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RESEARCH ARTICLE

MODELLING THE SINUSOIDAL CHARACTERISTICS OF DRY SEASON MOMENTUM TRANSFER IN BIODEGRADATION OF PETROLEUM HYDROCARBON IN POND SYSTEM

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ABSTRACT

The sinusoidal characteristics of petroleum hydrocarbon degradation upon the influence of momentum transfer in dry season of pond system were studied. Both frequency and time responses are used to elucidate the dynamics characteristics of sinusoidal input. The process used for illustration purposes is the continuous discharge of effluent wastewater into a batch reactor (pond) of which the transport of the contaminants within the system is dependent of velocity, frequency and time. Having noted that, the phase lag, output characteristics and process time influence the biodegradation process as well as the momentum transfer, this paper show how sinusoidal input of momentum transfer influence the biodegradation of petroleum hydrocarbon in pond system.

INTRODUCTION

The increase in petroleum hydrocarbon production and utilization, as a fuel as well as chemical industry raw materials, prompt more and more wastewater production daily, which is been discharged into the environment as a finally receiving end. The great production and use of petroleum hydrocarbons in technologically advanced societies provokes the release of many xeriobiotic substances in the aquatic environment. (Cui, and Zhang, 2004; Wu, Gui and Li, 2003; Ling, 2008; Guo and Wang, 2009 and Ukpaka, 2010). Such materials could promote toxic effects even at low concentrations. Although the chemical dispersion of petroleum hydrocarbon is conceptually simple, the influences of mixing energy and salinity are not fully understood. When describing the movement and spreading of a pollutant, the physical scientist are capable of providing analyses that are as accurate as possible. However, it is at least as important to known the significant of sinusoidal characteristics in biodegradation of petroleum hydrocarbon in a pond system upon the influence of momentum transfer, which is created as a result of continuous discharge of effluent in system (Ogoni and Ukpaka, 2004). Petroleum products that enter the marine environment have distinct effects, according to their composition, concentration and the elements in the environment that are taken into consideration. Some effects can be related to transformations of the chemical composition of the environment and alterations in its physical properties, the destruction of the nutritional capital of the marine biomass, danger to human health and changes in the environmental biological equilibrium (Lu, Sun and Jiang, 2005; Cui and Zhang, 2004 and Ukpaka 2006). The mechanism of toxic action depends, on the petroleum's characteristics. The toxicity of the various fractions of the pollutants is directly related to the distilled products, on a short-term basis, and related to the slow-action products, on a long-term basis. From the physical point of view, hydrocarbon directly influence the marine environment, since gas transfer mechanisms are disturbed by the presence of a pollutant layer on the surface (Pinho, Antunes and Vicira, 2002; Guo and Wang, 2009; Nazir, Khana, Anyutte and Sadiq, 2008). The sinusoidal characteristics are particularly efficient for monitoring biodegradation of petroleum hydrocarbon in pond system, since the nature of pollutants is easily detectable. They do not however, allow the corresponding concentrations to be determined. A system whereby samples are collected by containers provides more precise information about contaminants distribution and transport upon the influence of momentum transfer.

The Model

Sinusoidal Input

The individual petroleum hydrocarbon in the pond system was assumed to have been distributed from one point to another in the form of sinusoidal, i.e. the input vary from one point to another periodically. The general flow diagram for such behaviour is shown in Figure 1.

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These are inputs that vary periodically

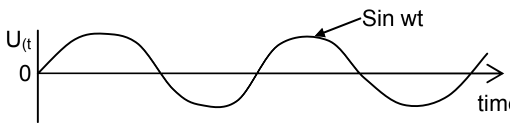

Figure 1: A simple wave formation due to continuous discharge of effluent in pond

The input discharge of wastewater into the pond and characteristics of individual hydrocarbon behaviour in the system is assumed to be in sinusoidal form. Therefore the general pollution to such form of behaviours is given as:

$U_{(t)} = A\sin wt$ for $t \geq 0$ (1)

where $w = 2\pi f$

f = frequency

Laplace of $U_{(t)}$ gives

$$U_{(s)} = \frac{Aw}{S^2 + w^2} \quad (1a)$$

Since the transfer function model equation is given as

$$y_{(s)} = g_{(s)} U_{(s)} \text{ or } g_s = \frac{y_{(s)}}{U_{(s)}} \quad (1b)$$

Therefore, assuming the behaviour of individual petroleum hydrocarbon in pond system to have a general solution of

$$y_{(s)} = \frac{KU_{(s)}}{\tau_{s+1}} \quad (1c)$$

By substituting equation (1a) into (1c) yields

$$Y_{(s)} = \frac{KAw}{(\tau s+1)\left(S^2 + w^2\right)} \quad (2)$$

Solving equation (2) by the method of partial fraction gives

$$\frac{KAw}{(\tau s+1)\left(S^2+w^2\right)} = \frac{B}{(\tau s+1)} + \frac{C}{S^2+w^2} + \frac{D}{S+w} \quad (3)$$

$$KAw = B(S^2 + w^2) + C(\tau s + 1) + D(S+w)(\tau s+1) \quad (4)$$

Let $S = -\frac{1}{\tau}$, substituting this assumption into equation (4) yields.

