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RESEARCH ARTICLE

MHD CONVECTIVE FLOW ALONG A VERTICAL ISOTHERMAL PLATE UNDER VARIABLE ELECTRICAL CONDUCTIVITY AND HEAT GENERATION IN POROUS MEDIUM

*,1Shyamanta Chakraborty and 2Nripen Medhi

1UGC-HRDC, Gauhati University, Guwahati -781014, Assam, India
2Department of Mathematics, Dispur College, Guwahati -781005, Assam, India

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ABSTRACT

Natural convective MHD flow of a viscous incompressible fluid along a vertical plate is discussed with the effect of variable electrical conductivity and heat generation under the action of transverse magnetic field in porous medium. It is supposed that there is an internal heat generation along the plate that decays exponentially while electrical conductivity of the fluid is a function of fluid temperature. The process of similarity transformation is used to transform the partial governing equations into ordinary. Considering fluid flow of low Prandlt Number {Pr << 1}, numerical solutions and results are obtained using Runge-Kutta method while Shooting method is used to find the missing initial conditions. The results are used to plot velocity and temperature profile near the plate, and variation of skin-friction and heat transfer at the plate for various values of physical parameters used. The results show significant effects of fluid electrical conductivity and medium porosity on the flow and heat transfer in presence of transverse magnetic field and heat generation.

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INTRODUCTION

The study of Magneto-hydrodynamic viscous flow has been to a great extent in the recent years because of its large applications in science, technology and industry. MHD thermal boundary layer flow with variable fluid properties in the presence of a transverse magnetic field has been to a great deal of attention in present days because of its scientific importance and wide ranging applications in geophysics, thermal insulation engineering, industrial fields such as chemical engineeringprocess, drying process etc., Magnetohydrodynamic (MHD) generators, Pumps, Accelerators, Flow-meters, and many others. By selecting fluids of suitable electrically conductivity and the magnetic field induction, one can control many metallurgical processes involving cooling of continuous strips etc. The effect of heat generation or absorption in MHD flows can be effectively dealt by taking into account the variation of fluid properties along with temperature field, Herwig, *et al*. (1986). The effect of internal heat generation is especially pronounced for low Prandtl number fluid e.g. liquid metal like Mercury, Bismuth, KCl solution, NaCl solution etc.

Corresponding author: Shyamanta Chakraborty,
UGC-HRDC, Gauhati University, Guwahati -781014, Assam, India.

This is because of the fact that they have smaller Prandtl number but higher thermal conductivity which provides them ability to transport heat even if small temperature difference exists. The MHD flow with suitable electrically conducting fluid under magnetic field can control the rate of cooling while achieved desired results, Chakrabarti *et al*. (1979). Some liquid metals have smaller Prandtl number, of order 0.01 to 0.1; e.g. Bismuth=0.01, Mercury =0.023 etc. They are generally used as coolants because of higher thermal conductivity. Many authors have studied problems of natural convection flow along vertical isothermal plate with such fluids of low Prandtl number. Flow of such kind of fluids at stagnation point have been discussed by Pai *et al*. (1956). Kay (1966) reported that thermal conductivity of liquids with low Prandtl number varies linearly with temperature in range of 0°F to 400°F. Arunachalam and Rajappa (1978) considered forced convection in liquid metals with variable thermal conductivity and capacity in potential flow and derived explicit closed form of analytical solution. Chen (1998) considered laminar mixed convection flow adjacent to vertical, continuously stretching sheet. Molla *et al*. (2004) studied the natural convection flow along a horizontal cylinder in the presence of heat generation. Recently, Gorla *et al*. (2013), (Alam, 2011) Chain,(1998), Hazen A. Allia (2002) and many others have studied MHD flow with heat generation problem with various geometries. Recently, Boracic *et al*. (2010), has studied natural convection MHD flow with variable electrical conductivity and heat generation along an isothermal plate. More recently, Sharma *et al*. (2010) have studied steady MHD natural convection flow with variable electrical conductivity and heat generation along an isothermal vertical plate. Motivated by the above referenced works and the numerous applications in various fields, it is our interest to investigate a steady, fully developed MHD convective heat and mass transfer problem of an incompressible fluid flow in porous medium where fluid is sucked through vertical plate and maintained at constant suction velocity under the action of heat generation and variable electrical conductivity in presence of transverse magnetic field. The effects of various flow parameters like fluid velocity, temperature, skin friction and heat transfer at the plate are analyzed graphically and thereby discussed.

