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RESEARCH ARTICLE

GENERALIZED HIGHER LEFT CENTRALIZER OF PRIME RINGS

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ABSTRACT

In this paper we introduce the concepts of generalized higher centralizer and generalized Jordan higher centralizer of rings R as well as we proved that every generalized Jordan higher centralizer of rings is generalized higher centralizer of R.

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1. INTRODUCTION

Throughout this paper R is a ring, R is called prime if aRb=(0) implies a=0 or b=0 with a,b$\in$ R and R is called semiprime if aRa=(0) with a$\in$R implies a=0 R is called 2-torsion free 2a=0 then a=0 with a$\in$R (Vokman and Ulbl, 2003) as usual [a,b] denotes the commutater ab-ba. An additive mapping d of R into itself is called derivation (resp. Jordan derivation) if d(ab) = d(a)b +ad(b) (resp. d(a^2)= d(a)a+ad(a)), for all a,b$\in$ Havala (1998) presented the concept of generalized derivation as follow an additive mapping f of R into itself is called generalized derivation (resp. generalized Jordan derivation) if there exists a derivation (resp. Jordan derivation) d of R such that f(ab) = f(a)b + ad(b) (resp. f(a^2) = f(a)a + ad(a)) hold for all a,b$\in$ R. The concept of derivation was extended to higher derivation by Hasse and Schmidt (1937) also see (Vokman and Ulbl, 2003) as follow, let D=$(d_n)_{n\in N}$ be a family of additive mappings of R into itself then D is called higher derivation (resp. Jordan higher derivation) of R if $d_n(ab)=\sum_{i+j=n} d_i(a)d_j(b)$ (resp. $d_n(a^2)=\sum_{i+j=n} d_i(a)d_j(a)$) hold for all a,b$\in$ R. In an attempt to generalize Herstein result for higher derivations, Haetinger (2000) proved that on a prime ring with 2-torsion free every Jordan higher derivation is a higher derivation. Cortes and Haetinger (2005) defined a generalized higher derivations as follows, let F=$(f_i)_{i\in N}$ be a family of additive mapping of R then F is called generalized higher derivation (resp. generalized Jordan higher derivation) if there exists higher derivation(resp. Jordan higher derivation) D=$(d_i)_{i\in N}$ such that $f_n(ab)=\sum_{i+j=n} f_i(a)d_j(b)$ and they prove that if R is 2-torsion free ring which has commutator right non-zero divisor and U a square closed Lie ideal of R then every generalized Jordan higher derivation is generalized higher derivation. Following Zalar (1991), an additive mapping T from R into itself is called a left (resp. right) centralizer of R if T(ab)= T(a)b (resp. T(ab) =aT(b)) holds for all a,b$\in$ R, if T is both left as well as right centralizer, then its called a centralizer. B. Zalar proved that any left (resp. right) Jordan centralizer on 2-tortion free semiprime ring is a left (resp. right) centralizer.

In this paper we present the concepts of higher left centralizer, Jourdan higher left centralizer also we introduce the concepts of generalized higher left centralizer, generalized Jordan higher left centralizer of R, as well as we prove that every Jordan higher left centralizer of 2-torsion free prime ring R is higher left centralizer of R. Also we prove every generalized Jordan higher left centralizer of 2-torsion free prime ring R is generalized higher left centralizer of R.

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2. Jordan Higher Left Centralizer

In this section we present the concepts of higher left centralizer, Jordan higher left centralizer of a ring R also we study the properties of them. We begin with the following definition:

Definition 2.1: Let T= $(t_i)_{i\in N}$ be a family of additive mappings of a ring R into itself. Then T is called higher left centralizer if for every $n\in N$

$t_n(ab) = \sum_{i=1}^{n} t_i(a)t_{i-1}(b)$ for all $a,b\in R$ …(1)

And T is called Jordan higher left centralizer of R if for every $n\in N$

$t_n(a^2) = \sum_{i=1}^{n} t_i(a)t_{i-1}(a)$ for all $a\in R$ …(2)

