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RESEARCH ARTICLE

USE OF RECURRENCE RELATION FOR BINOMIAL PROBABILITY COMPUTATION

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ABSTRACT

The binomial probability computation have since been made using the binomial probability distribution expressed as $\binom{n}{x}P^x(1 \quad P)^{n-x}$ for a fixed *n* and for *x*=0, 1, 2…, *n*. In this paper, a new formular, $f(x+1) = a\,\frac{n-x}{x+1}\,f(x)$, fashioned out of the existing binomial probability expression (distribution) is proposed where the expression is stated as a recurrence relations that is *f*(*x*+1) in terms of *f*(*x*).

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INTRODUCTION

The binomial distribution presently in use for computing binomial probabilities is defined by

$$\Pr(X=x) = \binom{n}{x}P^x(1 \quad P)^{n-x}, x=0, 1, 2, \ldots, n, \qquad (1)$$

see (2) and (6), Where *n* is the number of repeated independent and identical Bernoulli trials.

X is the number of success out of possible total *n*,
P is the probability of success in each trial, and
$\binom{n}{x}$ is the number of combinations of *n* things taking *x* at a time

Specifically,

$$\Pr(X=0) = \binom{n}{0}P^0(1 \quad P)^n, for\ x = 0,$$
$$\Pr(X=1) = \binom{n}{1}P^1(1 \quad P)^{n-1}, for\ x = 1.$$

Thus the probability for each value of x is computed independently by making substitution for *x* in the expression on the right-hand side of (1)

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Now, the expression on the right-hand side of (1) is derived from an important fact in algebra called the binomial expression theorem which deals with an expansion formulation for $(a+b)^n$.

The binomial expansion for any positive integer power n is given by

$$(a+b)^n = a^n + \binom{n}{1}ba^{n-1} + \binom{n}{2}b^2a^{n-2} +,\ldots,+ \binom{n}{x}b^xa^{n-x} + b^n, \quad \ldots\ldots\ldots\ldots(2)$$

Considering in particular, *a=q, b=p* and *q+p*=1, then the formula yields

$$(q+p)^n = q^n + \binom{n}{1}pq^{n-1} + \binom{n}{2}p^2q^{n-2} +,\ldots,+ \binom{n}{x}p^xq^{n-x} + p^n, \quad \ldots\ldots\ldots(3)$$

Successive terms on the right-hand side of (3) are precisely the binomial probabilities *b* (0), *b* (1), …, *b(n).* They are evaluated by making substitution for *x* one by one in the general expression (3).

But if we consider the binomial distribution in (1) again

$$\Pr(X=x) = \binom{n}{x}P^x(1 \quad P)^{n-x}$$
$$= \binom{n}{x}P^xq^{n-x} \quad q=1-p$$

and let $f(x) = \Pr(X=x)$

Then $f(x+1) = \binom{n}{x+1} P^{x+1} q^{n-x-1}$ (4)

and $\dfrac{f(x+1)}{f(x)} = \dfrac{\binom{n}{x+1} P^{x+1} q^{n-x-1}}{\binom{n}{x} P^{x} q^{n-x}}$

$= \dfrac{\binom{n}{x+1} P}{\binom{n}{q} q}$

$= \dfrac{p}{q} \left(\dfrac{n!}{(x+1)!(n-x-1)!} \quad X \quad \dfrac{(n-x)!}{n!} \right)$

$= \dfrac{p\,(n-x)}{q\,(x+1)}$

Thus $f(x+1) = \dfrac{p\,(n-x)}{q\,(x+1)} f(x)$ …………………(5)

Therefore, the binomial distribution used for evaluating the binomial probabilities can in fact be expressed as a recurrence relation to make the computation easier.