Formulation of the problem

We have considered steady laminar natural convection flow of a viscous incompressible fluid along a vertical non-conducting plate in porous medium. It is considered that the plate is at constant temperature that generates an internal volumetric heat within the fluid flow while the fluid electrical conductivity varies inversely with temperature (Boracic et al., 2010). The x-axis is taken along the plate and y-axis is normal to the plate. A uniform magnetic field of intensity Bo is applied normal to the plate. It is assumed that the electrical field due to polarization of charges and Hall Effect are negligibly small. Incorporating the Boussinesqs approximation within the boundary layer, the governing equations of continuity, momentum and energy, Schlichting (1968), respectively are given as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta\,(T_w - T_\infty) - \frac{\sigma_1 B_0^2}{\rho}\,u - \frac{\upsilon}{k_1}\,u = 0 \qquad (2)$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2} + Q \qquad (3)$$

$$\sigma_1 Z \frac{\sigma}{1+\varepsilon\theta} \qquad (4)$$

The boundary conditions are $y = 0 : u = 0 , v = 0 , T = T_w$

$$y \rightarrow \infty:\ u \rightarrow 0,\ T \rightarrow T_\infty \qquad (5)$$

Method of Solution

Introducing the stream function Ψ(x, y) such that

$$u = \frac{\partial \Psi}{\partial y} \text{ and } v = -\frac{\partial \Psi}{\partial x} \qquad (6)$$

Where,

$$\Psi(x,y) = 4\upsilon f(\eta)\left(\frac{Gr}{4}\right)^{\frac{1}{4}} \text{ and } \eta = \frac{y}{x}\left(\frac{Gr}{4}\right)^{\frac{1}{4}} \qquad (7)$$

Following Crepeau and Clarksean (1997), the volumetric rate of heat generation is given as

$$Q = Sk\left(\frac{T_w - T\infty}{x^2}\right)\left(\frac{Gr}{4}\right)^{1}{}^{2}e^{-n})\} \qquad (8)$$

Since equation (1) is identically satisfied equation (6), using equations (6), (7), and (8), equations (2) and (3), along with the equation (4), the resulted coupled non-linear ordinary differential equations, given are as follows

$$f''' - 2f'^2 + 3ff'' + \theta\ J\left(\frac{M}{1+\epsilon\theta} + \frac{1}{D_a}\left(\frac{4}{G_r}\right)^{\frac{1}{2}}\right)f' = 0 \qquad (9)$$

and

$$\theta' + 3P_r\theta' f + S\,e^{-n} = 0 \qquad (10)$$

Where,

$\upsilon = \frac{\mu}{\rho}$;

$Gr = \left(\frac{g\beta(T_w - T_x)x^3}{v^2}\right)$;

$M = \frac{\sigma B_0^2 x^2}{\mu}\left(\frac{Gr}{4}\right)^{-1}{}^{2}$

$Pr = \frac{\mu C_p}{K}$;

$Q = K\left(\frac{T_w - T_x}{x^2}\right)\left(\frac{G_r}{4}\right)^{\frac{1}{2}}e^{-n}$); $\theta = \frac{T - T_\infty}{T_w - T_\infty}$; $Da = \frac{K_1}{X^2}$

The boundary conditions are reduced to

$f(0) = 0, f'(0) = 0, f'\ \infty = 0, \theta\ 0 = 1$ and $\theta\ \infty = 0$

The governing boundary layer equations (9) and (10) with boundary conditions (11) are solved using Runge-Kutta fourth order technique along with double shooting technique.

Skin –Friction Coefficient

$$\tau = \frac{\tau}{\frac{1}{2}\rho u_o^2} = 2\left(\frac{Gr}{4}\right)^{\frac{1}{4}} f''(0)$$

Where,

$(\tau)_{y=0} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=0}$), shear stress at the plate

$u_0 = \sqrt{\{g\beta x(T_w - T\alpha)\}}$, convective fluid velocity near the plate.

