{d^2t_y}{d\xi^2}-\theta^*_{t_y}\left(\frac{d^2t_y}{d\xi^2}\right)\right) \quad \text{…(15)}$$

Combining these results and ignoring the *asterisks*, the following was obtained:

$$h\left(\theta_{t_x}+\theta_{t_y}\right)-q_{oy}t_x-q_{ox}t_y=\frac{d}{d\xi}\left(\int_0^{t_x}\int_0^{t_y}\theta dxdy\right)-\theta_{t_x}\left(\frac{dt_x}{d\xi}\right)-\theta_{t_y}\left(\frac{dt_y}{d\xi}\right)$$
$$-\left(\frac{1}{V}\right)^2\left(\frac{d^2}{d\xi^2}\left(\int_0^{t_x}\int_0^{t_y}\theta dxdy\right)-\theta_{t_x}\left(\frac{d^2t_x}{d\xi^2}\right)-\theta_{t_y}\left(\frac{d^2t_y}{d\xi^2}\right)\right) \quad \text{…………(16)}$$

Auxiliary function for $\theta(x, y)$

The result shown in equation (16) could not as yet be solved because the temperature distributions appearing in the integrals were not known. Some functions were required to represent these temperature distributions in the workpiece during cooling. Consequently it was imperative to design an accurate temperature profile θ which, in itself, satisfies the boundary conditions that prevail in the cooling of the bars during rolling and was a function of x, y and ξ. In considering the two-dimensional steady state heat conduction problem, the integral approaches proposed by Ritz and Kantorovich (Yang and Lu, 1986) report auxiliary functions with at least one unspecified parameter. Ritz method assumes a quadratic function in the dimension that runs across the width of the workpiece, and an exponential function in the dimension that runs along the length of the workpiece. The result of the two-dimensional profile was defined mathematically as:

$$\theta(x,y)=A\left(\ell^2-y^2\right)e^{-BX} \quad \text{…………….. (17)}$$

where A and B were determined from the boundary conditions, and ℓ represents the width of the workpiece of infinite length. Kantorovich's method was almost similar to Ritz's. The difference was that the form of the profile assumed in the dimension that runs along the length of the flat bar was an unknown function. This reduced the Ritz function to the form.

$$\theta(x,y)=(\ell^2-y^2)X(x) \quad \text{……………………… (18)}$$

where $X(x)$ was required to be determined from the boundary conditions.

$$\theta=a_o+a_1\left(\frac{x}{t}\right)+a_2\left(\frac{x}{t}\right)^2 \quad \text{…………………….. (19)}$$

In current investigation, a *2-D* auxiliary function based on the spatial cooling profiles reported by Obinabo (1991) was proposed for the surface and width dimensions of the workpiece as follows

$$\theta(x,y)=\left(a_o+a_2x^2+a_4x^4\right)\left(b_o+b_1y+b_2y^2\right) \quad \text{…….. (20)}$$

The spatial distribution was symmetrical in the surface dimension and asymmetrical in the width dimension. During air cooling the workpiece rested surface–wise on the cooling bed. In this condition, the top surface was exposed to the free air stream surrounding it while the bottom surface exchanged heat by conduction with the cooling bed. This condition gave rise to (20). The *a's* and *b's* were determined from the boundary conditions. Expanding (20) and ignoring terms containing powers of *x's* and *y's* higher than *2*, the following was obtained.

$$\theta(x,y)=a_ob_o+a_ob_1y+a_ob_2y^2+a_2b_ox^2 \quad \text{………….. (21)}$$

The justification for truncating (21) is embodied in the reasoning that the variables x and y became non-dimensionalised by defining the following:

$$x^*=\frac{t_x}{x} \quad \text{…………………………………...(22)}$$

$$y^*=\frac{t_y}{y} \quad \text{……………………………..….(23)}$$

where t_x and t_y are instantaneous spatial extents along the directions of x and y respectively. It then follows that the maximum value either t_x or t_y can take in (23) and (24) is x or y. Consequently, in analyzing the heat distributions within the workpiece, the values of x and y in the auxiliary function will always be fractional and higher powers of fractions reduced them to negligibly small quantities, and the terms containing them tend to zero. For all values of x and y, the expansion to power *2* obtained in equation (22) seemed quite reasonable, and therefore, represents the approximate auxiliary function required to compute the temperature distributions in the workpiece.

