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A MULTI-SERVER MARKOVIAN QUEUEING SYSTEM WITH DISCOURAGED ARRIVALS AND RETENTION OF RENEGED CUSTOMERS WITH CONTROLLABLE ARRIVAL RATES

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ABSTRACT

In this paper, a multi-server, finite Capacity, interdependent queueing model, controllable arrival rates with discouraged arrivals and retention of reneged customers is considered. The steady state probabilities of system size are derived explicitly. The effect of the probability of customer's retention on the expected system size has been studied. The analytical results are numerically illustrated and relevant conclusions are presented.

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INTRODUCTION

Queues with discouraged arrivals have applications in computer with batch job processing where job submissions are discouraged when the system is used frequently and arrivals are modeled as a Poisson process with state dependent arrival rate. The discouragement affects the arrivals rate of the queueing system. Morse (1968) considers discouragement in which the arrival rate falls according to a negative exponential Law. We consider c servers and the customer arrive in to a multi-server queueing system in a Poisson fashion with rates λ_0- faster arrival rate, λ_1- slower arrival rate and a customer finding every server busy arrive with arrival rate that depends on the number of customers present in the system at that time. That is, if there are $n(n > c)$ customers in the system, the new customer enters the system with faster arrival rate $\frac{\lambda_0}{(n-c)+1}$ and with the slower arrival rate $\frac{\lambda_1}{(n-c)+1}$. It is also assumed that whenever the queue size reaches a prescribed number R, the arrival rate reduces from λ_0 to λ_1 and it continues with the rate as long as the content in the queue was greater than some prescribed integer $r[(r \geq 0)$ and $r < R]$. when the content reached R, the arrival rate changed back to λ_0 and the same process is repeated. Queueing with impatience finds its origin during the early 1950's Haight (1959) studied a single server Markovian queuing system with reneging. Srinivasa Rao *et al*. (2000) have discussed $M/M/1/\infty$ interdependent queuing model with controllable arrival rates. Srinivasan and Thiagarajan (2006, 2007) have analysed $M/M/1/K$ interdependent Queuing model with controllable arrival rates and $M/M/C/K/N$ interdependent queuing model with controllable arrival rates balking, reneging and spares.

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Choudhury and Medhi (2010) have studied customer impatience in multi server queues. Kapodistria (2011) has studied a single server Markovian queue with impatient customers and considered the situations where customers abandon the system simultaneously. Kumar and Sharma (Kumar and Sharma, 2012) have studied $M/M/1/N$ queueing system with retention of reneged customers. Abou- El- Ata and Hariri (Abou-EL-Ata and Hariri, 1992) have analyzed some multi server queueing systems with balking and reneging. An attempt is made in this paper to obtain relevant results for the $M/M/C/K$ interdependent queueing models with discouraged arrivals, retention of reneged customers and controllable arrival rates. In section 2, the description of the model is given. In section 3, the steady state equations are obtained. In section 4, the characteristics of the model are derived. In section 5, numerical results are illustrated.

Description of the Model

Consider a Multi server finite capacity Markovian Queueing system with discouraged arrivals and Retention of reneged customers arrive according to the Poisson flow of rate λ_0 and λ_1. The mean service rate is given by $\mu_n = \{ n\mu; 0 \le n \le c-1$ and $c\mu; c \le n \le K \}$. A queue gets developed when the number of customers exceeds the number of servers, that is, when $n > c$. Each customers upon joining the queue will wait a certain length of time for his service to begin. If it has not begun by then, he will get impatient(reneged) and may leave the queue without getting service with probability p and may remain in the queue for his service with probability $q(=1-p)$ .The impatient times follow exponential distribution with parameter ξ .

It is assumed that the arrival process $[X_1(t)]$ and the service process $[X_2(t)]$ of the systems are correlated and follows a Bivariate Poisson process given by

$$P(X_1 = x_1, X_2 = x_2; t) = e^{-(\lambda_i+\mu-\in)t} \sum_{j=0}^{\min(x_1 x_2)} \frac{(\in t)^d \left[[\lambda_i - \in]t\right]^{x_1-j} \left[(\mu-\in)t\right]^{x_2-j}}{j!(x_1-j)!(x_2-j)!}$$

where $x_1, x_2 = 0,1,2,3,...;\quad \lambda_i, \mu > 0;\quad 0 < \in < \min(\lambda_i, \mu)\quad i = 0,1$

with parameters $\lambda_0, \lambda_1, \mu$ and $\in$ as mean faster rate of arrivals, mean slower rate of arrivals, mean service rate and mean dependence rate (covariance between the arrival and service processes) respectively.