$$KAw = B\left(\left(-\frac{1}{\tau}\right)^2 + w^2\right) + C\left(\tau\left(-\frac{1}{\tau}\right)+1\right) + D\left(-\frac{1}{\tau}+w\right)\left(\tau\left(\left(-\frac{1}{\tau}\right)+1\right)\right)$$

$$KAw = B\left(\frac{1}{\tau^2} + w^2\right) + C(-1+1) + D\left(-\frac{1}{\tau}+w\right)(-1+1)$$

$$KAw = B\left(\frac{1}{\tau^2} + w^2\right) + C(0) + D(0)$$

$$B\left(\frac{1}{\tau^2} + w^2\right) = KAw$$

$$B = \left(\frac{1+w^2\tau^2}{\tau^2}\right) = KAw$$

$$B = \frac{KAw\tau^2}{1+w^2\tau^2}$$

Let $S = \frac{1}{\tau}$, substituting into equation (4) yields

$$KAw = B\left(\frac{1}{\tau^2} + w^2\right) + C((1+1)) + D\left((1+1)\left(\frac{1}{\tau}+w\right)\right)$$

$$KAw = B\left(\frac{1+w^2\tau^2}{\tau^2}\right) + 2C + 2D\left(\frac{1}{\tau}+w\right) \quad (5)$$

Let S = 0, substituting into equation (4) gives,

$KAw = B(0 + w^2) + C(0+1) + D(1)(w)$

$KAw = Bw^2 + C + DW$ (6)

Solving equation (5) and (6) simultaneously that is:

$$KAw = B\left(\frac{1+w^2\tau^2}{\tau^2}\right) 2C + 2D\left(\frac{1}{\tau}+w\right) \quad (7)$$

From equation (6), make C the subject of the equation

$C = KAw - Bw^2 - Dw$ (8)

Substituting equation (8) into equation (5) gives

$$KAw = B\left(\frac{1+w^2\tau^2}{\tau^2}\right) + 2KAw - 2Bw^2 - 2Dw + 2D\left(\frac{1}{\tau}+w\right) \quad (9)$$

$$-KAw = B\left(\frac{1+w^2\tau^2}{\tau^2}\right) - 2Bw^2 - 2Dw + 2D\left(\frac{1}{\tau}+w\right)$$

Substituting the value of B into equation (9) yields

-KAw=

$$\frac{KAw\tau^2}{1+w^2\tau^2}\left(\frac{1+w^2\tau^2}{\tau^2}\right) - \frac{2KAw^3\tau^2}{1+w^2\tau^2} - 2Dw + 2D\left(\frac{1}{\tau}+w\right)$$

$$-KAw = KAw - \frac{2KAw\tau^2w^2}{1+w^2\tau^2} - 2Dw + 2D\left(\frac{1}{\tau}+w\right)$$

$$2Dw - 2D\left(\frac{1}{\tau}+w\right) = 2KAw - \frac{2KAw^3\tau^2}{1+w^2\tau^2} \quad (10)$$

Dividing through equation (10) by 2, gives

$$Dw - D\left(\frac{1}{\tau}+w\right) = KAw - \frac{KAw^3\tau^2}{1+w^2\tau^2}$$

$$D\left(w - \frac{1}{\tau} - w\right) = KAw \; \frac{-KAw^3\tau^2}{1+w^2\tau^2}$$

$$\frac{-D}{\tau} = \frac{KAw\left(1+w^2\tau^2\right) - KAw^3\tau^2}{1+w^2\tau^2}$$

$$\frac{-D}{\tau} = \frac{KAw + KAw^3\tau^2 - KAw^3\tau^2}{1+w^2\tau^2} \quad (11)$$

Equation (11) reduces to

$$\frac{-D}{\tau} = \frac{KAw}{1+w^2\tau^2}$$

Therefore

$$D = -\frac{KAw\tau}{1+w^2\tau^2}$$

To determine the constant C, substitute the values of B and D into equation (6), we have

$KAw = Bw^2 + C + Dw$

where

$$B = \frac{KAw\tau^2}{1+w^2\tau^2}$$

$$D = \frac{-KAw\tau}{1+w^2\tau^2}$$

Then

$$KAw = \frac{KAw\tau^2}{1+w^2\tau^2}\left(w^2\right)+C - \frac{KAw\tau}{1+w^2\tau^2}(w)$$

$$KAw = \frac{KAw^3\tau^2}{1+w^2\tau^2} + C - \frac{KAw^2\tau}{1+w^2\tau^2}$$

$$C = KAw - \frac{KAw^3\tau^2}{1+w^2\tau^2} + C - \frac{KAw^2\tau}{1+w^2\tau^2}$$

$$C = \left\{KAw\left(1+w^2\tau^2\right) - KAw^3\tau^2 + KAw^2\tau\right\}/\left(1+w^2\tau^2\right)$$

$$C = \left\{KAw + KAw^3\tau^2 - KAw^3\tau^2 + KAw^2\tau\right\}/\left(1+w^2\tau^2\right)$$