T is called Jordan triple higher left centralizer of R if for every $n\in N$

$t_n(aba) = \sum_{i=1}^{n} t_i(a)t_{i-1}(b)t_{i-1}(a)$ for all $a,b\in R$ …(3)

Lemma 1: Let T= $(t_i)_{i\in N}$ be Jordan higher left centralizer of a ring R into itself then for all $a,b,c\in R$, $n\in N$

1) $t_n(ab+ba) = \sum_{i=1}^{n} t_i(a)t_{i-1}(b) + t_i(b)t_{i-1}(a)$

2) $t_n(abc+cba) = \sum_{i=1}^{n} t_i(a)t_{i-1}(b)t_{i-1}(c) + t_i(c)t_{i-1}(b)t_{i-1}(a)$

In particular if R is 2-tortion free commutative ring

3) $t_n(abc) = \sum_{i=1}^{n} t_i(a)t_{i-1}(b)t_{i-1}(c)$

Proof: 1)

$t_n((a+b)(a+b)) = \sum_{i=1}^{n} t_i(a+b)t_{i-1}(a+b)$

$= \sum_{i=1}^{n} t_i(a)t_{i-1}(a) + t_i(a)t_{i-1}(b) + t_i(b)t_{i-1}(a) + t_i(b)t_{i-1}(b)$ …(1)

On the other hand

$t_n((a+b)(a+b)) = t_n(a^2+ab+ba+b^2)$

$= \sum_{i=1}^{n} t_i(a)t_{i-1}(a) + t_i(b)t_{i-1}(b) + t_n(ab+ba)$ …(2)

Comparing (1) and (2) we get

$t_n(ab+ba) = \sum_{i=1}^{n} t_i(a)t_{i-1}(b) + t_i(b)t_{i-1}(a)$

2) Replace a+c for a in Definition 2.1 (iii)

$t_n((a+c)b(a+c)) = \sum_{i=1}^{n} t_i(a+c)t_{i-1}(b)t_{i-1}(a+c) = \sum_{i=1}^{n} t_i(a)t_{i-1}(b)t_{i-1}(a) + t_i(a)t_{i-1}(b)t_{i-1}(c) + t_i(c)t_{i-1}(b)t_{i-1}(a) + t_{i-1}(c)t_{i-1}(b)t_{i-1}(c)$ …(1)

On the other hand

$t_n((a+c)b(a+c)) = t_n(aba+abc+cba+cbc) = \sum_{i=1}^{n} t_i(a)t_{i-1}(b)t_{i-1}(a) + t_{i-1}(c)t_{i-1}(b)t_{i-1}(c) + t_n(abc+cba)$ …(2)

Comparing (1) and (2) we get

$t_n(abc + cba) = \sum_{i=1}^{n} t_i(a)t_{i-1}(b)t_{i-1}(c) + t_i(c)t_{i-1}(b)t_{i-1}(a)$

3) Since R is commutative and from (2) we get

$2\, t_n(abc) = 2\sum_{i=1}^{n} t_i(a)t_{i-1}(b)t_{i-1}(c)$

Since R is 2-torsion free we get the require result.

Definition 2.2: Let T=(t_i) $_{i\in N}$ be a family of higher Jordan left centralizer of a ring R and n$\in$N

$\delta_n(a,b) = t_n - \sum_{i=1}^{n} t_i(a)t_{i-1}(b)$

Lemma 2: Let T=(t_i) $_{i\in N}$ be a family of higher Jordan left centralizer of a ring R then for all a,b,c $\in$ R and n$\in$N

1) $\delta_n(a+b,c) = \delta_n(a,c) + \delta_n(b,c)$
2) $\delta_n(a,b+c) = \delta_n(a,b) + \delta_n(b,c)$
3) $\delta_n(a,b) = -\delta_n(b,a)$

Proof :

1) $\delta_n(a+b,c) = t_n((a+b)c) - \sum_{i=1}^{n} t_i(a+b)t_{i-1}(c)$

$= t_n(ac+bc) - \sum_{i=1}^{n} t_i(a)t_{i-1}(c) + t_i(b)t_{i-1}(c)$

$= t_n(ac) - \sum_{i=1}^{n} t_i(a)t_{i-1}(c) + t_n(bc) - \sum_{i=1}^{n} t_i(b)t_{i-1}(c)$