The final form of the model

Apart from the roll gap where heat was generated within the workpiece due to deformation, no heat sources were known to exist in the mill train. Consequently, a zero heat flow condition across the centre line of the workpiece was assumed so that the following result was obtained from (16).

$$h\left(\theta_{t_x}+\theta_{t_y}\right)=\frac{d}{d\xi}\left(\int_0^{t_x}\int_0^{t_y}\theta dxdy\right)-\theta_{t_x}\left(\frac{dt_x}{d\xi}\right)-\theta_{t_y}\left(\frac{dt_y}{d\xi}\right)$$

$$-\left(\frac{1}{V}\right)^2\left(\frac{d^2}{d\xi^2}\left(\int_0^{t_x}\int_0^{t_y}\theta dxdy\right)-\theta_{t_x}\left(\frac{d^2t_x}{d\xi^2}\right)-\theta_{t_y}\left(\frac{d^2t_y}{d\xi^2}\right)\right) \quad \text{........(24)}$$

Difficulties associated with measurement of the surface temperature of the workpiece during the rolling process made direct measurement of the heat transfer coefficients at these locations almost impossible. Indirect method of measurement which involves use of radiation pyrometers has been adopted generally (Kim and Huh, 2000; Polukhin, 1975). The disadvantage of this technique of temperature measurement was that the other modes of cooling were not monitored. Consequently, the accuracy of the results so obtained depends largely on the effectiveness of the radiation mechanism, and the surface heat flux of the material becomes a direct function of the radiation mechanism. Harding (1976) argued that this is misleading since convection was a more important heat transfer mechanism than was generally thought. Polukhin (1975) and Hills (Obinabo, 1991) also considered a combined effect of convection and radiation mechanisms and related it to the surface heat flux of the workpiece. Meanwhile, in their classical experiments on heat flow in continuous casting of steel ingots, Savage and Pritchard (Hills, 1963) obtained a relationship that expresses the surface flux as a function of time. This was done by measuring the rise in the temperature of the cooling water. The data so generated was used to estimate the total quantity of heat removed from the surface of the cooling steel ingot. The expression obtained from the heat flux was of the form

$$q''_o = [q_o]_o - b\sqrt{\tau} \quad \text{..................................(25)}$$

for which the values of 2628 and 221.9 were obtained for q_o and b respectively; b is constant of linear relationship between the heat flux and dwell time. In terms of the dimensionless variables used in the development of this work, this expression reduces to

$$q* = 1-\beta\sqrt{\xi} \quad \text{..........................(26)}$$

where $\beta = \dfrac{221.9}{2628}\ (=0.08)$ and is a constant of linear relationship obtained by transforming (25) to its dimensionless form. This result was reduced in Obinabo (1991) to the following:

$$\frac{\partial q}{\partial \xi} = f_\xi = \frac{-\beta}{2\sqrt{\xi}} \quad \text{.................................... (27)}$$

From (27) the following result was obtained:

$$\frac{\partial^2 q}{\partial \xi^2} = f'_\xi = \frac{\beta}{4\sqrt{\xi^3}} \quad \text{..................................(28)}$$

Hills (1963) shows that the heat transfer coefficient at the surface of the workpiece bears a linear relationship with time, and gives the surface heat flux as:

$$q''_o = -h_o(1-\gamma\xi)\theta_o \quad \text{..................................(29)}$$

where the subscript o represents the values on the surface of the workpiece, and γ represents a constant of linear relationship. In terms of the dimensionless variables this result becomes:

$$q^* = (1-\gamma\xi)\theta \quad \text{.......................................(30)}$$

which yields:

$$\frac{\partial q}{\partial \xi} = f'_\xi = -\gamma\theta \quad \text{...................................... (31)}$$

The following result was deduced from (31)

$$\frac{\partial^2 q}{\partial \xi^2} = f''_\xi = 0 \quad \text{....................................... (32)}$$

For the modes of cooling the workpiece considered in this work, the surface heat flux was given by an equation of the form (Eckert *et al*., 1996)

$$q = -\left(\sigma F\left(\theta^4-\theta_A^4\right)+h(\theta-\theta_A)\right) \quad \text{.................(33)}$$

where θ_A = ambient temperature
θ = measurement surface temperature

σ = Stefan-Boltzman constant $= 56.7\times10^{-12}\ kWm^{-2}k^{-4}$
F = shape factor accounting for the geometry of the surface of the workpiece radiating heat.

In terms of the dimensionless variables (33) becomes:

$$q^* = \frac{\theta^4-\phi^4+h^*(\theta-\phi)}{1-\phi^4+h^*(1-\phi)} \quad \text{..........................(34)}$$

where $h^* = \dfrac{h}{\phi F\theta^3}$, ϕ = dimensionless absolute ambient temperature.