$$C = \left\{KAw + KAw^2\right\}/\left(1+w^2\tau^2\right)$$

$$C = \frac{KAw(1+w\tau)}{1+w^2\tau^2} \qquad (12)$$

If $(1 + \omega\tau) = 1$, therefore equation (12) becomes

$$C = \frac{KAw}{1+w^2\tau^2}$$

Substituting the value of B, C and D into equation (3) gives

$$\frac{KAw}{(\tau s+1)\left(S^2+w^2\right)} = \frac{\left(KAw\tau^2/1+w^2\tau^2\right)}{\tau s+1} - \frac{\left(KAw\tau/1+w^2\tau^2\right)}{S+w} + \frac{\left(KAw/1+w^2\tau^2\right)}{S^2+w^2}$$

Since $Y_{(s)} = \frac{KAw}{(\tau s+1)\left(S^2+w^2\right)}$

Therefore

$$Y_{(s)} = \frac{\left(KAw\tau^2/1+w^2\tau^2\right)}{\tau s+1} - \frac{\left(KAw\tau/1+w^2\tau^2\right)}{S+w} + \frac{\left(KAw/1+w^2\tau^2\right)}{S^2+w^2}$$

$$Y_{(s)} = \frac{KA}{1+w^2\tau^2}\left\{\frac{w\tau^2}{\tau s+1} - \frac{w\tau}{S+w} + \frac{w}{S^2+w^2}\right\} \qquad (13)$$

$$Y_{(t)} = \frac{KA}{1+w^2\tau^2}\left\{w\tau^2\ell^{-t/\tau} - w\tau\cos wt + \sin wt\right\} \qquad (14)$$

where ϕ is the phase lag = $-\tan^{-1(wt)}$

The Momentum Model

Force balance model on a fluid element in a pond: The conservation of the momentum equation can be expressed in a linear or angular momentum form. Momentum is defined as a product of mass and its velocity. However, is derived here in the interest to determine the effect of momentum transfer on biodegradation of individual hydrocarbon in pond system, considering the volume element or control volume as shown in Figure 2.

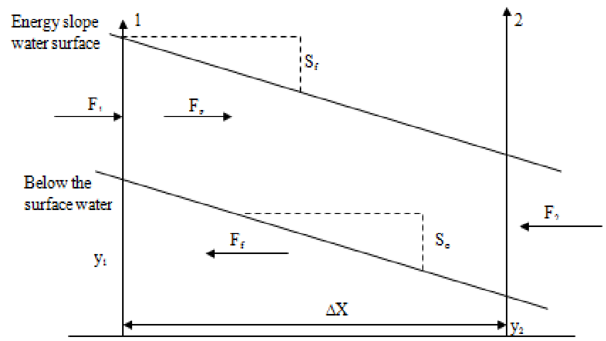

Figure 2: Force balance on a fluid element in a pond

From Figure 2 the force acting on the fluid element (petroleum hydrocarbons and other components) at length ΔX is express mathematically as:

Gravity $F_g = \rho g A, \Delta x S_o$ (15)

Friction $F_f = \rho g A \Delta x S_f$ (16)

Hydrostatic $F_1 - F_2 = ½ \rho g \frac{\partial}{\partial x}\left(\overline{y^2} A_1\right)$ (17)

The energy line represents the total head time at each section i.e.

Total head = datum + water depth + $\frac{V^2}{2g}$ (18)

where $\frac{V^2}{2g}$ is the velocity head.

The energy slope time, water surface slope and below the water surface slope are assumed not to be same in this paper. For this investigation it is assumed that the energy line has a slope of S_f and below the water surface level has a slope of S_o. Then, the conservation of momentum equation becomes

$$\begin{bmatrix}\text{The rate of change}\\ \text{of momentum for}\\ \text{the volume element}\end{bmatrix} = \begin{bmatrix}\text{The resultant of the}\\ \text{force acting on the}\\ \text{volume element}\end{bmatrix} \qquad (19)$$

Mathematical representation is given as:

$$\frac{dU}{dt} = F_g + \left(F_1 - F_2\right) - F_f \qquad (20)$$

Substituting equations (15, 16, and 17) into equation (20) yields

$$\frac{dU}{dt} = \rho g A \Delta x S_o + ½\rho\frac{\partial}{\partial x}\left(\overline{y}^2 A\right) - \rho g A \Delta x S_f \qquad (21)$$

Since

$$\frac{dU}{dt} = \rho A \Delta x\left(\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x}\right) \qquad (22)$$

Therefore substituting equation (22) into equation (21) and rearranging the equation yields

$$\rho A \Delta x\left(\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x}\right) = \rho g A \Delta x\left(S_o - S_f\right) + ½\rho g\frac{\partial}{\partial x}\left(\overline{y}^2 A\right) \qquad (23)$$

Since the investigation was conducted under the following conditions, a constant cross section and one-dimensional flow, thus equation (23) becomes;

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x} = g\left(S_o - S_f\right) \qquad (24)$$