$= \delta_n(a,c) + \delta_n(b,c)$

2) As the same way of (1)

3) By Lemma 2.1 (i)

$t_n(ab+ba) = \sum_{i=1}^{n} t_i(a)t_{i-1}(b) + t_i(b)t_{i-1}(a)$

$t_n(ab) - \sum_{i=1}^{n} t_i(a)t_{i-1}(b) = -t_n(ba) + \sum_{i=1}^{n} t_i(b)t_{i-1}(a)$

$\delta_n(a,b) = -\delta_n(b,a)$

Remark 2.3: Not that T=(t_i) $_{i\in N}$ is higher left centralizer of a ring R, iff $\delta_n(a,b) = 0.$

Lemma 3: Let T=(t_i) $_{i\in N}$ be a family of higher Jordan left centralizer of 2-torsion free prime ring R then for all a,b$\in$ R and n$\in$N

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]$

Proof: we prove by induction on n$\in$N.
If n=1,
Let w= abmba + bamab
t(w) = t(a(bmb)a + b(ama)b)
= t(a)(bmb)a t(b)(ama)b …(1)

On the other hand
T(w) = t((ab)m(ba) + (ba)m(ab)
= t(ab)m(ba) + t(ba)m(ab) …(2)
Compare (1) and (2) we get
0= (t(ab) – t(a) b)m (ba) + (t(ba) – t(b)a) m (ab)

$= \delta(a,b)mba + \delta(b,a)mab$
$= \delta(a,b)mba - \delta(a,b)mab$
$= \delta(a,b)m(ba - ab)$
$= \delta(a,b)m[a,b]$ for all a,b,m $\in$ R

Then we can assume that

$\delta_s(a,b)t_{s-1}(m)[t_{s-1}(a),t_{s-1}(b)] = 0$ for all a,b,m $\in$ R and n,s $\in$ N, s < n.

Now,

$T_n(w) = t_n$ (a(bamb+ b(ama)b)
$= \sum_{i=1}^{n} t_i(a)t_{i-1}(bmb)t_{i-1}(a) + t_i(b)t_i(ama)t_{i-1}(b)$
$= \sum_{i=1}^{n} t_i(a)t_{i-1}(b)t_{i-1}(m)t_{i-1}(b)t_{i-1}(a) + t_i(b)t_i(a)t_{i-1}(m)t_{i-1}(a)t_{i-1}(b)$
$= (\sum_{i=1}^{n} t_i(a)t_{i-1}(b))t_{n-1}(m)t_{n-1}(b)t_{n-1}(a) + t_n(b)t_{n-1}(a)t_{n-1}(m)\sum_{i=1}^{n} t_{i-1}(a)t_{i-1}(b)$
$+(\sum_{i=1}^{n} t_i(b)t_{i-1}(a))t_n(m)t_{n-1}(a)t_{n-1}(b) + t_n(b)t_{n-1}(a)\sum_{i=1}^{n} t_{i-1}(a)t_{i-1}(b)$ …(1)

On the other hand

$t_n(w)= t_n($ (ab)m(ba) + (ba)m(ab))
$= \sum_{i=1}^{n} t_i(ab)t_{i-1}(m)t_{i-1}(ba) + t_i(ba)t_{i-1}(m)t_{i-1}(ab)$
$= t_n(ab)t_{n-1}(m)t_{n-1}(b)t_{n-1}(a) + \sum_{i=1}^{n-1} t_i(ab)t_{n-1}(m)t_{n-1}(b)t_{n-1}(a)$
$+ t_n(ba)t_{n-1}(m)t_{n-1}(ab) + \sum_{i=1}^{n-1} t_i(ba)t_{i-1}(m)t_{i-1}(a)t_{i-1}(b)$ …(2)