Steady state equations

Let $P_n(0)$ denote the steady state probability that there are n customers in the system, when the system is in the faster rate of arrival. Let $P_n(1)$ denote the steady state probability that there are n customers in the system, when the system is in the slower rate of arrival. We observe that only $P_n(0)$ exists, when $n = 0,1,2,...c-1, c,...r-1, r$, both $P_n(0)$ and $P_n(1)$ exist when $n = r+1, r+2,...R-2, R-1$; $P_n(0) = P_n(1) = 0$ if $n > K$

The steady state equations are

$$-(\lambda_0 - \in)P_0(0) + (\mu - \in)P_1(0) = 0 \tag{3.1}$$

$$-[(\lambda_0 - \in) + n(\mu - \in)]P_n(0) + (\lambda_0 - \in)P_{n-1}(0) + (n+1)(\mu - \in)P_{n+1}(0) = 0 \quad (1 \le n \le c-1) \tag{3.2}$$

$$-\left[\left[\frac{(\lambda_0 - \in)}{(n-c+2)}\right] + c(\mu - \in) + (n-c)\xi p\right]P_n(0) + \left(\frac{(\lambda_0 - \in)}{(n-c+1)}\right)P_{n-1}(0) + [c(\mu - \in) + (n+1-c)\xi p]P_{n+1}(0) = 0 \tag{3.3}$$

$$-\left[\left[\frac{(\lambda_0 - \in)}{(r-c+2)}\right] + c(\mu - \in) + (r-c)\xi p\right]P_r(0) + \left(\frac{(\lambda_0 - \in)}{(r-c+1)}\right)P_{r-1}(0) + [c(\mu - \in) + (r+1-c)\xi p] P_{r+1}(0) + [c(\mu - \in) + (r+1-c)\xi p]P_{r+1}(0) = 0 \tag{3.4}$$

$$-\left[\left[\frac{(\lambda_0-\in)}{(n-c+2)}\right]+c(\mu-\in)+(n-c)\xi p\right]\mathrm{P}_n(0)+\left(\frac{(\lambda_0-\in)}{(n-c+1)}\right)\mathrm{P}_{n-1}(0)$$
$$+\left[c(\mu-\in)+(n+1-c)\xi p\right]\mathrm{P}_{n+1}(0)=0,\quad (r+1\le n\le R-2) \qquad (3.5)$$

$$-\left[\left[\frac{(\lambda_0-\in)}{(R-c+1)}\right]+c(\mu-\in)+(R-1-c)\xi p\right]\mathrm{P}_{R-1}(0)+\frac{(\lambda_0-\in)}{(R-c)}\mathrm{P}_{R-2}(0)=0 \qquad (3.6)$$

$$-\left[\left[\frac{(\lambda_1-\in)}{(n-c+3)}\right]+c(\mu-\in)+(r+1-c)\xi p\right]\mathrm{P}_{r+1}(1)+\left[c(\mu-\in)+(r+2-c)\xi p\right]\mathrm{P}_{r+2}(1)=0 \qquad (3.7)$$

$$-\left[\left[\frac{(\lambda_1-\in)}{(n-c+2)}\right]+c(\mu-\in)+(n-c)\xi p\right]\mathrm{P}_n(1)+\left(\frac{(\lambda_1-\in)}{(n-c+1)}\right)\mathrm{P}_{n-1}(1)$$
$$+\left[c(\mu-\in)+(n+1-c)\xi p\right]\mathrm{P}_{n+1}(1)=0,\qquad (r+2\le n\le R-1) \qquad (3.8)$$

$$-\left[\left[\frac{(\lambda_1-\in)}{(R-c+2)}\right]+c(\mu-\in)+(R-c)\xi p\right]\mathrm{P}_R(1)+\left(\frac{(\lambda_1-\in)}{(R-c+1)}\right)\mathrm{P}_{R-1}(1)+$$
$$+\left[\frac{(\lambda_0-\in)}{(R-c+1)}\right]\mathrm{P}_{R-1}(0)+\left[c(\mu-\in)+(R+1-c)\xi p\right]\mathrm{P}_{R+1}(1)=0$$
(3.9)