On the surface heat transfer coefficient of steel products cooling in air, a number of results has been deduced by in the existing literature. On the run-out table of a strip mill, Labiesh (1982) reported a wide range of total heat transfer coefficient in the range $60-120\,Wm^{-2}k^{-1}$ for a strip piece being transported from the roll stand to the cooling bed. Several other publications have been made on the predication of this value, and some of them have been discussed extensively in Obinabo (1991). A complete analysis of (24) was possible only when an auxiliary function was defined and the surface heat flux adequately accounted for. When the three forms of the

surface heat flux variation were considered, the following results were obtained.
From (26)

$$\theta(x,y)=\theta_o+(5y/2)\ \ (\theta_{17}-\theta_{18}) + (y/2)(1-\beta\sqrt{\zeta})+(x/2)(1-\beta\sqrt{\zeta}) \quad \ldots\ldots\ldots(35)$$

From (30), the auxiliary function becomes:

$$\theta(x,y)=\theta_O+(5y/2)\ \ (\theta_{17}-\theta_{18}) + (y/2)(1-\beta\sqrt{\zeta})+(x/2)(1-\beta\sqrt{\zeta}) \quad \ldots\ldots\ldots(36)$$

From (34), the auxiliary function becomes:

$$\theta(x,y)=\theta_o+(5y/2)(\theta_{17}-\theta_{18})+(y/2)(\theta)+(x/2)\theta \ ..\ (37)$$

Now taking the integrals:

$$\int_0^{t_x}\int_0^{tt_y}\theta dxdy=\int_0^{t_y}\left(\int_0^{t_x}\theta dx\right)dy$$

From (35) the following was obtained

$$\int_0^{t_x}\theta\,dx=t_x\theta_o+\frac{5}{2}t_x y(\theta_{17}-\theta_{18})+t_x\frac{y}{2}\left(1-\beta\sqrt{\xi}\right)+\frac{t_x^2}{4}\left(1-\beta\sqrt{\xi}\right)$$

and $\int_0^{t_y}\left(\int_0^{t_x}\theta dx\right)dy=t_xt_y\theta_o+\frac{5}{4}t_xt_y^2(\theta_{17}-\theta_{18})+\frac{1}{4}t_xt_y^2\left(1-\beta\sqrt{\xi}\right)+\frac{1}{4}t_xt_y\left(1-\beta\sqrt{\xi}\right)$ ...(38)

Similarly from (36) and (37) respectively the following were obtained:

$$\int_0^{t_x}\int_0^{t_y}\theta\,dxdy=t_xt_y\theta_o+\frac{5}{4}t_xt_y^2(\theta_{17}-\theta_{18})+\frac{1}{4}t_xt_y^2(1-\gamma\xi)\theta_{t_y}+\frac{1}{4}t_x(1-\gamma\xi)\theta_{t_x} \quad \ldots(39)$$

$$\int_0^{t_x}\int_0^{t_y}\theta\,dxdy=t_xt_y\theta_o+\frac{5}{4}t_xt_y^2(\theta_{17}-\theta_{18})+\frac{1}{4}t_xt_y^2\theta t_y+\frac{1}{4}t_x^2t_y\theta t_x \ \cdots \quad (40)$$

From these results, therefore, (16) is written for each of the cases considered above in the *x*-and *y*- dimensions as follows:

From (38), the following were obtained:

In the *x*-dimension:

$$h\theta_{t_x}=\frac{d}{d\xi}\left(t_xt_y\theta_o+\frac{1}{4}t_x^2t_y\left(1-\beta\sqrt{\xi}\right)\right)-\theta_{t_x}\frac{dt_x}{d\xi}$$
$$-\left(\frac{1}{V}\right)^2\left(\frac{d^2}{d\xi^2}\left(t_xt_y\theta_o+\frac{1}{4}t_x^2t_y\left(1-\beta\sqrt{\xi}\right)\right)\right)-\theta_{t_x}\frac{d^2t_x}{d\xi^2} \quad \ldots\ldots\ldots\ldots(41)$$