In the dry season it is assumed that there were no friction losses during the investigation, therefore S_f = 0 and equation (24) becomes

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x} = gS_o \qquad (25)$$

Mathematical Application on Equation (25)

The momentum transfer process for the oxidation system can be described as a simple batch phenomenon under conditions where organic sedimentation, sediment reactions and loss of organic volatiles component of petroleum hydrocarbon are unimportant. Therefore equation (25) can be resolved by the application of separation of variables and using the following boundary conditions, such as at $X_1 = 0$, $t = 0$ and $U = C_n$ similarly, at $x_1 = L$, $t = t$

$$\text{Let } U = T_1X_1 \tag{26}$$

Solutions obtained from equation (26) by the mathematical application of separation of variables are:

$$\frac{\partial u}{\partial t} = T_1' X_1 \tag{27}$$

$$\frac{\partial u}{\partial X} = T_1\, X_1' \tag{28}$$

Substituting equation (27) and (28) into equation (25) and dividing through by T_1X_1. The obtained equation can be expressed by considering both sides to be constant. In practice, it is convenient to write this real constant as either λ^2 or $-\lambda^2$. The mathematical representation of the above statement is given as

$$\frac{T'}{T_1} = -V_1\frac{X_1'}{X_1} = \frac{gS_o}{TX_1} = \lambda^2 \tag{29}$$

The possible solutions obtained from equation (29) are

$$T_1' - \lambda^2 T_1 = 0 \tag{30}$$

$$-V_1X_1' + \lambda^2 X = 0 \tag{31}$$

$$gS_o - \lambda^2 T_1X_1 = 0 \tag{32}$$

The general solution to equations (30), (31) and (32) are:

$$T_1 = C_1 e^{\lambda^2 t} \tag{33}$$

$$X_1 = C_2 e^{-\frac{\lambda^2 X_1}{V_1}} \tag{34}$$

$$\lambda^2 = \frac{gS_o}{T_1X_1} \tag{35}$$

Substituting equation (33) and (34) into equation (26) yield

$$U = \left(C_1 e^{\lambda^2 t}\right)\left(C_2 e^{-\frac{\lambda^2}{V_1}X_1}\right) \tag{36}$$

Since $\lambda^2 = \dfrac{gS_o}{T_1X_1}$ therefore equation (22) can be written as:

$$U = \left(C_1 e^{\frac{gS_o}{V_1}t}\right)\left(C_2 e^{-\frac{gS_o}{V_1}X}\right) \tag{37}$$

Considering the boundary condition, at X = L and t = t and

$$V_1 = \frac{L_1}{t} \tag{38}$$

Rearranging equation (38) yields

$t = \dfrac{L_1}{V_1}$ and $L_1 = V_1t$, therefore equation (37) becomes

$$U_{LI} = \left(C_1 e^{\frac{gS_o}{V_1}L_1}\right)\left(C_2 e^{-\frac{gS_o}{V_1}L_I}\right) \tag{39}$$

Substituting the boundary conditions of X = 0, t = 0, and $U = C_n$ into equation (36) yields

$$C_n = C_1C_2$$

Therefore

$$C_1 = \frac{C_n}{C_2} \tag{40}$$

Therefore substituting equation (40) into (39) becomes

$$U_{LI} = \left(\frac{C_n}{C_2} e^{\frac{gS_o}{V_1^2}L_1}\right)\left(C_2 e^{-\frac{gS_o}{V_1}L_I}\right) \tag{41}$$

Equation (41) can be written as

$$U_{LI} = \left(C_n e^{\frac{gS_o}{V_1^2}L_1}\right)\left(C_2 e^{-\frac{gS_o}{V_1}L_I}\right) \tag{42}$$

Equation (42) can be simplified to yield

$$\text{In}\,\frac{U_{LI}}{C_n} = \frac{gS_oL_I(1-V_1)}{V_1^2} \tag{43}$$

Making $\dfrac{V_I^2}{(1-V_I)}$ the subject of the formula yields

$$\frac{V_I^2}{1-V_I} = gS_oL_1\,\frac{\text{In}\,C_n}{U_{LI}} \tag{44}$$

Kinetics Model For Multiple Substrate and Single Enzyme Without Activator

In this paper mathematical equations were developed for multiple substrate and single enzyme reaction without activator under the influence of momentum transfer. The pond system contains different mixture of petroleum hydrocarbon. Assuming "n" number of petroleum hydrocarbons as the only source of carbon, using this as substrate, the reaction steps in the scheme involving multiple intermediate can be presented as follows:

$$\begin{array}{lcl} [E] + [Q_1] \underset{K_2}{\overset{K_1}{\rightleftharpoons}} [EQ_1] & \xrightarrow{K_3} & [P_1] + [E] \\ [E] + [Q_2] \underset{K_5}{\overset{K_4}{\rightleftharpoons}} [EQ_2] & \xrightarrow{K_6} & [P_2] + [E] \\ \vdots \quad \vdots \quad\quad \vdots & & \vdots \quad \vdots \\ [E_n] + [Q_n] \underset{K_8}{\overset{K_7}{\rightleftharpoons}} [EQ_n] & \xrightarrow{K_9} & [P_n] + [E_n] \end{array} \tag{45}$$