$$-\left[\left[\frac{(\lambda_1-\in)}{(n-c+2)}\right]+c(\mu-\in)+(n-c)\xi p\right]\mathrm{P}_n(1)+\left(\frac{(\lambda_1-\in)}{(n-c+1)}\right)\mathrm{P}_{n-1}(1)$$
$$+\left[c(\mu-\in)+(n+1-c)\xi p\right]\mathrm{P}_{n+1}(1)=0,\quad (R+1\le n\le K-1) \qquad (3.10)$$

$$\left[\frac{(\lambda_1-\in)}{(\mathrm{K}-c+1)}\right]\mathrm{P}_{\mathrm{K}-1}(1)+\left[c(\mu-\in)+(\mathrm{K}-c)\xi p\right]\mathrm{P}_{\mathrm{K}}(1)=0 \qquad (3.11)$$

From (3.1) and (3.2), we get

$$\mathrm{P}_n(0)=\frac{(\lambda_0-\in)^n}{n!(\mu-\in)^n}\mathrm{P}_0(0) \qquad n=0,1,2,\ldots c-1 \qquad (3.12)$$

From (3.3), we get

$$\mathrm{P}_n(0)=\frac{1}{(n-c+1)!}\prod_{l=c+1}^{n}\frac{(\lambda_0-\in)}{\left[c(\mu-\in+(l-c)\xi p\right]}\frac{(\lambda_0-\in)^c}{c!(\mu-\in)^c}\mathrm{P}_0(0) \qquad n=c,c+1,\ldots..r \qquad (3.13)$$

From (3.4) we get

$$\mathrm{P}_{r+1}(0)=\frac{1}{(r-c+2)!}\frac{(\lambda_0-\in)^c}{c!(\mu-\in)^c}\prod_{l=c+1}^{r+1}\frac{(\lambda_0-\in)}{\left[c(\mu-\in+(l-c)\xi p\right]}\mathrm{P}_0(0)-\mathrm{P}_{r+1}(1)$$

Using the above result and (3.13) in (3.5), we get

$$P_n(0)=\frac{1}{(n-c+1)}\prod_{l=c+1}^{n}\frac{(\lambda_0-\in)}{[c(\mu-\varepsilon)+(l-c)\xi p]}\frac{(\lambda_0-\varepsilon)^c}{c(\mu-\varepsilon)^c}P_o(0) \tag{3.14}$$

$$-\frac{P_{r+1}(1)}{\prod_{l=r+2}^{n}[c(\mu-\varepsilon)+(l-c)\xi p}\left[\begin{array}{l}\frac{(\lambda_0-\varepsilon)^{n-r-1}}{(n+1-c)P_{n-r-1}}+\frac{(\lambda_0-\varepsilon)^{n-r-2}}{(n+1-c)P_{n-r-2}}[c(\mu-\varepsilon)+(r+1-c)\xi p]\\+\ldots+[c(\mu-\varepsilon)+(r+1-c)\xi p]+\ldots+[c(\mu-\varepsilon)+(n-1-c)\xi p]\end{array}\right]$$

From (3.4), (3.5) and (3.14) we get

$$P_{r+1}(1)=\frac{\frac{(\lambda_0-\in)^R}{(R-c+1)!}\frac{(\lambda_0-\in)^c}{c!(\mu-\in)^c}\prod_{l=c+1}^{r+1}\left[\left[\frac{1}{(c(\mu-\in)+(l-c)\xi p}\right]P_0(0)\right]}{\begin{array}{l}\frac{(\lambda_0-\in)^{R-r-1}}{(R-c+1)P_{R-r-1}}+\frac{(\lambda_0-\in)^{R-r-2}}{(R-c+1)P_{R-r-2}}\left[c(\mu-\in)+\left(r+1-c\right)\xi p\right]\\+\ldots..+\left[c(\mu-\in)+(r+1-c)\xi p\right]\ldots\left[c(\mu-\in)+(R-1-c)\xi p\right]\end{array}} \tag{3.15}$$