In the *y*-dimension:

$$h\theta_{t_x}=\frac{d}{d\xi}(t_xt_y\theta_o+\frac{5}{4}t_xt_y^2(\theta_{17}-\theta_{18})+\frac{1}{4}t_xt_y\left(1-\beta\sqrt{\xi}\right)-\theta_{t_y}\frac{dt_y}{d\xi}$$
$$-(\frac{1}{V})^2(\frac{d^2}{d\xi^2}(t_xt_y\theta_o+\frac{5}{4}t_xt_y^2(\theta_{17}-\theta_{18}) + \frac{1}{4}t_xt_y(1-\beta\sqrt{\xi}))-\theta_{t_y}(\frac{d^2t_t}{d\xi^2})) \quad \ldots\ldots\ldots\ldots\ldots(42)$$

From (39), the following were obtained:

In the *x*-dimension:

$$h\theta_{t_x}=\frac{d}{d\xi}(t_xt_y\theta_o+\frac{1}{4}t_x^2t_y(1-\gamma\xi)\theta_{t_x})-\theta_{t_x}\frac{dt_x}{d\xi})$$
$$-(\frac{1}{V})^2(\frac{d^2}{d\xi^2}(t_xt_y\theta_o+\frac{1}{4}t_x^2t_y(1-\gamma\xi)\theta_{t_x})-\theta_{t_x}\frac{d^2t_x}{d\xi^2}) \quad \ldots\ldots\ldots(43)$$

In the *y*-dimenson:

$$h\theta_{t_y}=\frac{d}{d\xi}(t_xt_y\theta_o+\frac{5}{4}t_xt_y^2(\theta_{17}-\theta_{18}) + \frac{1}{4}t_xt_y^2(1-\gamma\xi)\theta_{t_y})-\theta_{t_y}\frac{dt_y}{d\xi}$$
$$-(\frac{1}{V})^2(\frac{d^2}{d\xi^2}(t_xt_y\theta_o+\frac{5}{4}t_xt_y^2(\theta_{17}-\theta_{18}) + \frac{1}{4}t_xt_y^2(1-\gamma\xi)\theta_{t_y})-\theta_{t_y}\frac{d^2t_y}{d\xi^2}$$
(44)

From (40), the following were obtained:

In the *x*-dimension:

$$h\theta_{t_x}=\frac{d}{d\xi}(t_xt_y\theta_o+\frac{1}{4}t_x^2t_y(q_{t_x}))-\theta_{t_x}(\frac{dt_x}{d\xi})$$

$$-(\frac{1}{V})^2(\frac{d^2}{d\xi^2}(t_xt_y\theta_o+\frac{1}{4}t_x^2t_y(q_{t_x}))-\theta_{t_x}(\frac{d^2t_x}{d\xi^2})) \quad \ldots\ldots(45)$$

In the *y*-dimension:

$$h\theta_{t_y}=\frac{d}{d\xi}(t_xt_y\theta_o+\frac{1}{4}t_xt_y^2q_{t_x})-\theta_{t_x}\frac{dt_y}{d\xi}$$
$$-(\frac{1}{V})^2(\frac{d^2}{d\xi^2}(t_xt_y\theta_o+\frac{1}{4}t_xt_y^2q_{t_y})-\theta_{t_y}\frac{d^2t_y}{d\xi^2} \quad \ldots\ldots\ (46)$$

From (41) and (42), the following were obtained:

$$(\frac{1}{V})^2\frac{d^2\theta_o}{d\xi^2}-\frac{d\theta_o}{d\xi}=-(\frac{1}{t_xt_y}(1-\beta\zeta^{\frac{1}{2}})$$

$$+\frac{1}{8}\beta t_x \xi^{-\frac{1}{2}}(1+2\left(\frac{1}{v}\right)^2\zeta^3)) - \frac{\theta_{t_x}}{t_x t_y}(\frac{dt_x}{d\xi}) - \frac{\theta_{t_x}}{t_x t_y}(\frac{d^2 t_x}{d\xi^2}) \quad \ldots\ldots(47)$$

and $(\frac{1}{V})^2 \frac{d^2\theta_o}{d\xi^2} - \frac{d\theta_o}{d\xi} = -\frac{1}{t_x t_y}(1-\beta\zeta^{\frac{1}{2}})$

$$+\frac{1}{8}\beta\xi^{\frac{1}{2}}(1+\frac{1}{2}(\frac{1}{V})^2\xi^3)) - \frac{\theta_{t_y}}{t_x t_y}(\frac{dt_y}{d\xi}) - \frac{\theta_{t_y}}{t_x t_y}(\frac{d^2 t_y}{d\xi^2}) \quad \ldots(48)$$