Binomial probability distribution

The binomial random variable is about the commonest of the discrete random variables (Bakouch *et al*., 2012) and (Good, 1965). This is because of its wide application to outcomes of statistical experiments in many daily life situations. When events, say the number of heads in a toss of a coin a fixed number of times or the number of 6's in a roll of a die are observed, the random variables that result in the two experiments defined above, possess the binomial distribution. And the random variables associated with the distribution are discrete (Wonnacott and Wonnacott, 1976), (Erricker, 1980) and (Cox and Hinkley, 1974). Discrete random variables are variables that can assume only a finite number of values or possibly an infinite number of values that can be arranged in a sequence. They are denoted by capital letters, say X, while the values they assume are represented by corresponding lower case characters, which in this case, is x(2), (8) B. Associated with each discrete random variable is a probability distribution which is a list of the distinct values x of X and the probability associated with each x. This can be expressed as a function. This function in this case is the binomial probability distribution function (Adamu and Tinulle, 1975). Essential feature of the binomial distribution is the Bernoulli trail which was first developed by Jacob Bernoulli, in his publication that contained the theory of permutations and combinations as well as the Binomial Theorem. A Bernoulli trial is a statistical experiment which has two possible outcomes, generally called success and failure. If now X represents the number of successes in n repeated independent Bernoulli trials with probability, I, of success in a given trial, then, X is a binomial random variable with parameters n and p (Aderson *et al*., 2003).

A Binomial Experiments

This is an experiment with the following four properties:

1. This experiment consists of a sequence of n identical trials.
2. Two outcomes are possible in each trial – one, referred to as success, and the other, a failure.
3. The probability of a success, denote by P, does not change from trial to trial. Consequently, the probability of a failure, denoted by $q(=1\text{-}P)$, does not change from trial to trial.
4. The trials are independent.

If properties 2, 3 and 4 are present, we say the trials are generated by a Bernoulli process. Any statistical experiment which has two possible outcomes generally called success or failure, yes or no, on or off, is called Bernoulli trial (process).

If in addition, property 1 is present, we say we have a binomial experiment. In a binomial experiment our interest is in the number of successes occurring in the trials. Considering the fact that it is possible to record no success in the n trials, the count of the number of successes starts from 0 and other values the count takes are 1, 2, in step of 1 to n. if we then denote the number of successes by letter x which is finite, x then is a discrete random variable (i.e. x = 0, 1, 2,…, n,). The probability distribution associated with this random variable is called the binomial probability distribution.

In applications involving binomial experiments, a special mathematical formula referred to is the expansion of $(a+b)^n$ for a special case with $a = p$, $b = q$, and $p+q=1$. This expansion is:

$$(p+q)^n = q^n + {}^{n}C_1\, p^1 q^{n-1} + \binom{n}{2} p^2 q^{n-2} + \text{---} + \binom{n}{x} P^x (1 \quad P)^{n-x} +,\ldots,+\, p^n$$

With this each expression of the expansion on the right hand side incidentally corresponds to probability of each of the value the random variable x takes (x = 0, 1, 2,…, x,…, n,) in a binomial experiment.

This mathematical formula linked to the binomial probability formula was proposed by Jacob Bernoulli (1654 – 1705), in his publication that contained the theory of permutations and combinations as well as the Binomial Theorem. Since then, the formula for computing each of the probabilities the value x takes in the binomial experiment is stated as:

$\Pr(X=x) = \binom{n}{x} P^x (1 \quad P)^{n-x}, x=0, 1, 2, \ldots, n,$

where n is the number of repeated Bernoulli trials;

x, the number of success out of possible total, n;
p, the probability of success in each trial;
$q = 1 - p$, the probability of failure in each trial;

and $\binom{n}{x}$, the number of combinations of n things taking x at a time. Because of the properties 2 and 3 of the Binomial Experiment, p and q feature prominently in the binomial probability formula. Also the expansion of $(p+q)^n$ introduces the combination symbol. For these two reasons, scholars of statistics have not been able to express explicitly the binomial probability formula as a function of x and n (Aderson *et al*., 2003) and (Erricker, 1980). The formula proposed below takes care of just that, by having an in-depth look into present formula, and fashioning out a constant, and a variable binomial coefficient. The constant factor is explained clearly in the proposed formula, while the variable binomial coefficient, $\binom{n}{x}$ is explained to be a recurrence relation as stated below.

