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RESEARCH ARTICLE

MARGRABE'S FORMULA IN FUZZY ENVIRONMENT

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ABSTRACT

In this paper researchers have discussed exchange call option valuation using Margrabe's formula in Fuzzy environment. To model the uncertainty in parameters of Margrabe's formula, the trapezoidal fuzzy numbers are used. By using case example, the range of option values is predicted to a certain extent, which eventually can help investors to decide the investment strategy.

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1. INTRODUCTION

An Exchange option between two stocks gives its holder the right, but not the obligation, to exchange one for the other at the time of expiration. Exchange options are used in the commodity market for hedging risk against the retail price of the good verses the cost of manufacture. In energy markets, such options are required for hedging the price for the consumption of electricity against the price of a natural gas.

The valuation of an Exchange option was derived independently by William Margrabe (1978) and Stanley Fisher (1978). The pricing model used by Margrabe has the same assumptions as those in Black-Scholes option pricing model (Black *et al*., 1973). Rubinstein (1991) formulated his binomial option pricing model to fit the exchange option model by taking relative asset prices. Boyle and Guthrie (2003) have applied exchange option valuation model to decide investment decision timing. Alos *et al.* (2015) have studied Margrabe options in a general stochastic volatility framework. Due to the market dynamics, some of the input parameters in Margrabe's formula cannot always be obtained in precise sense.

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Uncertainty may be caused by either lack of knowledge or inherent vagueness or both. There has been a growing interest in using fuzzy numbers to deal with uncertainty (Appadoo and Srimantoorao Semischetty, 2006; Thavaneswaran *et al*., 2007). In (Ostaszewski and Krzysztof, 1993), Ostaszewski has applied fuzzy set theory as an alternative to stochastic methods for modeling uncertainty. In this paper, we consider an Exchange call option between two correlated stocks; each follows a geometric Brownian motion. The paper is organized as follows: Section 2 introduces Exchange option and Margrabe's formula. In section 3, basic definitions and results of fuzzy set theory are discussed. In section 4, trapezoidal fuzzy numbers are used to model uncertain parameters in the Margrabe's formula and the model is presented by way of a numerical example to calculate the value of an Exchange call option. The paper concludes with section 5.

2. Exchange option and Margrabe's formula

The exchange option payoff at time t is given by $EO = (S_1(t) \quad S_2(t))^+$ where $S_1(t)$ and $S_2(t)$ are the stock prices at time t. Exchange options are also known as Outperformance options or Margrabe options.

We assume that stock prices follow a geometric Brownian motion under the risk-neutral measure. That is,

$$dS_i = \mu_i S_i dt + \sigma_i S_i dB_i, \quad i = 1, 2 \qquad (2.1)$$

where μ_i are instantaneous returns, σ_i denotes volatility of stock prices, $B_i's$ are standard Brownian motions correlated by ρ for $i = 1,2$.

Let r_1 and r_2 be the interest rates on S_1 and S_2 respectively and the expiration time is denoted by T.

The time t value of the exchange option is given by

$$EO = S_1(t)\, e^{(\mu_1 - r_1)(T-t)} N(d_1) \quad S_2(t)\, e^{(\mu_2 - r_2)(T-t)} N(d_2) \qquad (2.2)$$

where

$$d_1 = \frac{\log\left(\frac{S_1(t)}{S_2(t)}\right) + \left(\mu_1 \quad \mu_2 + r_2 \quad r_1 + \frac{\sigma^2}{2}\right)(T \quad t)}{\sigma\sqrt{T \quad t}}$$
$$d_2 = d_1 \quad \sigma\sqrt{T \quad t}$$
$$\sigma = \sqrt{\sigma_1{}^2 + \sigma_2{}^2 \quad 2\,\sigma_1\sigma_2\rho}$$

σ is the volatility of $\frac{S_1(t)}{S_2(t)}$ and $N(\,)$ denotes cumulative standard normal density.