Similarly from (43) and (44), the following were obtained:

$$-(\frac{1}{V})^2 \frac{d^2\theta_o}{d\xi^2} - \frac{d\theta_o}{d\xi} = \frac{1}{t_x t_y}(-\frac{1}{4}t_x^2 t_y \gamma\theta_{t_x} - ((1-\gamma\xi)\theta_{t_x})$$

$$-\theta_{t_x}(\frac{dt_x}{d\xi}) - \theta_{t_x}(\frac{d^2 t_x}{d\xi^2}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(49)$$

and

$$(\frac{1}{V})^2 \frac{d^2\theta_o}{d\xi^2} - \frac{d\theta_o}{d\xi} = \frac{1}{t_x t_y}(-\frac{1}{4}t_x t_y^2 \gamma\theta_{t_y} - ((1-\gamma\xi)\theta_{t_x})$$

$$-\theta_{t_y}(\frac{dt_y}{d\xi}) - \theta_{t_y}(\frac{d^2 t_y}{d\xi^2})) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(50)$$

From (45 and (46), the following were obtained:

$$(\frac{1}{V})^2 \frac{d^2\theta_o}{d\xi^2} - \frac{d\theta_o}{d\xi} = \frac{1}{4}t_x \frac{d}{d\xi}(q_{t_x}) - (\frac{1}{4V^2})t_x \frac{d^2}{d\xi^2}(q_{t_x})$$

$$-\frac{k}{t_x t_y}(q_{t_x}) - \theta_{t_x}(\frac{dt_x}{d\xi}) - \theta_{t_x}(\frac{d^2 t_x}{d\xi^2}) \quad \ldots\ldots\ldots\ldots\ldots...(51)$$

$$(\frac{1}{V})^2 \frac{d^2\theta_o}{d\xi^2} - \frac{d\theta_o}{d\xi} = \frac{1}{4}t_y \frac{d}{d\xi}(q_{t_y}) - (\frac{1}{4V^2})t_y \frac{d^2}{d\xi^2}(q_{t_x})$$

$$-\frac{1}{t_x t_y}(q_{t_y}) - \theta_{t_y}(\frac{dt_y}{d\xi}) - \theta_{t_y}(\frac{d^2 t_y}{d\xi^2}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(52)$$

The problem was finally represented globally using matrix notation as follows:

$$\begin{bmatrix} (\frac{1}{v})^2 & 0 \\ 0 & (\frac{1}{v})^2 \end{bmatrix} \begin{bmatrix} D_x{}^2\theta_o \\ D_x{}^2\theta_o \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} D_x\ \theta_o \\ D_y\ \theta_o \end{bmatrix} = \begin{bmatrix} \Gamma \\ \Lambda \end{bmatrix} \quad \ldots...(53)$$

where the D's represent derivatives with respect to time. Γ and Λ were deduced directly from the preceding equations as:

$$\Gamma = -(\frac{1}{t_x t_y}((1-\beta\xi^{\frac{1}{2}}) + \frac{1}{8}\beta t_x \xi^{-\frac{1}{2}}(1+2(\frac{1}{v})^2\xi^3))$$

$$-\frac{\theta_{t_x}}{t_x t_y}(\frac{dt_x}{d\xi}) - \frac{\theta_{t_x}}{t_x t_y}(\frac{d^2 t_x}{d\xi^2}))$$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(54)$

$$\Lambda = -(\frac{1}{t_x t_y}((1-\beta\xi^{\frac{1}{2}}) + \frac{1}{8}\beta\xi^{-\frac{1}{2}}(1+\frac{1}{2}(\frac{1}{v})^2\xi^3))$$

$$-\frac{\theta_{t_y}}{t_x t_y}(\frac{dt_y}{d\xi}) - \frac{\theta_{t_y}}{t_x t_y}(\frac{d^2 t_y}{d\xi^2}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (55)$$

We now let the state variable $x_1 = \theta_o(t)$ in (39) and (40) so that

$$\frac{d}{dt}x_1 = x_2$$

$$\frac{d}{dt}V^2 \ (x_2 + \Gamma)$$

or, using the matrix notation, the above result becomes

$$\frac{d}{d\xi}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & V^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \Phi \end{bmatrix}$$