Proposed new binomial probability function (Formula)

From a careful study of the existing binomial probability formula, it is discovered that the formula can be factored into three distinct parts. A constant, a variable factor and *f(x),* where f is a probability distribution function with *f(x)* relating to *f(x+1)*. This suggest that a recurrence relation can in fact be established where *f(x+1)* can be expressed in terms of f(x) to give a sequence of binomial probabilities for x greater than 0 and less or equal to n. The first term in the sequence is *f*(0), where *f*(0) = q^n. Once *f*(0) is computed, the subsequent probabilities each computed as function of the preceding one. Thus, it will no longer be necessary to fall back on the binomial probability formula in order to compute the probability for x, ($0 < x \leq n$). A recurrence relation is an equation that recursively defines its sequence or multi dimensional array of values, once one or more initial terms are given, each further term of the sequence or array is defined as a function of the preceding the term. This is the case of the binomial distribution formula

Proposed formula

The proposed new binomial probability formula, composing of three parts with *f*(*x*+1) expressed in terms of *f*(*x*), as one of the component parts, is:

$$f(x+1) = \left(\frac{p}{q}\right)\frac{\binom{n}{x+1}}{\binom{n}{x}}f(x)\,; \qquad x = 0,1,2,\ldots\ldots..,n$$

$$= a\,\frac{\binom{n}{x+1}}{\binom{n}{x}}\,f(x)\,; \qquad a = \frac{p}{q}$$

$$= a\,\frac{n-x}{x+1}\,f(x); \qquad \frac{\binom{n}{x+1}}{\binom{n}{x}} = \frac{n-x}{x+1},\ x = 0,1,2,\ldots\ldots..,n\text{-}1$$

With $f(0) = p(0) = q^n \qquad q = 1 \quad p$

Computations

Comparison of binomial probabilities computed using the binomial probability formula and the proposed formula.

Example

Suppose a biased coin comes up heads with probability 0.3 when tossed what is the probability of achieving 0,1,…, 6 heads after six tosses?

$$6\binom{n}{x} \quad (0.3^x)(0.7)^{6-x} \qquad \left(\frac{.3}{.7}\right)\frac{n \quad x}{x+1}f(x)$$

Computation	Pr (0 heads)	=	0.1176	0.1176
	Pr (1 head)	=	0.3025	0.3025
	Pr (2 heads)	=	0.3240	0.3241
	Pr (3 heads)	=	0.1852	0.1852
	Pr (4 heads)	=	0.0595	0.0595
	Pr (5 heads)	=	0.0102	0.0102
	Pr (6 heads)	=	0.0007	0.0007

The probabilities computed using the proposed formula are virtually the same as those computed using the binomial probability formula. Most students of statistics, as in most elementary statistics text books, the probability for each value of x is computed separately/independently by substitution in the binomial probability formula. This is so because each of the probability values, *P*(0), *P*(1), *P*(2), *P*(3),……*P(n)*, corresponds to each of the expressions in the binomial expansion $(p+q)^n$.

i.e $(p+q)^n = P(0) + P(1) + P(2) +,\ldots,+P(n) = (n\mathcal{C}_0)p^0q^n + (n\mathcal{C}_1)p^1q^{n-1} + (n\mathcal{C}_2)p^2q^{n-2} + \quad +(n\mathcal{C}_n)p^nq^{n-n}$

In the proposed method of computation once *P*(0) is determined for any binomial distribution, other probabilities are calculated in steps, with each $f(x+1)$, a function of $f(x)$

Conclusion

The proposed formula enables students of statistics to compute the binomial probabilities progressively with each probability as a function of the preceding one, once *f*(0) is computed. This makes the computation easier, faster and easily computerized. The only condition for this proposed formula to be applicable is for *f*(0) not to be equal to zero.

A table for the values of the ratio:

$\frac{n-x}{x+1}$; n = 2, 3, - - - , 20; x + 1 = 1, 2, - - - - -, 20, is attached as an appendix to make easier the computation of the probabilities, for the fixed value of n. Also attached are tables of computed probabilities for two values of p, (p = 0.1, and p = 0.2) with n fixed at 10.

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APPENDIX

Table 1.