The mathematical expression for a typical aerobic pond reactor (batch reactor) and the material balance can be expressed to yield

$$-\frac{1}{X}\frac{dQ}{dt} = \mu^Q \frac{X}{Y} \tag{46}$$

Similarly, the mathematical expression for Monod equation for this typical aerobic pond reactor is given as

$$\frac{dQ}{dt} = \left(\mu_{\max}^Q \frac{Q}{K_m + Q}\right)\frac{X}{Y} \tag{47}$$

The mathematical expression in terms of microbial substrate relationship is given as

$$Y = \frac{X - X_o}{Q_o - Q} = \frac{dX}{dQ} \tag{48}$$

Substituting equation (48) into (47) yields

$$\frac{dQ}{dt} = \left(\mu_{\max}^Q \frac{Q}{K_m + Q}\right)\frac{X}{dX/dQ} \tag{49}$$

The mathematical expression obtained in equation (49) is only for single component of the system. Therefore defining equation (49) in terms of multiple component system yields

$$\frac{dQ}{dt} = \mu_{\max}^{Q}\left(\frac{Q_1}{K_{m_1}+Q_1}\bullet\frac{Q_2}{K_{m_2}+Q_2}\bullet\frac{Q_3}{K_{m_3}+Q_3}\bullet\bullet\bullet\frac{Q_n}{K_{m_n}+Q_n}\right)\frac{X}{dX/dQ} \quad (50)$$

Rearranging equation (50) yields

$$\frac{dX}{dt} = \mu_{\max}^{Q}\left(\frac{Q_1}{K_{m_1}+Q_1}\bullet\frac{Q_2}{K_{m_2}+Q_2}\bullet\frac{Q_3}{K_{m_3}+Q_3}\bullet\bullet\bullet\frac{Q_n}{K_{m_n}+Q_n}\right)X \quad (51)$$

Similarly equation (51) can be rearranged to becomes

$$\frac{dX}{X} = \mu_{\max}^{Q}\left(\frac{Q_1}{K_{m_1}+Q_1}\bullet\frac{Q_2}{K_{m_2}+Q_2}\bullet\frac{Q_3}{K_{m_3}+Q_3}\bullet\bullet\bullet\frac{Q_n}{K_{m_n}+Q_n}\right)dt \quad (52)$$

Integrating equation (52) becomes

$$\text{In}\frac{X}{X_o} = \mu_{\max}^{Q}\left(\frac{Q_1}{K_{m_1}+Q_1}\bullet\frac{Q_2}{K_{m_2}+Q_2}\bullet\frac{Q_3}{K_{m_3}+Q_3}\bullet\bullet\bullet\frac{Q_n}{K_{m_n}+Q_n}\right)t \quad (53)$$

Therefore, equation (53) becomes

$$\mathrm{X} = X_o\ e^{\mu_{\max}^{Q}\left(\frac{Q_1}{K_{m1}+Q_1}\bullet\frac{Q_2}{K_{m2}+Q_2}\bullet\frac{Q_3}{K_{m3}+Q_3}\bullet\bullet\bullet\frac{Q_n}{K_{mn}+Q_n}\right)t} \quad (54)$$

Further simplification of equation (54) yields

$$\mu_{\max}^{Q}\ t\left(\frac{Q_1Q_2Q_3\bullet\bullet\bullet Q_n}{K_{m_1}+Q_1\bullet K_{m_2}+Q_2\bullet K_{m_3}+Q_3\bullet\bullet\bullet K_{m_n}+Q_n}\right) = \text{In}\frac{X}{X_o} \quad (55)$$

$$\mu_{\max}^{Q}\ t = \frac{\text{In}\frac{X}{X_o}\left(K_{m_1}+Q_1\bullet K_{m_2}+Q_2\bullet K_{m_3}+Q_3\bullet\bullet\bullet K_{m_n}+Q_n\right)}{Q_1Q_2Q_3\bullet\bullet\bullet Q_n} + C$$

If $X_o = 0$ and expressing equation (55) in terms of component i, yields

$$\mu_{\max}^{Q}\ t = \left[\begin{matrix} n \\ \pi \\ i=1 \end{matrix}\left(\frac{[K_m]_i+[Q]_i}{Q_i}\right)\right]\ \frac{dX}{X} \quad (56)$$

Integrating equation (42) yields

$$\mu_{\max}^{Q}\ t = \left[\begin{matrix} n \\ \pi \\ i=1 \end{matrix}\left(\frac{[K_m]_i+[Q]_i}{Q_i}\right)\right]\ \text{In}\,\mathrm{X} + \mathrm{C} \quad (57)$$

Model for Correlation of Momentum Transfer and Biokinetic Model for Multiple Substrate and Single Enzyme without Activation for Dry Season