Rate of Heat Transfer

(6)

The rate of heat transfer in terms of the Nusselt number at the plate is given by

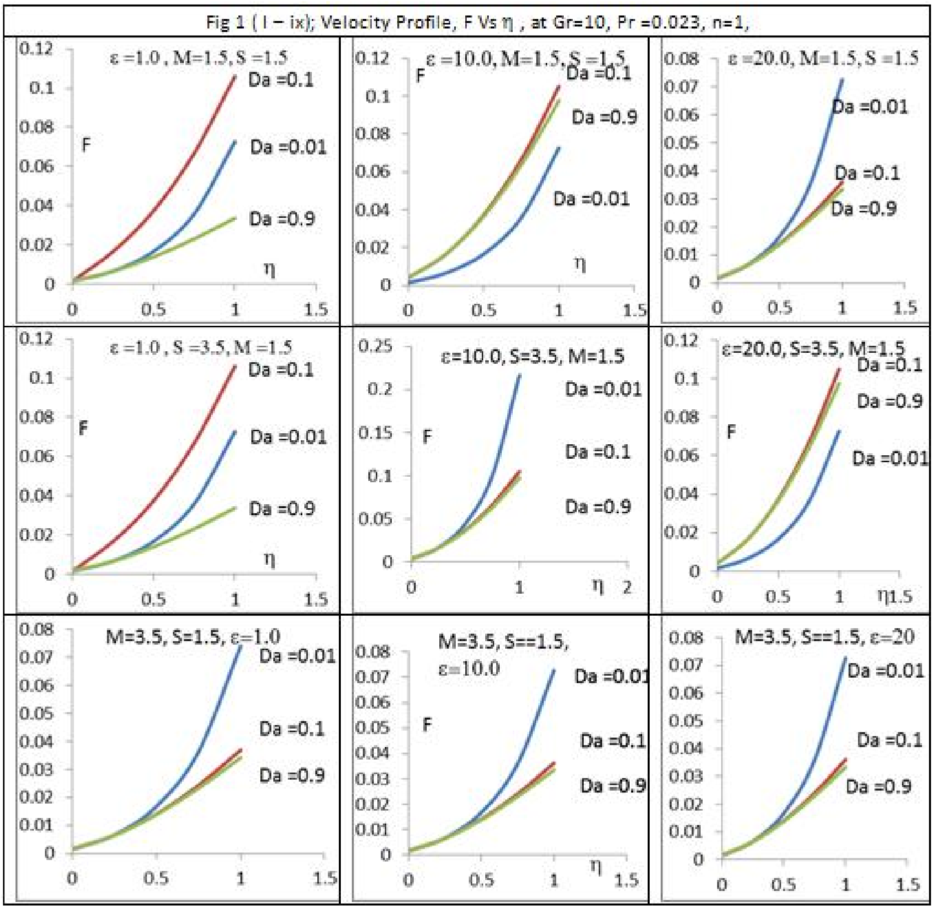

Fig. 1. (I-III), near the plate (¦>0), for constant values of £=& Da, F increases. The rate of variation of F with ¦=for different values of Da, depends upon values of £K= Within smaller value of £=E=¼=NKMFI=F increases for Da=0.01 to 0.1, while decreases for Da=0.1 to 0.9, fig1(i); for moderate value of £=E=¼==NMKMFI=F increases with the rise of Da=0.01 to 0.1, while decreases within Da=0.1 to 0.9,fig1(ii); for higher value of £=E=¼==OMKMFI=F decreases within Da=0.01 to 0.1 , also decreases slowly within Da=0.1 to 0.9,fig1(iii)K==When heat generation S is increased (1.5 to 3.5), the variation of F away from the plate, is same as above for smaller value of £E=¼=NKMF=within Da=0.01 to 0.1 and Da=0.1 to 0.9fig1(iv) ; for moderate value of £=E=¼=NMKMFI=F decreases with the rise of Da=0.01 to 0.1, while decreases slowly for Da=0.1 to 0.9,.fig1(v); for higher value of £=E=¼==OMKMFI=F increases within Da=0.01 to 0.1 , also decreases slowly within Da=0.1 to 0.9,fig1(vi)KWhen M is increased (1.5 to 3.5), for all values of £I=F decreases for Da =0.01 to 0.1 while decreases slowly for Da=0.1 to 0.9 Fig 1(vii -ix)