The formula (2.2) is known as Margrabe's formula.

3. Basic definitions and results of fuzzy set theory

In this paper, fuzzy numbers are used to model uncertainty in stock prices, interest rates, volatility and dividends. We have used definitions and results from Thaigarajah *et al.* (2007).

We define a trapezoidal fuzzy number which we are going to use in our option pricing model.

Definition 3.1 Let be the set of all real numbers. A trapezoidal fuzzy number $A(x), x \in$ is of the form

$$A(x) = \begin{cases} \frac{x-a}{b-a} & \text{when } x \in [a,b) \\ 1 & \text{when } x \in [b,c) \\ \frac{d-x}{d-c} & \text{when } x \in [c,d) \\ 0 & \text{otherwise} \end{cases} \qquad (3.1)$$

where a, b, c, d are real numbers such that $a < b < c < d$.

The above fuzzy number will be denoted by $A = (a, b, c, d)$.

If $A = (a, b, c, d)$, then its α level sets (α cuts), for all $\alpha \in [0,1]$, are given by

$$A(\alpha) = (a + (b \quad a)\alpha,\ d \quad (d \quad c)\alpha). \qquad (3.2)$$

For trapezoidal fuzzy numbers $A = (a_1, a_2, a_3, a_4)$ and B= (b_1, b_2, b_3, b_4), their possibilistic mean and possibilistic covariance are as defined in Thaigarajah (2007) (see formulas (3.20) and (3.24)).

4. Fuzzy exchange call option valuation model

In this section, we have used trapezoidal fuzzy numbers to write the Margrabe's formula (2.2). In the formula (2.2), uncertain parameters are stock prices, interest rates and volatilities of two correlated assets. Time to expiry, $T \quad t$, is always known and is a non-fuzzy. The following is our fuzzy Exchange call option formula:

$$\widehat{FECO} = \hat{S}_1(t) \quad e^{(\mu_1 \quad r_1)(T-t)} N(d_1) \quad \hat{S}_2(t) \quad e^{(\mu_2 \quad r_2)(T-t)} N(d_2) \qquad (4.1)$$

where

$$d_1 = \frac{\log\left(\mathrm{E}\,(\hat{S}_1(t) \quad \hat{S}_2(t))\right) + \left(E(\mu_1 \quad \mu_2 \oplus r_2 \quad r_1) + \frac{\sigma^2}{2}\right)(T \quad t)}{\sigma\sqrt{T \quad t}}$$
$$d_2 = d_1 \quad \sigma\sqrt{T \quad t}$$
$$\sigma = \sqrt{E(\hat{\sigma}_1{}^2) + E(\hat{\sigma}_2{}^2) \quad 2\,COV(\hat{\sigma}_1, \hat{\sigma}_2)}$$

where

$\hat{S}_i(t)$ $(for\ i = 1,\ 2)$ are fuzzy stock prices
r_i $(for\ i = 1,\ 2)$ are fuzzy interest rates
μ_i $(for\ i = 1,\ 2)$ are drift rates of assets
$E\left(\hat{S}_i(t)\right)$ $(for\ i = 1,\ 2)$ are possibilistic mean values of fuzzy stock prices
$E(\hat{\sigma}_i{}^2)$ $(for\ i = 1,\ 2)$ are possibilistic mean values of fuzzy variances of stocks
σ is the volatility of $\frac{S_1(t)}{S_2(t)}$
$COV(\hat{\sigma}_1, \hat{\sigma}_2)$ is the covariance of $\hat{\sigma}_1$ and $\hat{\sigma}_2$
ρ is the correlation coefficient between the Brownian motions defined in the equation (2.1)
$N(\,)$ denotes cumulative standard normal density.

Notations:

$\oplus$, , , represents addition, subtraction, multiplication and division of two trapezoidal fuzzy numbers. We have used these definitions as given in Banerjee and Roy (2012) by taking $w = 1$. A real number $'a'$ in fuzzy form is written as (a, a, a, a). Here we have used Removable Area Defuzzification method described in (Banerjee *et al*., 2012).