The correlation model was developed with the influence of momentum transfer in the pond system. Recalling the mathematical expression obtained in equation (50), which states that

$$\frac{dQ}{dt} = \mu_{\max}^{Q}\left(\frac{Q_1}{K_{m_1}+Q_1}\bullet\frac{Q_2}{K_{m_2}+Q_2}\bullet\frac{Q_3}{K_{m_3}+Q_3}\bullet\bullet\bullet\frac{Q_n}{K_{m_n}+Q_n}\right)\frac{X}{dX/dQ}$$

Recalling the general equation obtained in equation (42) for mathematical application of dry season momentum transfer is given as

$$U_{LI} = \left(C_n e^{\frac{gS_o}{V_1^2}L_1}\right)\left(e^{-\frac{gS_o}{V_1}L_I}\right)$$

The mathematical expression for momentum is given as

$$\hat{M} = U_L = \text{mass x velocity} = \text{MV} \quad (58)$$

But velocity $= \frac{dQ}{dt} = \frac{\text{change in substrate concentration}}{\text{change in time}}$ (59)

Substituting equation (58) into equation (42) yield

$$\text{MV} = \left(C_n e^{\frac{gS_o}{V_1^2}L}\right)\left(e^{-\frac{gS_o}{V_1}L}\right) \quad (60)$$

Where M = mass and V $= \frac{dQ}{dt}$ = velocity or specific rate

$$\text{Since V} = \frac{dQ}{dt} = \frac{dX}{dt} \quad (61)$$

Therefore substituting equation (61) into equation (60) yields

$$M\frac{dQ}{dt} = \left(C_n e^{\frac{gS_o}{V_1^2}L_1}\right)\left(e^{-\frac{gS_o}{V_1}L}\right) \quad (62)$$

Therefore rearranging equation (62) and then substituting the obtained solution into equation (58) yields

$$\frac{1}{M}\left[\left(C_n e^{\frac{gS_o}{V_1^2}L_1}\right)\left(e^{-\frac{gS_o}{V_1}L_1}\right)\right] = \mu_{\max}^{Q}\left(\frac{Q_1}{K_{m_1}+Q_1}\bullet\frac{Q_2}{K_{m_2}+Q_2}\bullet\frac{Q_3}{K_{m_3}+Q_3}\bullet\bullet\bullet\frac{Q_n}{K_{m_n}+Q_n}\right)\Big]\frac{X}{dX/dQ} \quad (63)$$

$$M_{Q_m}\left[\left(C_n e^{\frac{gS_o}{V_1^2}L}\right)\left(e^{-\frac{gS_o}{V_1}L_1}\right)\right] = \mu_{\max}^{Q}\left(\frac{Q_1}{K_{m_1}+Q_1}\bullet\frac{Q_2}{K_{m_2}+Q_2}\bullet\frac{Q_3}{K_{m_3}+Q_3}\bullet\bullet\bullet\frac{Q_n}{K_{m_n}+Q_n}\right)\Big]\frac{X}{dX/dQ} \quad (64)$$

Similarly, equation (51) becomes

$$\frac{dX}{dt} = \mu_{\max}^{Q}\left[\left(\frac{Q_1}{K_{m_1}+Q_1}\bullet\frac{Q_2}{K_{m_2}+Q_2}\bullet\frac{Q_3}{K_{m_3}+Q_3}\bullet\bullet\bullet\frac{Q_n}{K_{m_n}+Q_n}\right)\right]X$$

Therefore

$$M\frac{dX}{dt} = \left(C_n e^{\frac{gS_o}{V_1^2}L_1}\right)\left(e^{-\frac{gS_o}{V_1}L_1}\right) \quad (65)$$

Substituting equation (51) into equation (65) yields

$$M_{xm} = \left[\left(C_n e^{\frac{gS_o}{V_1^2}L_1}\right)\left(e^{-\frac{gS_o}{V_1}L_1}\right)\right] \Big/ \mu_{\max}^{Q}\left(\begin{matrix}\frac{Q_1}{K_{m_1}+Q_1}\bullet\frac{Q_2}{K_{m_2}+Q_2} \\ \bullet\frac{Q_3}{K_{m_3}+Q_3}\bullet\bullet\bullet\frac{Q_n}{K_{m_n}+Q_n}\end{matrix}\right)X \quad (66)$$

$$\text{But}\ \hat{M} = U_{LSm} = MV = M_{xm}V = M_{xm}\frac{dx}{dt} \quad (67)$$

Therefore equation (66) becomes

$$\hat{M} = U_{LSm}\left[\left[\left(C_n e^{\frac{gS_o}{V_1^2}L_1}\right)\left(e^{-\frac{gS_o}{V_1}L_1}\right)\right] \Big/ \mu_{\max}^{Q}\left(\begin{matrix}\frac{Q_1}{K_{m_1}+Q_1}\bullet\frac{Q_2}{K_{m_2}+Q_2} \\ \bullet\frac{Q_3}{K_{m_3}+Q_3}\bullet\bullet\bullet\frac{Q_n}{K_{m_n}+Q_n}\end{matrix}\right)\right]\frac{dX}{dt} \quad (68)$$