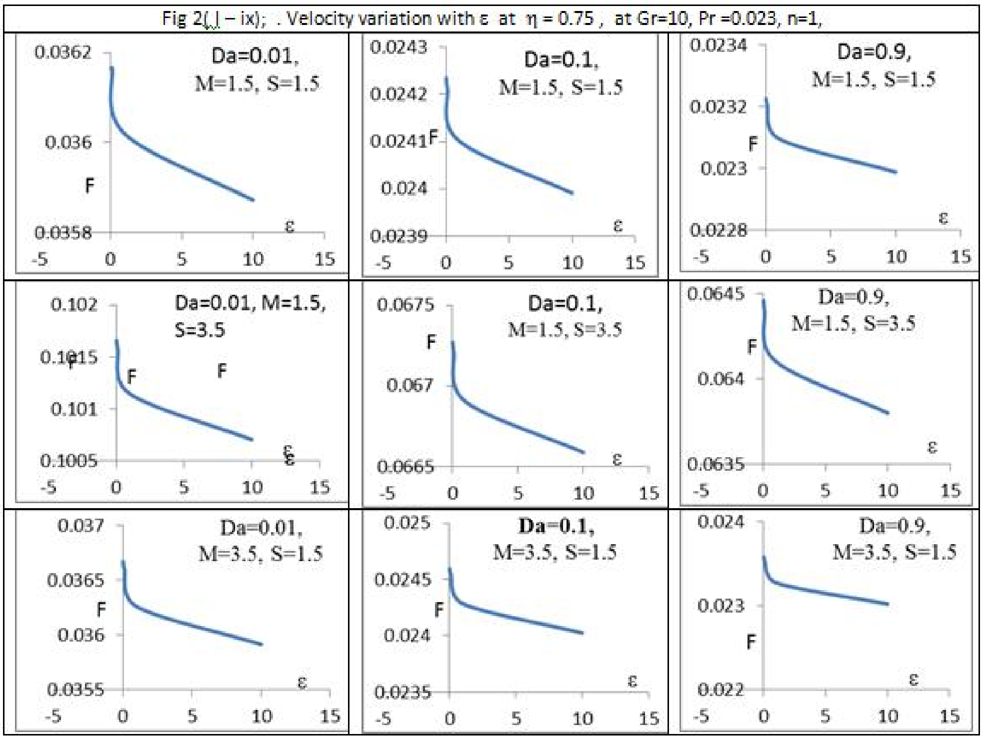

Fig. 2. (i-ix), for all values of Da, M & S, fluid velocity F decreases with the increase of £; the rate of decrease is more within smaller values of £ (á 0.1) while it is less for £>0.1. With the rise of Da, the variation of F with £I=decreases slowly fig 2(i-iii); this is increased when S increases (1.5 to 3.5), similarly for M, fig2(i & iv) & fig2(i & vii)respectively