By truncating Taylor series expansion, a fuzzy number $e^{(a,b,c,d)}$ is approximated as

$$(1 + a, 1 + b, 1 + c, 1 + d)$$

We write equation (4.1) as a fuzzy number as follows:

$$\widehat{FECO} = (S_{11},\ S_{12}, S_{13},\ S_{14}) \quad e^{(\mu_1 - r_{14}, \mu_1 - r_{13}, \mu_1 - r_{12}, \mu_1 - r_{11})(T-t)} N(d_1)$$

$$(S_{21},\ S_{22}, S_{23},\ S_{24}) \quad e^{(\mu_2 - r_{24}, \mu_2 - r_{23}, \mu_2 - r_{22}, \mu_2 - r_{21})(T-t)} N(d_2)$$

$$= (S_{11},\ S_{12}, S_{13},\ S_{14})$$

$\left(1+(\mu_1 \quad r_{14})N(d_1), 1+(\mu_1 \quad r_{13})N(d_1), 1+(\mu_1 \quad r_{12})N(d_1), 1+(\mu_1 \quad r_{11})N(d_1)\right)$

$(S_{21}, S_{22}, S_{23}, S_{24})$

$$\left(1+(\mu_2 \quad r_{24})N(d_2), 1+(\mu_2 \quad r_{23})N(d_2), 1 + (\mu_2 \quad r_{22})N(d_2), 1+(\mu_2 \quad r_{21})N(d_2)\right)$$

Let the fuzzy numbers

$\left(1+(\mu_1 \quad r_{14})N(d_1), 1+(\mu_1 \quad r_{13})N(d_1), 1+(\mu_1 \quad r_{12})N(d_1), 1+(\mu_1 \quad r_{11})N(d_1)\right)$ and

$$\left(1+(\mu_2 \quad r_{24})N(d_2), 1+(\mu_2 \quad r_{23})N(d_2), 1 + (\mu_2 \quad r_{22})N(d_2), 1+(\mu_2 \quad r_{21})N(d_2)\right)$$

be denoted by $(e_{11}, e_{12}, e_{13}, e_{14})$ and $(e_{21}, e_{22}, e_{23}, e_{24})$ respectively. The above equation can be written as

$$\widehat{FECO} = (S_{11}, S_{12}, S_{13}, S_{14}) \qquad (e_{11}, e_{12}, e_{13}, e_{14})$$

$$(S_{21}, S_{22}, S_{23}, S_{24}) \qquad (e_{21}, e_{22}, e_{23}, e_{24})$$

$$= (S_{11}e_{11}, S_{12}e_{12}, S_{13}e_{13}, S_{14}e_{14}) \qquad (S_{21}e_{21}, S_{22}e_{22}, S_{23}e_{23}, S_{24}e_{24})$$

According to membership function (3.4.3) given in (Banerjee *et al.*, 2012), both these fuzzy numbers are non-trapezoidal. These non-trapezoidal numbers can be approximated to trapezoidal fuzzy numbers by using approximation $\sqrt{1 \quad x} = 1 \quad \frac{x}{2}$. This way $\widehat{FECO}$ can be considered as a difference of two trapezoidal fuzzy numbers which results in a trapezoidal fuzzy number. We denote this fuzzy number by $(FECO_1, FECO_2, FECO_3, FECO_4)$.