Compare (1) and (2) we get

$0= (t_n(ab) - \sum_{i=1}^{n} t_i(a)t_{i-1}(b))t_{n-1}(m)t_{n-1}(b)t_{n-1}(a) + (t_n(ba) - \sum_{i=1}^{n} t_i(b)t_{i-1}(a))t_{n-1}(m)t_{n-1}(a)t_{n-1}(b)$
$+t_n(a)t_{n-1}(b)\sum_{i=1}^{n-1} t_{i-1}(m)t_{i-1}(b)t_{i-1}(a) + t_n(b)t_n(a)\sum_{i=1}^{n-1} t_i(m)t_{i-1}(a)t_{i-1}(b) + \sum_{i=1}^{n-1} t_i(ab)t_{n-1}(m)t_{n-1}(b)t_{n-1}(a) + \sum_{i=1}^{n-1} t_i(ba)t_{i-1}(m)t_{i-1}(a)t_{i-1}(b)$

On our hypothesis, we have

$0= \delta_n(a,b)t_{n-1}(m)t_{n-1}(b)t_{n-1}(a) + \delta_n(b,a)t_{n-1}(m)t_{n-1}(a)t_{n-1}(b)$
$= \delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]$

Theorem 4: Let $T=(t_i)_{i \in N}$ be a Jordan higher left centralize of a prime ring R then for all a,b,c,d,m $\in$ R ,n $\in$ N

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(d)] = 0$

Proof: Replacing a+c for a in Lemma 3

$\delta_n(a+c,b)t_{n-1}(m)[t_{n-1}(a+c),t_{n-1}(b)] = 0$ $\delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]$ + $\delta_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)]$
+ $\delta_n(c,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]$ + $\delta_n(c,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)]$ =0

By Lemma2.3 we get

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)]$ + $\delta_n(c,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]$ =0

Therefore, we get

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)]\ t_{n-1}(m)\ \delta_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)]$ =0

$-\delta_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)]\ t_{n-1}(m)\ \delta_n(c,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]=0$

Hence by primeness of R

$$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)]=0 \qquad \ldots (1)$$

Now, replace b+d for b in Lemma 3, we get

$\delta_n(a,b+d)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b+d)]=0$

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)] + \delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)]$

$\delta_n(a,d)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)] + \delta_n(a,d)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)]=0$

By Lemma3 we get

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)] + \delta_n(a,d)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]=0$

Then we get

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)]\ t_{n-1}(m)\ \delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)]=0$

$-\delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)]\ t_{n-1}(m)\ \delta_n(a,d)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]=0$

Since R is prime then

$$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)]=0 \qquad \ldots (2)$$

Thus

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(a+c),t_{n-1}(b+d)]=0$

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)] + \delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)] +$

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)] + \delta_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(d)]=0$

By (1) and (2) and Lemma 3, we get

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(d)]=$

Theorem 5: Every Jordan higher left centralizer of 2-torsion free prime ring R is higher left centralizer of R.

Proof: Let T= $(t_i)_{i\in N}$ be Jordan higher left centralizer of prime ring R.

Since R is prime, we get from Theorem 4 , either $\delta_n(a,b)=0$ or $[t_{n-1}(c), t_{n-1}(d)]=0$ for all a,b,c,d $\in$ R and n $\in$ N. If $[t_{n-1}(c), t_{n-1}(d)]\neq 0$ for all c,d $\in$ R, n $\in$ N then $\delta_n(a,b)=0$ by Remark 2.3 we get T is higher left centralizer of R. If $[t_{n-1}(c), t_{n-1}(d)]=0$ for all c,d $\in$ R, n $\in$ N then R is commutative ring and by Lemma 1 we get

$$t_n(2ab)=2\sum_{i=1}^{n} t_i(a)t_{i-1}(b)$$

Since R is 2-torsion free, we obtain T is a higher left centralizer of R.

Proposition 6: Let T= $(t_i)_{i\in N}$ be Jordan higher left centralizer of 2-torsion free ring R, then T is Jordan triple higher left centralizer of R.