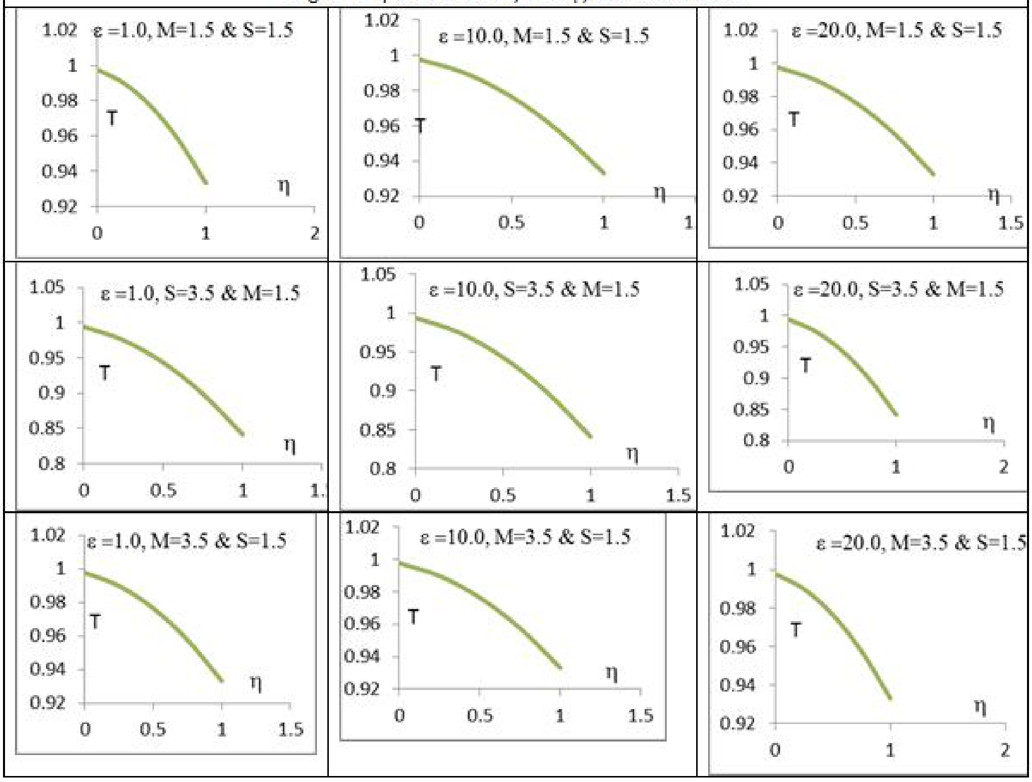

Fig 3. Temperature Profile, T Vs η , at M=1.5 & S=1.5

For all values of S, M & £l=near the plate (¦>0), T decreases slowly, fig 3(i & ix). For a value of ¦l=T decreases with the increase of Da, (e.g, at ¦=Z=MKOR=l=q=Z=MKVUVUPV=for Da =0.01 whereas, =q=Z=MKVUVTVM=for Da =0.1); similarly with the rise of £l=(e.g, at ¦=Z=MKRMl=q=Z=MKVTSQUN=for £ =1.0 whereas, =q=Z=MKVTSQUM=for £==10.0). =when S is increased (1.5 to3.5), T decreases; but when M is increased (1.5 to 3.5) T increases slowly