From (3.7) and (3.8), we recursively derive

$$P_n(1)=\frac{P_{r+1}(1)}{\prod_{l=r+2}^{n}\left[c(\mu-\in)+(l-c)\xi p\right]}\left[\begin{array}{l}\frac{\left[\lambda_{1-}\in\right]^{n-r-1}}{(n+1-c)P_{n-r}}+\frac{\left[\lambda_1-\in\right]^{n-r-2}}{(n+1-c)P_n}\left[c(\mu-\in)+r+1-c)\xi p\right]\\+\ldots+\left[c(\mu-\in)+(r+1-c)\xi p\right]\ldots\left[c(\mu-\in)+(n-1-c)\xi p\right]\end{array}\right] \tag{3.16}$$

where $P_{r+1}(1)$ is given by (3.15)

From (3.9), we get

$$P_{R+1}(1)=\frac{P_{r+1}(1)}{\prod_{l=r+2}^{R+1}\left[c(\mu-\in)+(l-c)\xi p\right]}\left[\begin{array}{l}\frac{\left[\lambda_1-\in\right]^{R-r}}{(R+2-c)P_{R-r}}+\frac{\left[\lambda_1-\in\right]^{R-r-1}}{(R+2-c)P_{R-r-1}}\left[c(\mu-\in)+(r+1-c)\xi p\right]\\+\ldots.+\frac{(\lambda-\in)}{(R+2-c)P_1}\left[c(\mu-\in)+(r+1-c)\xi p\right]\ldots.\left[c(\mu-\in)+(R-1-c)\xi p\right]\end{array}\right]$$

Using the above Result in (3.10) and (3.11), we recursively derive

$$P_n(1)=\frac{P_{r+1}(1)}{\prod_{l=r+2}^{K}\left[c(\mu-\in)+(l-c)\xi p\right]}\left[\begin{array}{l}\frac{\left[\lambda_{1-}\in\right]^{K-r-1}}{(K-c+1)P_{K-r}}+\frac{\left[\lambda_1-\in\right]^{K-r-2}}{(K-c+1)P_{K-r-2}}\left[c(\mu-\in)+\left(r+1-c\right)\xi p\right]\\+\ldots+\frac{(\lambda_1-\in)^{K-R}}{(K-c+1)P_{K-r}}\left[c(\mu-\in)+(r+1-c)\xi p\right]\ldots\left[c(\mu-\in)+(R-1-c)\xi p\right]\end{array}\right]$$

where $p_{r+1}(1)$ is given by (3.15)

All the steady state probabilities are expressed in terms of $P_0(0)$ from (3.12) to (3.17).

Characteristics of the model

The following system characteristics are considered and their analytical results are derived in this section.

- The probability P(0) that the system is in faster rate of arrivals with retention of reneged customers
- The probability P(1) that the system is in slower rate of arrivals with retention of reneged customers.
- The probability $P_0(0)$ that the system is empty.
- The expected number of customer in the system L_{s0} when the system is in the faster rate of arrivals with retention of reneged customers
- The expected number of customer in the system L_{s1} when the system is in the slower rate of arrivals with retention of reneged customers
- The expected waiting time of the customer in the system W_s

The Probability that the system is in faster rate of arrivals is

$$P(0) = \sum_{n=0}^{c-1} P_n(0) + \sum_{n=c}^{r} P_n(0) + \sum_{n=r+1}^{R-1} P_n(0) \quad (4.1)$$

From (3.12), (3.13),(3.14), (3.15) and (3.17), we get

$$\begin{aligned} P(0) = P_0(0) + \sum_{n=1}^{c} \frac{1}{n!} \frac{(\lambda_0 - \in)^n}{(\mu - \in)^n} P_0(0) + \sum_{n=c+1}^{R-1} \frac{1}{(n-c+1)!} \frac{(\lambda_0 - \in)^c}{c!(\mu - \in)^c} \prod_{l=c+1}^{R-1} \frac{(\lambda_0 - \in) P_0(0)}{[c(\mu - \in) + (l-c)\xi p]} \\ - \sum_{n=r+1}^{R-1} \frac{[A]}{B} \frac{(\lambda_0 - \in)^R}{(R-c+1)!} \frac{(\lambda_0 - \in)^c}{c!(\mu - \in)^c} \prod_{l=c+1}^{R-1} \frac{1}{[c(\mu - \in) + (l-c)\xi p]} P_0(0) \end{aligned} \quad (4.2)$$