Proof : Replace b by ab + ba in Definition 2.1, then :

$$t_n(a(ab+ba)+(ab+ba)a)=\sum_{i=1}^{n} t_i(a)t_{i-1}(ab+ba)+t_i(ab+ba)t_{i-1}(a)$$

$$=\sum_{i=1}^{n} t_i(a)t_{i-1}(a)t_{i-1}(b)+t_i(a)t_{i-1}(b)t_{i-1}(a)+t_i(a)t_{i-1}(b)t_{i-1}(a)+t_i(b)t_{i-1}(a)t_{i-1}(a)$$

$$= \sum_{i=1}^{n} t_i(a)t_{i-1}(a)t_{i-1}(b) + t_{i-1}(b)t_{i-1}(a)t_{i-1}(a) + 2\sum_{i=1}^{n} t_i(a)t_{i-1}(b)t_{i-1}(a) \qquad \ldots(1)$$

On the other hand :

$t_n(a(ab+ba) + (ab+ba)a) = t_n(aab + aba + aba+ baa)$
$= t_n (aab + baa) + 2t_n(aba)$

$$= \sum_{i=1}^{n} t_i(a)t_{i-1}(a)t_{i-1}(b) + t_i(b)t_{i-1}(a)t_{i-1}(a) + 2t_n(aba) \qquad \ldots(2)$$

Now, compare (1) and (2) we get

$$2t_n(aba) = 2\sum_{i=1}^{n} t_i(a)t_{i-1}(b)t_{i-1}(a)$$

Since R is 2-torsion free we obtain that T is Jordan triple higher left centralizer of R.

3) Generalized higher left centralizer of rings

In this section we present the concepts of generalized higher left centralizer and generalized Jordan higher left centralizer of rings also we present some properties of them.

Definition 3.1: Let F= $(f_i)_{i\in N}$ be a family of additive mappings of a ring R into itself. F is called generalized higher left centralizer of R if there exists a higher left centralizer T=$(t_i)_{i\in N}$ of R such that for every n$\in$N we have

$$f_n(ab) = \sum_{i=1}^{n} f_i(a)t_{i-1}(b) \qquad \ldots(1)$$

for all a,b$\in$R where T is called the relating higher left centralizer.

F is called Jordan generalized higher left centralizer of R if there exists a Jordan higher left centralizer of R if for every n$\in$N

$$f_n(a^2) = \sum_{i=1}^{n} f_i(a)t_{i-1}(a) \qquad \ldots(2)$$

for all a$\in$R where T is called the relating Jordan left centralizer.

F is called Jordan generalized triple higher left centralizer of R if there exists a Jordan triple higher left centralizer of R if for every n$\in$N

$$f_n(aba) = \sum_{i=1}^{n} f_i(a)t_{i-1}(b)t_{i-1}(a) \qquad \ldots(3)$$

for all a,b$\in$R where T is called the relating Jordan triple left centralizer,

Lemma 7: Let F= $(f_i)_{i\in N}$ be Jordan generalized higher left centralizer of a ring R into itself then for all a,b,c$\in$R , n$\in$N

1) $f_n(ab+ba) = \sum_{i=1}^{n} f_i(a)t_{i-1}(b) + f_i(b)t_{i-1}(a)$

2) $f_n(abc + cba) = \sum_{i=1}^{n} f_i(a)t_{i-1}(b)t_{i-1}(c) + f_i(c)t_{i-1}(b)t_{i-1}(a)$

In particular if R is 2-tortion free commutative ring

3) $f_n(abc) = \sum_{i=1}^{n} f_i(a)t_{i-1}(b)t_{i-1}(c)$

Proof: 1) $f_n((a+b)(a+b)) = \sum_{i=1}^{n} f_i(a+b)t_{i-1}(a+b)$

$$= \sum_{i=1}^{n} f_i(a)t_{i-1}(a) + f_i(a)t_{i-1}(b) + f_i(b)t_{i-1}(a) + f_i(b)t_{i-1}(b) \qquad \ldots(1)$$