Where $\Phi\ (=\Gamma - V^2 x_2)$

and $\Gamma = V^2 \left(\frac{t_x}{4}\frac{dq_n}{d\xi} - \frac{t_x}{4V^2}\frac{d^2 q_n}{d\xi^2} - \frac{q_n}{t_x t_y} - \theta_\alpha \frac{d^2 t_x}{d\xi^2}\right)$ in the x-dimension

$$\Gamma = V^2 \left(\frac{t_y}{4}\frac{dq_y}{d\xi} - \frac{t_y}{4V^2}\frac{d^2 q_y}{d\xi^2} - \frac{q_y}{t_x t_y} - \theta_\alpha \frac{d^2 t_y}{d\xi^2} - \theta_y \frac{d^2 t_y}{d\xi^2}\right)$$

in the y-dimension

$$\frac{d}{d\xi}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & V^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \Phi \end{bmatrix}u$$

or generally

$$\frac{d}{dt}\{x\} = A(u_o(\mu), \mu)x$$

Optimal Control of the State Model

In general, stability is a very important characteristic of the transient performance of dynamic systems. Almost every functional system is designed to be stable, and within the boundaries of parameter variations the system performance can be improved. The system represented by (52) can be investigated for asymptotic stability by studying the eigenvalues of the system when $A(u_o(\mu, \mu))$ is constant

(Mayne, 1973; Obinabo, 2008), or by studying the solution system $\phi(u)$ of (52) with the initial condition $\phi(0)=I$ where I is the unit diagonal matrix if $A(u_o(\mu,\mu))$ happens to be an arbitrary function of μ. Here, an optimal feedback control law $u(\mu)$ was obtained for the linear inhomogeneous system based on the quadratic performance index, and was expressed as a function of x given by $u(\mu)=f(x)$, which assures asymptotic stability $(x(\mu)\to 0)$ as $\mu\to\infty$. The system was represented by

$$\frac{d}{dt}\{x\}=A(\mu)x+B(\mu)u, \qquad x(x)=x_o$$

Minimizing the quadratic performance index

$$J=\int_0^\infty\left[x^r(\mu)Q(\mu)x(\mu)+u^T(\mu)R(\mu)u(\mu)\right]d\mu$$

where $Q(\mu)and\,R(\mu)$ are positive and semi-definite and positive respectively, leads to an optimal control which us a linear function of the state and is given by

$$u(\mu)=-R^{-1}(\mu)B^T(\mu)S(\mu)x(\mu)$$

where $S(\mu)$ is the solution of the matrix Riccati equation given by

$$S=-S(\mu)A(\mu)-A^T(\mu)S(\mu)+S(\mu)B(\mu)R^{-T}(\mu)S(\mu)-Q(\mu),\ S(\mu\to\infty)=0$$

$Q(\mu)\ and\ R(\mu)$ may be chosen as unit diagonal matrices for convenience. Such a choice also implies that all the control and state variables are equally weighted in the cost function. Now from (50) we derive equations for static cooling of the work piece on the cooling bed of the mill (where V = 0 and assumed unit) as follows:

$$\frac{d}{d\xi}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}u$$

The performance criteria is rewritten as

$$J(u)\begin{bmatrix}1\\0\end{bmatrix}^T x(1)+\int_0^1\left\{\begin{bmatrix}1\\0\end{bmatrix}^T x(t)+\lambda u^2(t)\right\}dt$$

or.

$$J(u)\begin{bmatrix}1\\0\end{bmatrix}^T\begin{bmatrix}x_1&(1)\\x_2&(1)\end{bmatrix}+\int_0^1\left\{\begin{bmatrix}1\\0\end{bmatrix}^T\begin{bmatrix}x_1&(t)\\x_2&(t)\end{bmatrix}+\lambda u^2(t)\right\}dt$$

which, on comparison with the general form,

$$J(u)=\alpha^T x(T)+\int_0^T\{\beta^T(t)x(t)+g(u(t),t\}dt$$

give $\alpha=\begin{bmatrix}1\\0\end{bmatrix}$ Therefore $\alpha^T=[1\quad 0]$, $\beta=\begin{bmatrix}1\\0\end{bmatrix}$ giving $\beta^T=[1\quad 0]\ and\ \ g=\lambda u^2$.