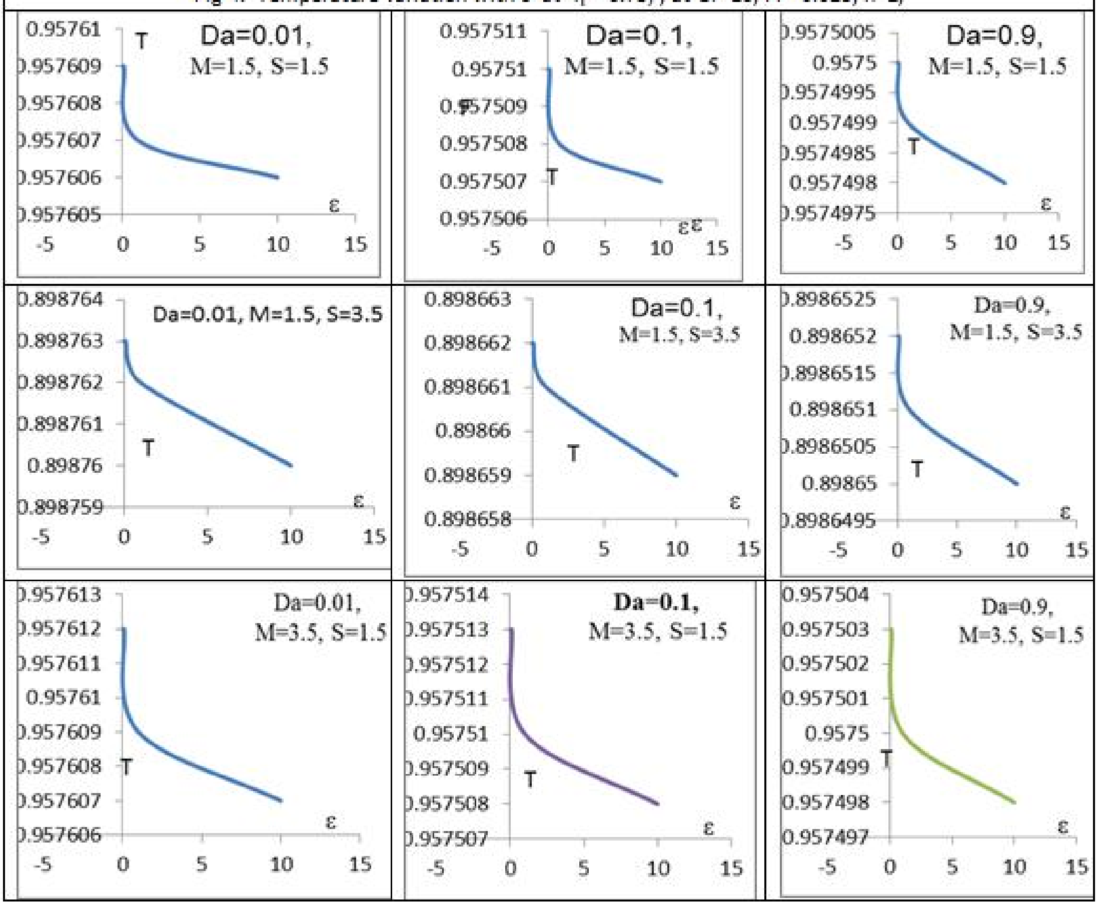

Fig 4. Temperature variation with ε at η = 0.75, , at Gr=10, Pr =0.023, n=1,

Fig. 4(i-ix), for all values of Da, M & S ; T decreases with the increase of £=; the rate of decrease of T is more within smaller values of £ (á 0.1). The variation of T with=£==slowly goes down with the rise of Da, fig 4(i-iii); this is more when S increases (1.5 to 3.5), similarly for increase of M, fig4(i & iv) & fig4(i & vii)respectively

Fig 5. Skin friction variation with ε at η = 0.00 , M=1.5 & S=1.5, , at Gr=10, Pr =0.023, n=1,