where

$$A = \frac{(\lambda_0 - \in)^{n-r-1}}{(n+1-c)P_{n-r-1}} + \frac{(\lambda_0 - \in)^{n-r-2}}{(n+1-c)P_{n-r-2}} [c(\mu - \in) + (r+1-c)\xi p] + ... + [c(\mu - \in) + (r+1-c)\xi p]...[c(\mu - \in) + (n-1-c)\xi p]$$

$$B = \frac{(\lambda_0 - \in)^{R-r-1}}{(R-c+1)P_{R-r-1}} + \frac{(\lambda_0 - \in)^{R-r-2}}{(R-c+1)P_{R-r-2}} [c(\mu - \in) + (r+1-c)\xi p] + ... + [c(\mu - \in) + (r+1-c)\xi p].....[c(\mu - \in) + (R+1-c)\xi p]$$

The probability that the system is in slower rate of arrivals is

$$P(1) = \sum_{n=r+1}^{R} P_n(1) + \sum_{n=R+1}^{K} P_n(1) \quad (4.3)$$

From (3.16) and (3.17) we get

$$\begin{aligned} P(1) = \frac{\left[\sum_{n=r+1}^{R} C\right]}{\mathbf{B}} \frac{(\lambda_0 - \in)^R}{(R-c+1)!} \frac{(\lambda_0 - \in)^c}{c!(\mu - \in)^c} \prod_{l=c+1}^{n=K-1} \frac{1}{[c(\mu - \in) + (l-c)\xi p]} P_0(0) + \\ \frac{\left[\sum_{n=R+1}^{K} D\right]}{\mathbf{B}} \frac{(\lambda_0 - \in)^R}{(R-c+1)!} \frac{(\lambda_0 - \in)^c}{c!(\mu - \in)^c} \prod_{l=c+1}^{n=K} \left[\frac{1}{[c(\mu - \in) + (l-c)\xi p]}\right] P_0(0) \end{aligned} \quad (4.4)$$

where

$$C=\frac{(\lambda_1-\in)^{n-r-1}}{(n+1-c)P_{n-r-1}}+\frac{(\lambda_1-\in)^{n-r-2}}{(n+1-c)P_{n-r-2}}\left[c(\mu-\in)+(r+1-c)\xi p\right]$$
$$+\ldots+\left[c(\mu-\in)+(r+1-c)\xi p\right]\ldots\left[c(\mu-\in)+(n-1-c)\xi p\right]$$

$$D=\frac{(\lambda_1-\in)^{K-r-1}}{(K-c+1)P_{K-r-1}}+\frac{(\lambda_1-\in)^{K-r-2}}{(K-c+1)P_{K-r-2}}\left[c(\mu-\in)+(r+1-c)\xi p\right]+$$
$$+\ldots+\frac{(\lambda_1-\in)^{K-R}}{(K-c+1)P_{K-R}}\left[c(\mu-\in)+(r+1-c)\xi p\right]\ldots\ldots\left[c(\mu-\in)+(R-1-c)\xi p\right]$$

and B is given by (4.2)

The probability ($P_0(0)$) that the system is empty can be calculated from the normalizing condition