From (57)

$$A=\begin{bmatrix}0&1\\0&0\end{bmatrix}\ and\ \ B=\begin{bmatrix}0\\1\end{bmatrix}.$$

Now the costate variable is

$$\frac{d}{dt}\{p(t)\}=-A^T(t)p(t)-\beta(t), \qquad p(T)=\alpha\ \ldots\ldots(61)$$

NOW substitute for A^T (t) and B (t) so that the following may be obtained

$$\frac{d}{dt}\{p(t)\}=-\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}p_1(t)\\p_2(t)\end{bmatrix}-\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}0&0\\-1&0\end{bmatrix}\begin{bmatrix}p_1(t)\\p_2(t)\end{bmatrix}-\begin{bmatrix}1\\0\end{bmatrix}$$

from which

$$\frac{d}{dt}\{p_1(t)\}=-1\ (62)$$

and $\frac{d}{dt}\{p_2(t)\}=-p_1(t)$ (63)

From (62) $p_1(t)=-t+C$ (64)

$$p(T)=\alpha,\ p(1)=\begin{bmatrix}0\\1\end{bmatrix}, p_1(1)=1\Rightarrow p_1(t)=1\,and\,p_2(1)=1\Rightarrow p_2(t)=0$$

Now substitute these for $p_1(t)$ in (64) to yield 1+t= C, that is, 1+1=C :.C = 2 giving

$$p_1(t)=2-t$$

From (63)

$$\frac{d}{dr}\{p_2(t)\}=-p_1(t)=-(2-t)=t-2\ \therefore p_2(t)=\frac{t^2}{2}-2t+\frac{3}{2}$$

Now we define the Hamiltonian

$$H(x,p,u,t)=\beta^T(t)x+g(u,t)+p^T(Ax+Bu)$$

$$\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}\lambda u^2 + p^T(Ax+Bu)$$
$$= x_1 + \lambda u^2 + p^T(Ax+Bu)$$
$$= x_1 + \lambda u^2 + p^T\left(\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}x + \begin{bmatrix}0\\1\end{bmatrix}u\right)$$
$$= x_1 + \lambda u^2 + p_1 x_2 + p_2 u \quad (65)$$

u is unconstrained hence we find $\frac{\partial H}{\partial u} = 0$

From (65)

$$\frac{\partial H}{\partial u} = 2\lambda u + p_2$$

Equating to zero gives

$$2\lambda u = -p_2 = -\left(\frac{t^2}{2} - 2t + \frac{3}{2}\right) \therefore u = -\frac{1}{2\lambda}\left(\frac{t^2}{2} - 2t + \frac{3}{2}\right)$$

Hence the control law for the static cooling of the rolled steel on the cooling bed of the mill is

$$u^{\cdot} = -\frac{1}{2\lambda}\left(\frac{t^2}{2} - 2t + \frac{3}{2}\right) \text{ which is optimal.}$$

Conclusion

This study has established an optimal control law for static cooling conditions, and an experimental validation test which enabled a two-dimensional heat flow model to be obtained as a function of the rolling speed for rectangular cross-sectional bars rolled from plain carbon steel. The model, which was based on the Hills' generalized integral profile method is of the form $\frac{d^2\theta}{d\zeta^2} - \frac{d\theta}{d\zeta} = f(q_{t,y}\varepsilon_{x,y}, v)$, and applies to both interstand cooling and cooling of the final products on the cooling bed of the mill. The terms $q_{t_{x,y}}$, $\dot{\varepsilon}_{x,y}$ and V characterize the surface heat flux, rate of change of the dimensions of the workpiece during cooling and the rolling speed respectively. The validity of the model was confirmed in Obinabo (1991) by comparing the profiles of the heat flow determined by experiment for static models with the theoretical results. The study shows that a good functional correspondence exists between the model and the data reported in the literature.

REFERENCES

Bryant, G. F., and M. O. Heselton, 1982. Roll-Gap Temperature Models for Hot mills, *Metals Technol.*, Vol. 9, 469 – 477.

Chao, B. T. and Chen, J. L. S., 1970, Series Solution of Unsteady Heat or Mass Transfer to a Translating Fluid Sphere, Int. J. Heat Mass Transfer, Vol. 13, p. 359 - 367.

Chapman, A. J., 1974, Heat Transfer", 3rd Ed., McMillan Co., New York, p. 165

Cook, W. J. 1970, Determination of Heat–Transfer Rates form Transient Surface Temperature measurements, *AIAA Journal*, Vol. 8, No. 7, p. 1366 - 1368.

Cooper, L. Y., 1969, Constant Surface Heating of a Variable Conductivity Half Space, Q. App. Math., Vol. 27, p.173 - 183.

Eckert, E. R. G., R. J. Goldstein, W. E. Ibele, S. V. Patankar, T. W. Simon, P. J. Strykowski, Tamma K. K., T. H. Kuehn, A. Bar-Cohen, J. V. R. Heberlein, *et al*., 1996. Heat Transfer – a Review of 1993 Literature, *Int. J. Heat Mass Transfer*, vol. 39, Issue 5, 885 – 963.

