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RESEARCH ARTICLE

ON THE STRUCTURE EQUATION $F^{3K}+F^{K}=0$

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ABSTRACT

In this paper, we have studied various properties of the structure equation $F^{3K} + F^{K}=0$, where K is a positive integer. Nijenhuis tensor and metric F-structure have also been discussed.

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INTRODUCTION

Let M^n be a C^∞ differentiable manifold and F be a C^∞ (1,1) tensor defined on M^n, and satisfying

$$F^{3k} + F^{k} = 0, \quad F^{k} \neq 0. \tag{1.1}$$

we define the operators l and m on M^n by

$$l = - F^{2K}, m = I + F^{2K}, \tag{1.2}$$

where I is the identity operator.
From (1.1) and (1.2) we have

$$l+m=I,\ l^2=l,\ \ m^2=m,\ \ lm=ml=0$$

(1.3)

$$F^{K}l = lF^{K} = F^{K}, \quad F^{K}m = mF^{K} = 0$$

Theorem (1.1) Let the (1,1) tensors p and q be defined by

$$p = m + F^{K}, \quad q = m - F^{K}, \quad then \tag{1.4}$$

$$pq = 1 \quad p^2 = q^2, \quad p^2 - p - q + I = 0 \tag{1.5}$$

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$$p^3 = q \quad q^3 = p \qquad p^4 = I = q^4$$
$$pl = -ql = F^K, \; p^2 l = q^2 l = -l, \; pm = qm = p^2 m = q^2 m = m$$

Proof: Using (1.2), (1.3) and (1.4), we have

$$pq = m - F^{2K} = m + l = I$$
$$p^2 = m - l, \; p^3 = m - F^K = q, \; p^4 = pq = I \; etc \tag{1.6}$$

Theorem (1.2) Define the (1, 1) tensors α and β by

$\alpha = l + F^K, \quad \beta = l - F^K$, then (1.7)

$$\alpha^2 + \beta^2 = 0, \; \alpha^3 + 2\beta = 0 = \beta^3 + 2\alpha \tag{1.8}$$

Proof: Using (1.2), (1.3), (1.3) and (1.7), we have

$$\alpha^2 = 2F^K, \beta^2 = -2F^K, \; \alpha^2 + \beta^2 = 0,$$
$$\alpha^3 = 2F^K\left(l + F^K\right) = 2F^K + 2F^{2K} = 2F^K - 2l = -2\beta \; etc. \tag{1.9}$$

Theorem (1.3): If $rank\left(\left(F\right)\right) = n,$

$l = I, \; m = 0, \; \left\{F^K\right\}$ is an almost complex structure (1.10)

Proof: From the result

$$Rank \; of \; F + Nulity \; of \; F = Dim\left(M^n\right) \tag{1.11}$$

$$\Rightarrow Nulity \; of \; F = 0$$
$$\Rightarrow Ker \; F \; contains \; only \; 0$$

$FX = 0$ has the only solution $X = 0$ (1.12)

Let $FX_1 = FX_2 \Rightarrow F\left(X_1 - X_2\right) = 0$ (1.13)

Using (1.12) in (1.13), we get $X_1 = X_2$. Thus *F* is 1-1, also an operator on a finite dimensional differntiable manifold is onto also. Thus

F is invertible

$\Rightarrow F^K$ is invertible

$\Rightarrow \left(F^K\right)^{-1}$ exists

Applying this result (1.3) gives $l = I, m = 0$ and (1.1) gives

$$F^{2K} + I = 0 \tag{1.14}$$

Thus $\left\{F^K\right\}$ is an almost complex structure

2. NIJENHUIS TENSOR

Let $\underset{F}{N}, \underset{l}{N}, \underset{m}{N}$ denote the Nijenhuis tensor corresponding to the operators *F, l* and *m* respectively. Then

$$\underset{F}{N}(X,Y)=[FX,FY]+F^2[X,Y]-F[FX,Y]-F[X,FY] \quad (2.1)$$

$$\underset{L}{N}(X,Y)=[lX,lY]+l^2[X,Y]-l[X,Y]-l[X,lY]. \quad (2.2)$$

$$\underset{m}{N}(X,Y)=[mX,mY]+m^2[X,Y]-m[mX,Y]-m[X,mY] \quad (2.3)$$

Theorem (2.1) For the structure *F* satisfying (1.1), we have

$$\underset{F^K}{N}(mX,mY)=l[mY,mX] \quad (2.4)$$

$$m\underset{F^K}{N}(mX,mY)=0 \quad (2.5)$$

$$\underset{l}{N}(mX,mY)=l[mX,mY] \quad (2.6)$$

$$\underset{m}{N}(lX,lY)=m[lX,lY] \quad (2.7)$$

$$\underset{l}{N}(lX,mY)=0=\underset{m}{N}[mX,lY] \quad (2.8)$$

Proof: Using (1.2) and (1.3) in (2.1), (2.2), (2.3) we get all these results.

3. METRIC F-STRUCTURE

Let the Riemannian metric *g* satisfies

$$`F(X,Y)=g(FX,Y) \text{ is skew symmetric} \quad (3.1)$$

then

$$g(FX,Y)=-g(X,FY) \text{ and} \quad (3.2)$$

$\{F,g\}$ is called a metric F-structure.

Theorem (3.1) With the structure F satisfying (1.1), we have

$$g\left(F^K X,F^K Y\right)=(-1)^{K+1}\left[g(X,Y)-`m(X,Y)\right] \quad (3.3)$$

Where

$$`m(X,Y)=g(mX,Y)=g(X,mY) \quad (3.4)$$

Proof: Using (1.1), (1.2) and (3.2), (3.4) we get (3.3)

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