On the other hand

$f_n((a+b)(a+b)) = f_n(a^2 + ab + ba + b^2)$

$= \sum_{i=1}^{n} f_i(a)t_{i-1}(a) + f_i(b)t_{i-1}(b) + f_n(ab + ba)$ …(2)

Comparing (1) and (2) we get

$f_n(ab+ba) = \sum_{i=1}^{n} f_i(a)t_{i-1}(b) + f_i(b)t_{i-1}(a)$

2) Replace a+c for a in Definition 2.1 (iii)

$f_n((a+c)b(a+c)) = \sum_{i=1}^{n} f_i(a+c)t_{i-1}(b)t_{i-1}(a+c)$

$= \sum_{i=1}^{n} f_i(a)t_{i-1}(b)t_{i-1}(a) + f_i(a)t_{i-1}(b)t_{i-1}(c) + f_i(c)t_{i-1}(b)t_{i-1}(a) + f_{i-1}(c)t_{i-1}(b)t_{i-1}(c)$ …(1)

On the other hand

$f_n((a+c)b(a+c)) = f_n(aba + abc + cba + cbc)$

$= \sum_{i=1}^{n} f_i(a)t_{i-1}(b)t_{i-1}(a) + ft_{i-1}(c)t_{i-1}(b)t_{i-1}(c) + f_n(abc + cba)$ …(2)

Comparing (1) and (2) we get

$f_n(abc + cba) = \sum_{i=1}^{n} f_i(a)t_{i-1}(b)t_{i-1}(c) + f_i(c)t_{i-1}(b)t_{i-1}(a)$

3) Since R is 2-torsion free commutative ring and from (2) we get the require result.

Definition 3.2: Let $F=(f_i)_{i\in N}$ be a family of Jordan generalized higher left centralizer of a ring R with relating Jordan higher left centralizer $T=(t_i)_{i\in N}$ of R , then for all $a,b\in R$ and $n\in N$,we define

$$\Phi_n(a,b) = f_n(ab) - \sum_{i=1}^{n} f_i(a)t_{i-1}(b)$$

Lemma 8: Let $F=(f_i)_{i\in N}$ be a family of higher Jordan left centralizer of a ring R with relating Jordan higher left centralizer $T=(t_i)_{i\in N}$ of R then for all $a,b,c \in R$ and $n\in N$

1) $\Phi_n(a+b,c) = \Phi_n(a,c) + \Phi_n(b,c)$
2) $\Phi_n(a,b+c) = \Phi_n(a,b) + \Phi_n(b,c)$
3) $\Phi_n(a,b) = -\Phi_n(b,a)$

Proof :

1) $\Phi_n(a+b,c) = f_n((a+b)c) - \sum_{i=1}^{n} f_i(a+b)t_{i-1}(c)$

$= f_n(ac+bc) - \sum_{i=1}^{n} f_i(a)t_{i-1}(c) + f_i(b)t_{i-1}(c)$

$= f_n(ac) - \sum_{i=1}^{n} f_i(a)t_{i-1}(c) + f_n(bc) - \sum_{i=1}^{n} f_i(b)t_{i-1}(c)$

$= \Phi_n(a,c) + \Phi_n(b,c)$

2) As the same way of (1)
3) By Lemma 7 (i)

$f_n(ab+ba) = \sum_{i=1}^{n} f_i(a)t_{i-1}(b) + f_i(b)t_{i-1}(a)$

$f_n(ab) - \sum_{i=1}^{n} f_i(a)t_{i-1}(b) = -f_n(ba) + \sum_{i=1}^{n} f_i(b)t_{i-1}(a)$

$\Phi_n(a,b) = -\Phi_n(b,a)$

Remark 3.3: Not that $F=(f_i)_{i\in N}$ is generalized higher left centralizer of a ring R with relating Jordan higher left centralizer $T=(t_i)_{i\in N}$ of R , iff $\Phi_n(a,b)=0.$