Grinberg, G. A. 1969. Solution of Diffusion-Type Problems for Expanding or Contracting Regions, *Appl. Math. Mech*., Vol.33, No. 2, p. 269 - 273.

Harding, R. A., 1976. Ph. D. Thesis, university of Sheffied.

Hills, A. W. D., 1963. An Integral Profile Method for the Analysis of Heat Flow in the Continuous Casting of Metals, Symp. Chem. Eng. Met. Ind., Instn. Chem. Engrs., p. 128 – 140.

Hills, A. W. D., 1965, Simplified Theoretical Treatment for the Transfer of Heat in Continuous-Casting Machine Moulds, *J. Iron steel Inst*., Vol. 203, pp. 18 - 29.

Karman, T. V., 1934. Some Aspects of the Turbulence Problem, 4th Int. *Cong. Appl. Mech*., p. 54 - 91.

Kim, J. G., and K. Y. Huh, 2000. Prediction of Transient Slab Temperature Distribution in the Reheating Furnace of a Walking-beam Type for Rolling of Steel Slabs, ISIJ International, Vol. 40, No. 11, 1115 – 1123.

Kwon, Y. W. and H. C. Bang, 2000. The Finite Element Method Using MATLAB, 2nd Ed. CRC Press.

Labeish, V. G., 1982. Air and Liquid cooling of steel strip During Hot Rollong, steel in the USSR Vol. 12, No. 5, 201 – 212.

Letcher Jr., J. S. 1969. On the Boundary Layers in the Theory of Heat Conduction in Solids, SIAM Review, Vol. 1, p. 20 - 29.

Mayne, D. Q. 1973. The Design of Linear Multivariable Systems. Automatica 9, 201 – 207

Obinabo, E. C. and Chijioke, M. O. 1991, Determination of the Heat Flow Distributions in Flat Bar Samples Rolled in Delta Steel Mill, Int. Congress on Iron and Steel Technology for Developing Countries, February 3 - 7, pp. 221 - 228.

Obinabo, E. C., 2007. Design of Shape Control Systems for Profiled-roll Mills, *Journal of Science, Engineering and Technology*, Vol. 14, No. 1, March, 2007, pp. 7137 – 7148.

Pedersen. L. M., 1999. Ph. D thesis, Lund Intitute of Technology.

Peletier, L. A., 1970, Asymptotic Behaviour of Temperature Profiles of a Class of Non-linear Heat Conduction Problems, Q. J. Mech. Appl. Math., Vol. 23, Part 3, p. 441 - 471.

Pepper, D. W. and J. C. Heinrich, 1992. The Finite Element Method, Basic Concepts and Applications, Hemisphere Pub. Corp.

Pohlhausen, K., 1921, Zeitschrift fur Angewandte Mathematic und Mechanic, Vol.1, p.252 - 268.

Polukhin, V. P. 1975. Mathematical Simulation and computer Analysis of Thin Strip Rollong Mills, MIR Publishers, Moscow, 202 -208.

Process Models in Iron-and Steel-making. The Metals Society, Amsterdam, 300 – 307.

Soliman, J. R. and Fakhroo, E. A., 1972. Finite Element Solution of Heat Transmission in Steel Ingots, *J. Mech. Eng. Sci*., Vol. 14, No. 1, p. 19 - 24.

Sorimachi, K. and Brimacombe, J. K., 1977, Improvements in Mathematical Modeling of Stresses in Continuous Casting of Steel, ibid, No. 4, p. 240 - 245.

Wartnann, R., and Mertes, H., 1973. Calculation of Temperature Distribution in a Hot Rolled Steel on the Run-out Table, with Emphasis on the α to γ Transition, Proc. Conf. Math.

Welty, J. R. Wilson, R. E. and Wicks, C. E., 1976, Fundamentals of Momentum Heat and Mass Transfer, John Wiley and Sons, New York, 2nd Ed.

Yang, Y. Y. and Y. Z. Lu, 1986. Development of a Computer Control Model for Slab Reheating Furnaces, Computers in Industry, Vol. 7, No. 2, 145 – 154.

Yu J. J. and W. K. Sang, 2007. An Estimation of a Billet Temperature During Reheating Furnace Operation, *Int J. Control, Automation and Sys*., Vol. 5, No. 1, 43 – 50,.

Zorzi, M. and Mazzantini, P., 1982, Continuous Casting Solidification Model by Finite Element Method, J. Num. Meth. Ind. Forming Processes, Pineridge Press, Swansea, p. 401- 411.

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