$\frac{n-x}{x+1}$, n = 2, 3,-----------------, 20
x = 0, 1, 2, 3----------------, 20

$x+1$ \ n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	½	2/2	3/2	4/2	5/2	6/2	7/2	8/2	9/2	10/2	11/2	12/2	13/2	14/2	15/2	16/2	17/2	18/2	19/2
3		1/3	2/3	3/3	4/3	5/3	6/3	7/3	8/3	9/3	10/3	11/3	12/3	13/3	14/3	15/3	16/3	17/3	18/3
4			1/4	2/4	3/4	4/4	5/4	6/4	7/4	8/4	9/4	10/4	11/4	12/4	13/4	14/4	15/4	16/4	17/4
5				1/5	2/5	3/5	4/5	5/5	6/5	7/5	8/5	9/5	10/5	11/5	12/5	13/5	14/5	15/5	16/5
6					1/6	2/6	3/6	4/6	5/6	6/6	7/6	8/6	9/6	10/6	11/6	12/6	13/6	14/6	15/6
7						1/7	2/7	3/7	4/7	5/7	6/7	7/7	8/7	9/7	10/7	11/7	12/7	13/7	14/7
8							1/8	2/8	3/8	4/8	5/8	6/8	7/8	8/8	9/8	10/8	11/8	12/8	13/8
9								1/9	2/9	3/9	4/9	5/9	6/9	7/9	8/9	9/9	10/9	11/9	12/9
10									1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10	10/10	11/10
11										1/11	2/11	3/11	4/11	5/11	6/11	7/11	8/11	9/11	10/11
12											1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12
13												1/13	2/13	3/13	4/13	5/13	6/13	7/13	8/13
14													1/14	2/14	3/14	4/14	5/14	6/14	7/14
15														1/15	2/15	3/15	4/15	5/15	6/15
16															1/16	2/16	3/16	4/16	5/16
17																1/17	2/17	3/17	4/17
18																	1/18	2/18	3/18
19																		1/19	2/19
20																			1/20

$\left(\frac{0.1}{0.9}\right)\frac{\square-\square}{\square+1}\square(\square)$

p = 0.1

Table 2.

$f(x)$ \ n	2	3	4	5	6	7	8	9	10
$\square_0$	.81	.729	.6561	.59049	.531441	.4782969	.43046721	.387420489	.3486784400
$\square_1$	.18	.234	.2916	.32805	.354294	.3720087	.38263752	.387420489	.3874204890
$\square_2$	.01	.027	.0486	.07290	.098415	.1240029	.14880348	.172186884	.1937102440
$\square_3$		.001	.0036	.00810	.014580	.0229635	.03306744	.044641044	.0573956280
$\square_4$			.0001	.00045	.001215	.0025515	.00196830	.007440174	.0111602610
$\square_5$				.0001	.00054	.001701	.0040824	.000826686	.0014880348
$\square_6$					.00001	.000063	.00002268	.00001236	.0001377810
$\square_7$						.000001	.00000072	.000002916	.0000037480
$\square_8$							.00000001	.000000081	.0000003645
$\square_9$								.000000001	.0000000090
$\square_{10}$									.0000000001

P = 0.2

Table 3.

$f(x)$ \ n	2	3	4	5	6	7	8	9	10
$\square_0$	.64	.512	.4096	.32768	.262144	.2097152	.16777216	.134217728	.1073741820
$\square_1$	.32	.384	.4096	.40960	.396216	.3670016	.33554432	.301989888	.2684354560
$\square_2$	.04	.095	.1536	.20480	.245760	.2752512	1.29360128	.301989888	.3019898880
$\square_3$		.005	.0256	.05120	.081920	.1146880	.14680064	.176160768	.2013265920
$\square_4$			.006	.00640	.015360	.0286720	.04587520	.066060288	.0880803840
$\square_5$				.00032	.001536	.0043008	.00917504	.016515072	.0234881020
$\square_6$					.000064	.0003584	.00114688	.002752512	.0055050240
$\square_7$						.000128	.00008192	.000294912	.0007864320
$\square_8$							.00000256	.000018432	.0000737280
$\square_9$								.000000152	.0000040960
$\square_{10}$									.0000001024

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