Lemma 9: Let $F=(f_i)_{i\in N}$ be a family of Jordan generalized higher left centralizer of 2-torsion free prime ring R then for all a,b $\in$ R and n$\in$N

$$\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]=0$$

Proof: we prove by induction on n$\in$N.
If n=1,
Let w= abmba + bamab
f(w) = f(a(bmb)a + b(ama)b)
 = f(a)(bmb)a+ t(b)(ama)b …(1)
On the other hand
f(w) = f((ab)m(ba) + (ba)m(ab)
 =f(ab)m(ba) + f(ba)m(ab) …(2)
Compare (1) and (2) we get
0= (f(ab) –f(a) b)m (ba) + (f(ba) – f(b)a) m (ab)

$= \Phi(a,b)mba+\Phi(b,a)mab$

$= \Phi(a,b)mba-\Phi(a,b)mab$

$= \Phi(a,b)m(ba-ab)$

$= \Phi(a,b)m[a,b]$ for all a,b,m$\in$R

Then we can assume that

$\Phi_s(a,b)t_{s-1}(m)[t_{s-1}(a),t_{s-1}(b)]=0$ for all a,b,m $\in$R and n,s$\in$N, s < n.

Now,

$f_n(w) = f_n$ (a(bamb+ b(ama)b)

$$= \sum_{i=1}^{n} f_i(a)t_{i-1}(bmb)t_{i-1}(a)+f_i(b)t_i(ama)t_{i-1}(b)$$

$$= \sum_{i=1}^{n} f_i(a)t_{i-1}(b)t_{i-1}(m)t_{i-1}(b)t_{i-1}(a)+f_i(b)t_i(a)t_{i-1}(m)t_{i-1}(a)t_{i-1}(b)$$

$$= (\sum_{i=1}^{n} f_i(a)t_{i-1}(b))t_{n-1}(m)t_{n-1}(b)t_{n-1}(a)+f_n(b)t_{n-1}(a)t_{n-1}(m)\sum_{i=1}^{n} t_{i-1}(a)t_{i-1}(b) +$$

$$(\sum_{i=1}^{n} f_i(b)t_{i-1}(a))t_n(m)t_{n-1}(a)t_{n-1}(b)+f_n(b)t_{n-1}(a)\sum_{i=1}^{n} t_{i-1}(a)t_{i-1}(b) \quad …(1)$$

On the other hand
$f_n(w)= f_n$((ab)m(ba) + (ba)m(ab))

$$= \sum_{i=1}^{n} f_i(ab)t_{i-1}(m)t_{i-1}(ba)+f_i(ba)t_{i-1}(m)t_{i-1}(ab)$$

$$= f_n(ab)t_{n-1}(m)t_{n-1}(b)t_{n-1}(a)+\sum_{i=1}^{n-1} f_i(ab)t_{n-1}(m)t_{n-1}(b)t_{n-1}(a)$$

$$+ f_n(ba)t_{n-1}(m)t_{n-1}(ab)+\sum_{i=1}^{n-1} f_i(ba)t_{i-1}(m)t_{i-1}(a)t_{i-1}(b) \quad …(2)$$

Compare (1) and (2) we get 0= $f_n(ab)-\sum_{i=1}^{n} t_i(a)t_{i-1}(b))t_{n-1}(m)t_{n-1}(b)t_{n-1}(a)+(f_n(ba)-\sum_{i=1}^{n} t_i(b)t_{i-1}(a))t_{n-1}(m)t_{n-1}(a)t_{n-1}(b)$

$+ f_n(a)t_{n-1}(b)\sum_{i=1}^{n-1} t_{i-1}(m)t_{i-1}(b)t_{i-1}(a)+f_n(b)t_n(a)\sum_{i=1}^{n-1} t_i(m)t_{i-1}(a)t_{i-1}(b) + \sum_{i=1}^{n-1} f_i(ab)t_{n-1}(m)t_{n-1}(b)t_{n-1}(a)+\sum_{i=1}^{n-1} f_i(ba)t_{i-1}(m)t_{i-1}(a)t_{i-1}(b)$

On our hypothesis, we have

0= $\Phi_n(a,b)t_{n-1}(m)t_{n-1}(b)t_{n-1}(a)+\Phi_n(b,a)t_{n-1}(m)t_{n-1}(a)t_{n-1}(b)$