Therefore equation (68) becomes

$$\hat{M} = U_{LQm} = V\left[\left[\left(C_n e^{\frac{gS_o}{V_1^2}L_1}\right)\left(e^{-\frac{gS_o}{V_1}L_1}\right)\right] \Big/ \mu_{\max}^{Q}\left(\begin{matrix}\frac{Q_1}{K_{m_1}+Q_1}\bullet\frac{Q_2}{K_{m_2}+Q_2} \\ \bullet\frac{Q_3}{K_{m_3}+Q_3}\bullet\bullet\bullet\frac{Q_n}{K_{m_n}+Q_n}\end{matrix}\right)\right] \quad (69)$$

Model Correlation of Sinusoidal Input with Momentum Transfer

Recalling the sinusoidal input responses of $Y_{(t)}$ =

$$\frac{KA}{1+w^2\tau^2}\left\{w\tau^2 e^{-\frac{t}{t}} - w\tau\cos wt + \sin wt\right\}$$

Similarly, recalling the expression of equation (44) which defines the momentum transfer in terms of velocity is given as

$$\frac{V^2}{1-V_1} = gS_o L_i \frac{In\ C_n}{U_{L_1}}$$

Considering when the output responses is dependent of velocity, therefore equation (14) can be expressed as

$$y_{(t)} = \frac{V_1^2}{1-V_1};$$ Thus equation (44) can be written as

$$gS_o L_i\, In\frac{C_n}{U_{L1}} = \frac{KA}{1+w^2\tau^2}\left\{w\tau^2 e^{-\frac{t}{\tau}} - w\tau\cos wt + \sin wt\right\} \quad (70)$$

Therefore,

$$In\frac{C_n}{U_{L1}} = \frac{KA}{gS_o L_i\left(1+w^2\tau^2\right)}\left\{w\tau^2 e^{-\frac{t}{\tau}} - w\tau\cos wt + \sin wt\right\} \quad (71)$$

Computational Procedure

The following parameters were used in evaluating the functional parameters such as time constant (τ) = 0.1min, amplitude (A) = 2°C, process gain (K) = 1, angular velocity (w) = 2πf, pond temperature (T) = 32°C, frequency of oscillation (f) = $\frac{w}{\pi}$ cycles/min, time (t) = 0-1.0min and length of the pond considered (L) = 1-5m. These values were used in simulating the developed models in this paper.

RESULTS AND DISCUSSION

The results obtained from the investigation are presented in tables and figures as shown below: The output response and the phase lag was examined upon the influence of momentum and time. The results obtained indicate sinusoidal characteristics on the output response, which leads to increase in output responses within the range of 0 to 0.2min and decreases within the range of 0.4 to 0.6 and later increase within the range of 0.8 to 1.0min as shown in Table 1.

Table 2 illustrates the relationship of time and output characteristics of the system as well as time and phase lag. The results presented in table 2 indicate increase in output value within the range of time of 0.02 to 0.2min and a decrease in output value within the range of time of 0.4 to 0.6min and sudden increase was observed in output value within the range of time of 0.8 to 1.0min. Similarly, the phase lag increases with increase in time. Although the phase lag values are negative, its significant objective was achieved as – $\tan^{-wt}$.

Table 1: Theoretical computation of output response and phase lag value of the function

Time (min)	$y_{(t)} = \frac{KA}{1+\omega^2\tau^2}\left\{\omega\tau^2\ell^{-\frac{t}{\tau}} - \omega\tau\cos\right.$	ϕ phase lag = $-\tan^{-wt}$
0	0	0
0.02	0.0277	-0.3805
0.04	0.1020	-0.6747
0.06	0.1415	-0.8761
0.08	0.1732	-1.0122
0.1	0.1067	-1.1071
0.2	0.1279	-1.3258
0.4	-0.0550	-1.4464
0.6	-0.0636	-1.4877
0.8	0.0406	-1.5084
1.0	0.2337	-1.5208

Table 2: Theoretical computation on momentum transfer effect on substrate concentration at various distance and time in the pond system

Time (min)	$In\frac{C_n}{U_{L_1}}$	$In\frac{C_n}{U_{L2}}$	$In\frac{C_n}{U_{L_3}}$	$In\frac{C_n}{U_{L4}}$	$In\frac{C_n}{U_{L_5}}$
Distance (m)	L=1	L=2	L=3	L=4	L=5
0	0	0	0	0	0
0.02	0.5018	0.2509	0.1672	0.1255	0.1004
0.04	1.8478	0.9239	0.6159	0.4620	0.3696
0.06	2.5634	1.2817	0.8545	0.6409	0.5127
0.08	3.1377	1.5688	1.0458	0.7844	0.6275
0.01	1.9330	0.9665	0.6443	0.4832	0.3866
0.2	2.3170	1.1585	0.7723	0.5793	0.4634
0.4	-0.0996	-0.4982	-0.3321	-0.2491	-0.1993
0.6	-1.1522	-0.5761	-0.3841	-0.2880	-0.2304
0.8	0.7355	0.3678	0.2452	0.1839	0.1471
1.0	4.2337	2.1168	1.4113	1.0584	0.8467