$$\mathrm{P}(0)+\mathrm{P}(1)=1 \tag{4.5}$$

$$P_0(0)=\frac{1}{\begin{array}{l}1+\sum_{n=1}^{c}\frac{1}{n!}\frac{(\lambda_0-\in)^n}{(\mu-\in)^n}+\sum_{n=c+1}^{R-1}\frac{1}{(n-c+1)!}\frac{(\lambda_0-\in)^c}{c!(\mu-\in)^c}\prod_{l=c+1}^{R-1}\frac{(\lambda_0-c)}{\left[c(\mu-\in)+(l+c)\xi p\right]}-\frac{\left[\sum_{n=r+1}^{R-1}A\right]}{\mathbf{B}}\frac{(\lambda_0-\in)R}{(R-c+1)!}\\ \frac{(\lambda_0-\in)^c}{c!(\mu-\in)^c}\prod_{l=c+1}^{R-1}\frac{1}{\left[c(\mu-\in)+(l-c)\xi p\right]}+\sum_{n=r+1}^{R}\frac{C}{B}\frac{(\lambda_0-\in)^R}{(R-c+1)!}\frac{(\lambda_0-\in)^c}{c!(\mu-\in)^c}\\ \prod_{l=c+1}^{K-1}\frac{1}{\left[c(\mu-\in)+(l-c)\xi p\right]}+\frac{\sum_{n=R+1}^{K}D}{B}\frac{(\lambda_0-\in)^R}{(R-c+1)!}\frac{(\lambda_0-\in)^c}{c!(\mu-\in)^c}\prod_{l=c+1}^{K}\frac{1}{\left[c(\mu-\in)+(l-c)\xi p\right]}\end{array}}$$

where A, B, C, D is given by (4.2) and (4.4)

The average number of customers in the system is given by

$$L_s=L_{s_0}+L_{s_1} \tag{4.6}$$

where

$$L_{s_0}=\sum_{n=0}^{c-1}n\mathrm{P}_n(0)+\sum_{n=c}^{r}n\mathrm{P}_n(0)+\sum_{n=r+1}^{R-1}n\mathrm{P}_n(0) \tag{4.7}$$

$$L_{s_1}=\sum_{n=r+1}^{R-1}n\mathrm{P}_n(1)+\sum_{n==R}^{K}n\mathrm{P}_n(1) \tag{4.8}$$

From (4.2) and (4.4) we get

$$L_{s_0} = \sum_{n=0}^{c-1} \frac{n}{n!}\frac{(\lambda_0 - \in)^n}{(\mu - \in)^n} + \sum_{n=c+1}^{R-1} \frac{n}{(n-c+1)!}\frac{(\lambda_0 - \in)^c}{c!(\mu - \in)^c}\prod_{l=c+1}^{R-1}\frac{(\lambda_0 - c)}{[c(\mu - \in)+(l-c)\xi p]}$$

$$-\frac{\left(\sum_{n=r+1}^{R-1} A\right)}{B} n \frac{(\lambda_0 - \in)^R}{(R-c+1)!}\frac{(\lambda_0 - \in)^c}{c!(\mu - \in)^c}\frac{1}{[c(\mu - \in)+(l-c)\xi p]}$$

$$L_{s_1} = n\sum_{n=r+1}^{R} \frac{C}{B}\frac{(\lambda_0 - \in)^R}{(R-c+1)!}\frac{(\lambda_0 - \in)^c}{c!(\mu - \in)^c}\prod_{l=c+1}^{K-1}\frac{1}{[c(\mu - \in)+(l-c)\xi p]}$$

$$-\frac{n\sum_{n=R+1}^{R-1} D}{B}\frac{(\lambda_0 - \in)^R}{(R-c+1)!}\frac{(\lambda_0 - \in)^c}{c!(\mu - \in)^c}\prod_{l=c+1}^{K}\frac{1}{[c(\mu - \in)+(l-c)\xi p]}$$

Using Little's formula $W_s = \frac{L_s}{\overline{\lambda}}$ where, $\overline{\lambda} = \lambda_0 \mathrm{P}(0) + \lambda_1 \mathrm{P}(1)$

the expected waiting time of the customer in the system can be calculated.

This model includes the models studied earlier as particular cases. For example, when $c = 1$, this model reduces to $M/M/1/K$ queueing model with discouraged arrivals and retention of reneged customer with controllable arrival rates.