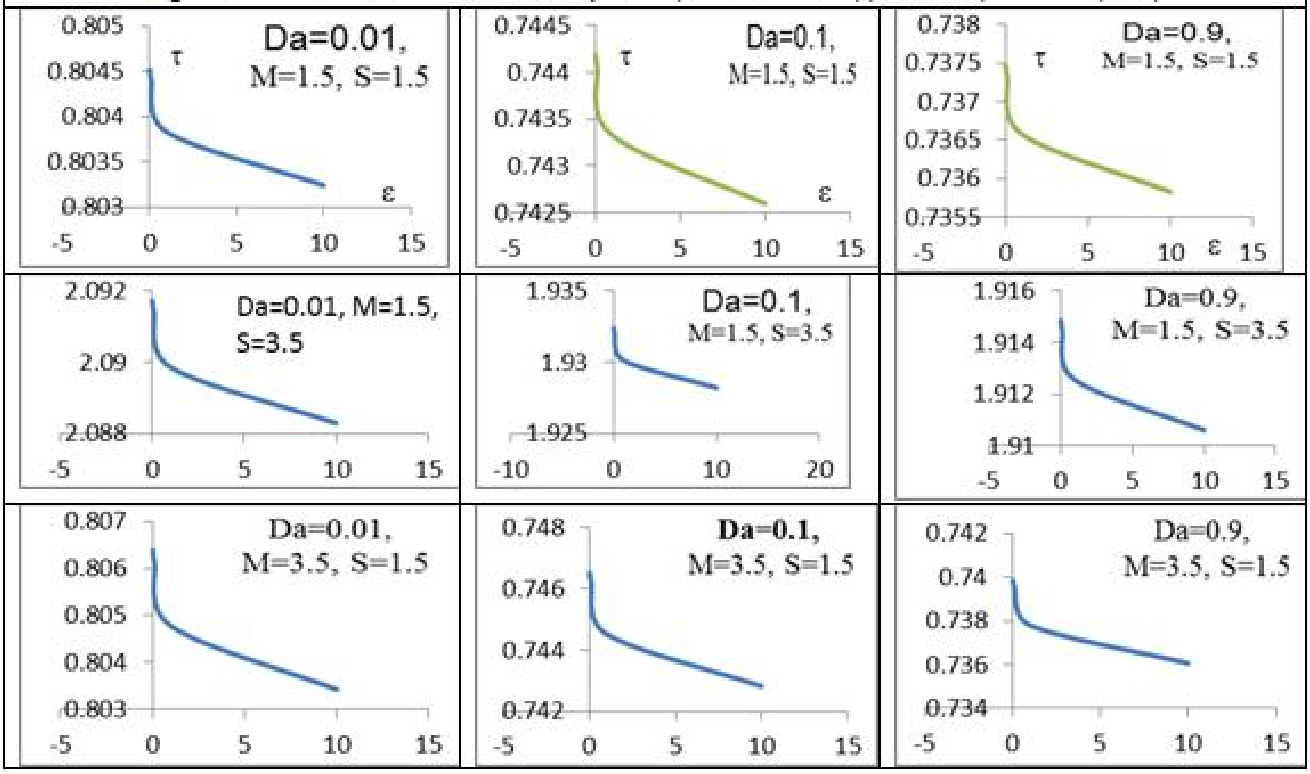

Fig. 5 (i-ix), for all values of Da, M & S, Skin friction ² decreases with the increase of £; the rate of decrease is more within smaller values of £ (á 0.1) while it is less for £>0.1. When Da is increased, the variation of ² with £decreases slowly fig 2(i-iii); this is increased when S increases (1.5 to 3.5), but as M is increased, it is decreased, fig2(i & iv) & fig2(i & vii)respectively