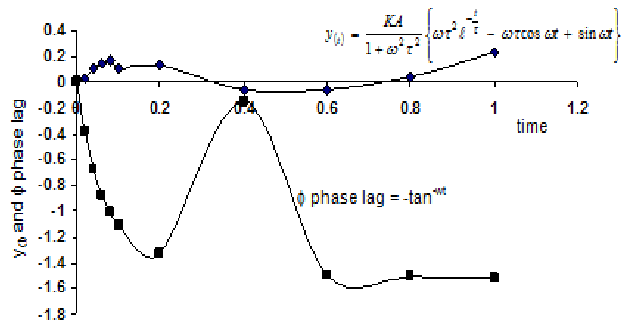

Figure 3: Graph of output and lag phase versus time

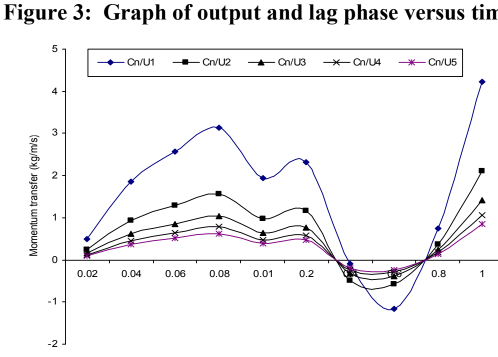

Figure 4: Graph of momentum transfer effect on substrate concentration $In\frac{C_n}{U_{Ln}}$ versus time

The results presented in Figure 3 illustrate the output and phase lag characteristics upon the influence of momentum transfer on biodegradation of petroleum hydrocarbon in pond system. The phase lag curve obtained is parabolic in shape within the range of time interval of 0 to 0.4min as well as 0.2 to 0.4min, before achieving a linear decrease in the phase lag. All the phase lag value of $-\tan^{-wt}$ are negative indicating how faster the output characteristics will be. similarly, the output characteristics (y_t) in Figure 3 illustrates an increase within the time range of 0 to 0.2 and decreases within 0.2 to 0.6 and sudden increase was observed within 0.6 to 1.0 as presented in figure 3.

Figure 4 illustrates the change in $In\frac{C_n}{U_L}$ against time at various distances. The variation in $In\frac{C_n}{U_L}$ can be attributed to the variation in time as well as distance. An increase in $In\frac{C_n}{U_L}$ was observed within the time range of 0.02 to 0.04min and a decrease occurred within the time range of 0.4 to 0.8min and a sudden increase was observed with a time range of 0.8 to 1min. The shape of the curve is sinusoidal and the substrate characteristics are also sinusoidal in behaviour as presented in Figure 4.

Conclusion

The following conclusions were drawn from the research work, such as:

1. The effect of momentum transfer on output characteristics was examined.
2. The effect of momentum transfer on phase lag was examined.
3. The substrate characteristics upon the influence of momentum transfer and microbial growth rate.
4. The significant effect of frequency on the wave formation as well as its contribution in improving the biodegradation of petroleum hydrocarbon discharged into the pond system.
5. The velocity of distribution is also another significant factor that influence the substrate concentration in the pond system.
6. The relationship between the momentum transfer and substrate concentration upon the influence of distance or the spreading rate in the pond system was examined and the results obtained indicate decrease in the ratio of momentum to substrate concentration.
7. The output characteristics of the system at 0.4 to 0.6min indicate a negative value which means less effect of momentum transfer was observed.
8. The energy slope below the surface water is also another contributor to the sinusoidal characteristics of petroleum hydrocarbon degradation in pond system upon the influence of momentum transfer as well as time dependent.
9. The degradation of the petroleum hydrocarbon depends on input concentration of the wastewater discharged into the pond as well as the velocity, momentum, volume, amplitude and frequency of the wave generated as a result of continuous discharge of wastewater into the pond system. These components mention above is other functional parameters that influence the biodegradation of petroleum degradation.

Nomenclature

f	=	Frequency (cycle/min)
t	=	Time (min)
τ	=	Time constant (min^{-1})
T	=	Temperature (^oC)
A	=	Amplitude (^oC)
K	=	Process gain (dimensionless)
w	=	Angular velocity (radius/min)
L	=	Length (m)
U_L	=	Momentum transfer (kg/m/s)
B,C & D	=	Constants
ρ	=	Density (kg/m^3)
A_l	=	Cross-sectional area (m^2)
F_g	=	Gravity force (N)
n	=	Integral number, 1, 2, 3, ---
K_m	=	Equilibrium constant of microbial growth rate 9dimensionless)
X_o	=	Initial biomass concentration (cfu/ml)
E	=	Enzyme (cfu/ml)
Fr	=	Frictional force (N)
g	=	Acceleration due to gravity (m/s^2)
V	=	Velocity (m/s)
S_o	=	Slope of the surface
C_1 & C_2	=	Constants
C_n	=	Hydrocarbon concentration (mg/l)
Q	=	Substrate concentration (mg/l)
X	=	Biomass concentration (cfu/ml)
μ^Q_{max}	=	Maximum specific rate (mg/l/min)

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