The membership function for the fuzzy Exchange call option is given below:

$$m(\mathrm{C}) = \begin{cases} \frac{\mathrm{C}-\mathrm{FECO}_1}{\mathrm{FECO}_2-\mathrm{FECO}_1} & \text{when } x \in [\mathrm{FECO}_1, \mathrm{FECO}_2) \\ 1 & \text{when } x \in [\mathrm{FECO}_2, \mathrm{FECO}_3) \\ \frac{\mathrm{FECO}_4-\mathrm{C}}{\mathrm{FECO}_4-\mathrm{FECO}_3} & \text{when } x \in [\mathrm{FECO}_3, \mathrm{FECO}_4) \\ 0 & \text{otherwise} \end{cases} \qquad (4.2)$$

Example 4.1 *Consider an Exchange option written on two stocks with the following assumptions:*

The current stock prices, stock price volatilities, the risk-free interest rates and dividends are assumed to be trapezoidal fuzzy numbers.

Let

$\hat{S}_1 = [117, 119, 121, 123], \hat{S}_2 = [119, 121, 123, 125]$,

$r_1 = [0.03, 0.04, 0.05, 0.06], r_2 = [0.02, 0.04, 0.06, 0.08]$,

$\hat{\sigma}_1 = [0.17, 0.19, 0.21, 0.23], \hat{\sigma}_2 = [0.32, 0.34, 0.36, 0.38]$,

$\mu_1 = [0, 0, 0, 0], \hat{\mu}_2 = [0, 0, 0, 0]$
$T \quad t = 0.25\ (in\ years)$

Possibilistic mean and possibilistic covariance are calculated by using formulas in Thaigarajah (2007) ((3.20) and (3.24)). d_1, d_2 *and* σ *are calculated from equations in (4.1). The* α *cuts* $FECO1(\alpha)$ *and* $FECO2(\alpha)$ *are calculated using (3.2).*

$E(\mu_1) = 0$ $\quad E(\mu_2) = 0$
$E(\hat{\sigma}_1) = 0.2$ $\quad E(\hat{\sigma}_2) = 0.35$
$E(\hat{\sigma}_1{}^2) = 0.040366667$ $\quad E(\hat{\sigma}_2{}^2) = 0.122866667$ $\quad COV(\hat{\sigma}_1, \hat{\sigma}_2) = 0.0003$
$\sigma = 0.403278233$ $\quad \sigma^2 = 0.162633333$
$d_1 = 0.027508542$ $\quad d_2 = 0.174130574$
$N(d_1) = 0.510972937$ $\quad N(d_2) = 0.430881423$

The following table gives fuzzy call option values for different values of α.

n	1	1
α	FECO1(α)	FECO2(α)
0	3.413829292	13.79128291
0.1	3.761147572	13.44677095
0.2	4.108465853	13.10225899
0.3	4.455784133	12.75774703
0.4	4.803102413	12.41323507
0.5	5.150420694	12.06872311
0.6	5.497738974	11.72421115
0.7	5.845057255	11.37969919
0.8	6.192375535	11.03518723
0.9	6.539693815	10.69067526
1	6.887012096	10.3461633

Next we compare our fuzzy exchange call prices with the theoretical Margrabe's formula. The data taken for comparison is the average of four components of the above fuzzy numbers.

Let $S_1 = 120,\ S_2 = 122,\ r_1 = 0.045,\ r_2 = 0.05,\ \sigma_1 = 0.20,\ \sigma_2 = 0.35,\ \mu_1 = 0,\ \mu_2 = 0,$

$T \quad t = 0.25\ and\ \rho = 0.004285714$

(For comparison purpose, we have kept the same value for correlation coefficient as calculated by $\rho = \frac{COV(\sigma_1, \sigma_2)}{E(\sigma_1)\,E(\sigma_2)} = 0.004285714$*).*

The values obtained are $\sigma = 0.402367991, d_1 = 0.024645092, d_2 = 0.176538904, N(d_1) = 0.509830974, N(d_2) = 0.429935294.$

The value of call option is 8.694766765 which lie in the above range.

5. Conclusion

In this paper, we have used trapezoidal fuzzy numbers to model parameter uncertainty in Margrabe's option pricing formula. This methodology gives the range for option values with a reasonable certainty. We are of the opinion that the fuzzy number theory is practical and useful technique which can help investor to decide his/her investment strategy.

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