Fig 6. Heat Transfer variation with Nu at η = 0.00

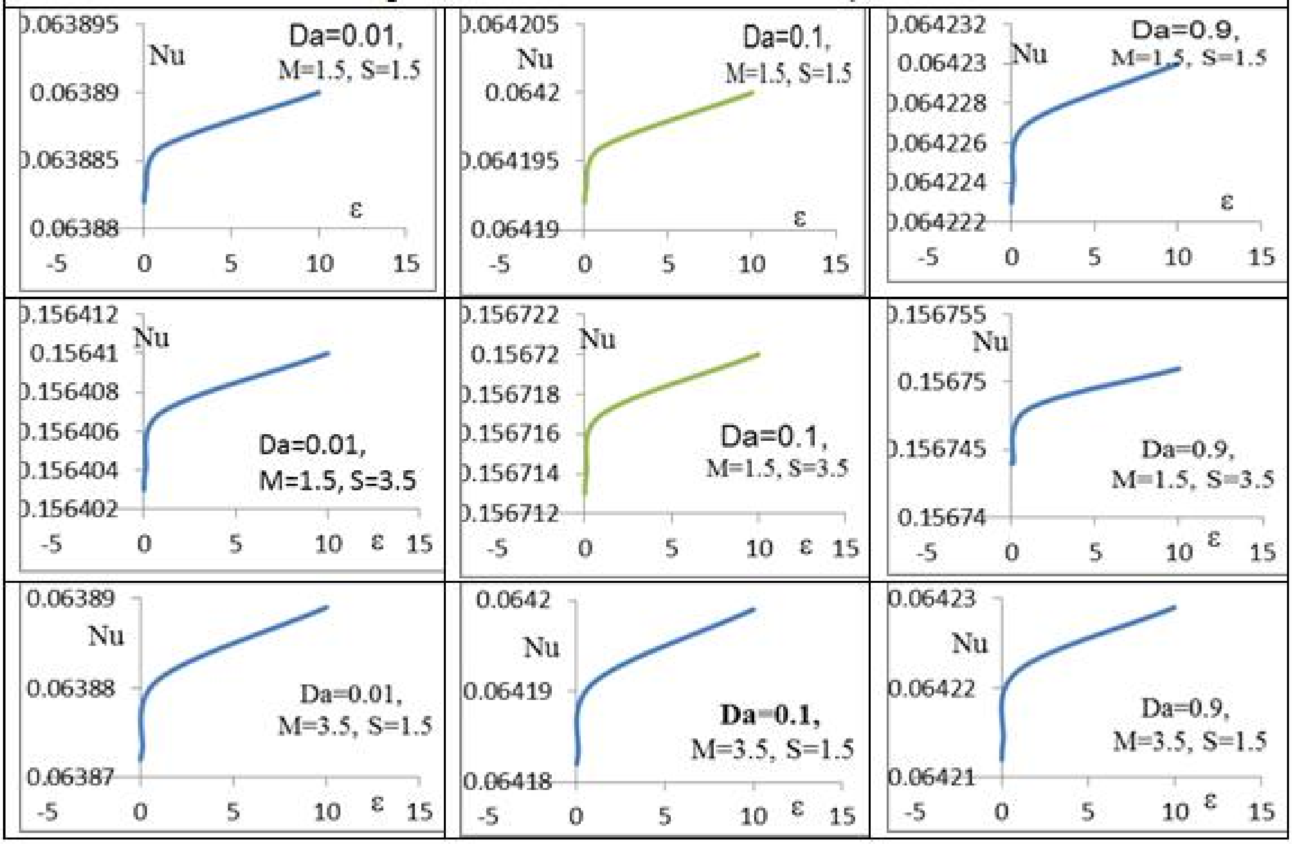

Fig. 6 (i-ix), for all values of Da, M & S, the rate of heat transfer Nu increases with the increase of £; the increased, the magnitude of Nu increases slowly fig 2(i-iii). Keeping Da constant, the magnitude of Nu increases with the rise of S; this is decreased when M is increased (1.5 to 3.5), fig5(i & iv) & fig5(i & vii)respectively

$$Nu = \frac{qx}{K(T_w - T_\infty)} = -(\frac{Gr}{4})^{\frac{1}{4}} \theta' \ 0$$

Where, $q = -K(\frac{\partial T}{\partial y})_{y=0}$

Solutions of equations

Solution for the equations (9 & 10) subject to the boundary condition (11) are obtained using Shooting iteration technique (guessing the missing values) along with fourth order Runge-Kutta method for different values of physical parameters. In calculating numerical results for physical quantities f, T, τ & Nu we have considered, Gr= 10 because it relates to the problems of cooling in nuclear reactors; Pr=0.023 since it is connected to the popular liquids metal mercury at 20^0c ; n= 1.0 (choosen arbitarily). The physical parameters whose effects on flow motion are the objectives of this study, varied as Da = 0.01 to 0.9 ; ε= 1.0 (KCl solution ε = 1.05 at 15^oc), to 20.0 (NaCl solution ε= 20.14 at 15^oc); M = 10.5 to 3.5; S = 1.5 to 3.5. We suppose that the electrical conductivity of the liquid (electrolyte) stands ε = 1.0 as smaller, ε = 10.0 as moderate and ε = 20.0 as larger. The various values of non-dimensional parameters fluid-velocity (f), fluid-temperature (T), Skin-friction at the plate (τ) and the rate of heat transfer (Nu) at the plate, as obtained from the numerical solutions are plotted for above mentioned values of Da, ε, M and S; the results are shown in the figures 1-6.

Technique for Numerical Solutions

The system of non-linear ordinary differential equations (9 - 10) together with the boundary conditions (11) are solved numerically using Nachtsheim-Swigert shooting iteration technique (guessing the missing values) along with fourth order Runge- Kutta initial value solver. Chakraborty *et al*. (2001), Hazarika *et al*. (2002), Alam *et al*. (2011) have also used same technique to solve their problems.

RESULTS AND DISCUSSION

Conclusions

- The nature of variation of fluid velocity with medium porosity depends upon fluid electrical conductivity; within smaller values of conductivity fluid velocity increases, whereas, for higher values it decreases. With the increase of fluid electrical conductivity, fluid velocity decreases. Higher the heat generation, the nature of variation fluid flow is opposite to that for moderate and higher values of it. Higher the magnetic field, fluid velocity decreases for all values of electrical conductivity and porosity of the medium.
- Fluid temperature, decreases with the increase of medium porosity; similarly for electrical conductivity. Higher the heat generation, rate of decrease is more which is unlikely when magnetic field is increased.
- Skin friction at the plate, decreases with the increase of electrical conductivity; the rate of decrease is more within smaller range of it compare to higher values. This is almost similar in nature when skin friction varies with medium porosity. At higher heat generation, the rate decrease is higher; similarly when magnetic field is higher.
- The rate of heat transfer at the plate increases with the increase of electrical conductivity; similarly for the variation with medium porosity. At higher heat generation, the rate of increase is higher; but decreases when magnetic field is higher.

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