Numerical Illustration

Table 5.1

C	R	R	K	λ_0	λ_1	μ	$\in$	ξ	P	$P_0(0)$	$P(0)$	$P(1)$
1	4	6	10	4	3	5	1	1	1	0.5631	0.9993	0.00003
2	4	6	10	4	3	5	1	1	1	0.4910	0.9994	0.00005
1	4	6	10	4	3	5	0.5	1	1	0.5511	0.9987	0.00006
2	4	6	10	4	3	5	0.5	1	1	0.4791	0.9995	0.00009
1	4	6	10	4	3	5	0	1	1	0.5409	0.9997	0.000060
2	4	6	10	4	3	5	0	1	1	0.4698	1.000	0.00016
1	4	6	10	4	3	6	0.5	1	1	0.6046	0.998	0.00001
2	4	6	10	4	3	6	0.5	1	1	0.5429	0.9999	0.00004
1	4	6	10	4	3	8	0.5	1	1	0.6794	0.999	$5.20*10^{-7}$
2	4	6	10	4	3	8	0.5	1	1	0.6344	0.9998	$9.59*10^{-6}$
1	4	6	10	4	4	5	0.5	1	1	0.5511	0.9999	$3.72*10^{-5}$
2	4	6	10	4	4	5	0	1	1	0.4791	0.9999	0.00010
1	4	6	10	4	4	5	0	1	1	0.5409	1.000	0.00006
2	4	6	10	4	4	5	0	1	1	0.4698	1.000	0.00017
1	4	6	10	6	5	4	0.5	1	1	0.2920	0.998	0.00110
2	4	6	10	6	5	4	0.5	1	1	0.2500	1.002	0.00671
1	4	6	10	5	5	5	0	1	1	0.4655	0.9992	0.00024
2	4	6	10	5	5	5	0	1	1	0.3960	1.0006	0.00085
2	4	6	10	5	5	5	0	1	1	0.3928	1.0072	0.00697
1	4	6	10	5	5	5	0	0	1	0.3677	0.9998	0.00309
1	4	6	10	5	5	5	0	0	0	0.4978	0.9982	0.00005
2	4	6	10	4	3	6	0.5	2	1	0.5433	1.0012	0.000019
1	4	6	10	6	5	5	1	2	1	0.3256	0.9990	0.000820
2	4	6	10	6	5	5	1	2	1	0.4260	0.9997	0.0000916
1	4	6	10	6	5	5	1	3	1	0.3271	0.9994	0.000394
2	4	6	10	6	5	5	1	3	1	0.4369	0.9997	0.000026

Table 5.2

C	R	R	K	λ_0	λ_1	μ	$\in$	ξ	P	L_s	W_s
1	4	6	10	4	3	5	1	1	1	0.45765663	0.114491
2	4	6	10	4	3	5	1	1	1	0.65487546	0.163810
1	4	6	10	4	3	5	0.5	1	1	0.47784013	0.119610
2	4	6	10	4	3	5	0.5	1	1	0.67642846	0.169180
1	4	6	10	4	3	5	0	1	1	0.49701504	0.124285
2	4	6	10	4	3	5	0	1	1	0.69451554	0.173608
1	4	6	10	4	3	6	0.5	1	1	0.41035225	0.102792
2	4	6	10	4	3	6	0.5	1	1	0.57171604	0.142939
1	4	6	10	4	3	8	0.5	1	1	0.3255318	0.081464
2	4	6	10	4	3	8	0.5	1	1	0.4359169	0.109000
1	4	6	10	4	4	5	0.5	1	1	0.47784013	0.119467
2	4	6	10	4	4	5	0	1	1	0.67642846	0.169182
1	4	6	10	4	4	5	0	1	1	0.49701504	0.124248
2	4	6	10	4	4	5	0	1	1	0.69451554	0.173621
1	4	6	10	6	5	4	0.5	1	1	1.08553789	0.1811190
2	4	6	10	6	5	4	0.5	1	1	1.36149434	0.225206
1	4	6	10	5	5	5	0	1	1	0.63667779	0.127406
2	4	6	10	5	5	5	0	1	1	0.84986276	0.169726
2	4	6	10	5	5	5	0	1	1	0.92565447	0.1825442
1	4	6	10	5	5	5	0	0	0	0.54705654	0.109603
2	4	6	10	4	3	6	0.5	2	1	0.56963379	0.245207
1	4	6	10	6	5	5	1	2	1	0.98201939	0.163721
2	4	6	10	6	5	5	1	2	1	0.63369095	0.105638
1	4	6	10	6	5	5	1	3	1	0.95883693	0.159849
2	4	6	10	6	5	5	1	3	1	0.58695886	0.0978537

Conclusion

It is observed from the Table 5.1 and 5.2 that when the mean dependence rate increases and the other parameters are kept fixed L_s and W_s decrease. When the service rate increases and the other parameters are kept fixed, $P_0(0)$ and $P(0)$ increase, $P(1)$, L_s *and* W_s decrease.

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