$= \Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]$

Theorem 10: Let $F=(f_i)_{i\in N}$ be a Jordan generalized higher left centralize of a prime ring R then for all a,b,c,d,m $\in$ R ,n$\in$N

$\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(d)]=0$

Proof: Replacing a+c for a in Lemma 9

$\Phi_n(a+c,b)t_{n-1}(m)[t_{n-1}(a+c),t_{n-1}(b)]=0$

$\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)] + \Phi_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)] + \Phi_n(c,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)] + \Phi_n(c,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)] =0$

By Lemma 9 we get

$\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)] + \Phi_n(c,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)] =0$

Therefore, we get

$\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)]\, t_{n-1}(m)\, \Phi_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)] =0$

$-\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)]\, t_{n-1}(m)\, \Phi_n(c,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)] =0$

Hence by primeness of R

$\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)] =0$ … (1)

Now, replace b+d for b in Lemma 9, we get

$\Phi_n(a,b+d)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b+d)]=0$

$\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)] + \Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)]$

$\Phi_n(a,d)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)] + \Phi_n(a,d)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)] =0$

By Lemma 9 we get

$\delta_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)] + \Phi_n(a,d)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)]=0$

Then we get

$\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)]\, t_{n-1}(m)\, \Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)] =0$

$-\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)]\, t_{n-1}(m)\, \Phi_n(a,d)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)] =0$

Since R is prime then

$\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)] =0$ …(2)

Thus

$\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a+c),t_{n-1}(b+d)] =0$

$\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(b)] + \Phi_n(a,b)t_{n-1}(m)[t_{n-1}(a),t_{n-1}(d)] +$

$\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(b)] + \Phi_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(d)] =0$

By (1) and (2) and Lemma 2.3, we get $\Phi_n(a,b)t_{n-1}(m)[t_{n-1}(c),t_{n-1}(d)]=$

Theorem 11: Every Jordan generalized higher left centralizer of 2-torsion free prime ring R is generalized higher left centralizer of R.

Proof: Let F= $(f_i)_{i\in N}$ be Jordan generalized higher left centralizer of prime ring R.

Since R is prime, we get from Theorem 10 , either $\Phi_n(a,b)=0$ or $[t_{n-1}(c), t_{n-1}(d)]=0$ for all a,b,c,d$\in$R and n$\in$N.

If $[t_{n-1}(c), t_{n-1}(d)]\neq 0$ for all c,d$\in$R, n$\in$N then $\Phi_n(a,b)=0$ by Remark 2.3 we get T is higher left centralizer of R.

If $[t_{n-1}(c), t_{n-1}(d)]=0$ for all c,d$\in$R, n$\in$N then R is commutative ring and by Lemma 7 we get

$$t_n(2ab) = 2\sum_{i=1}^{n} t_i(a)t_{i-1}(b)$$

Since R is 2-torsion free, we obtain T is a higher left centralizer of R.

Proposition 12: Let F= (f_i) $_{i \in N}$ be Jordan generalized higher left centralizer of 2-torsion free ring R, then T is Jordan generalized triple higher left centralizer of R.
Proof : Replace b by ab + ba in Definition 3.1, then :

f_n(a(ab+ba) + (ab+ba)a) = $\sum_{i=1}^{n} f_i(a)t_{i-1}(ab+ba) + f_i(ab+ba)t_{i-1}(a)$ $=\sum_{i=1}^{n} f_i(a)t_{i-1}(a)t_{i-1}(b) + f_i(a)t_{i-1}(b)t_{i-1}(a) + f_i(a)t_{i-1}(b)t_{i-1}(a) + f_i(b)t_{i-1}(a)t_{i-1}(a)$

$$= \sum_{i=1}^{n} f_i(a)t_{i-1}(a)t_{i-1}(b) + f_i(b)t_{i-1}(a)t_{i-1}(a) + 2\sum_{i=1}^{n} t_i(a)t_{i-1}(b)t_{i-1}(a) \quad \ldots(1)$$

On the other hand :

f_n(a(ab+ba) + (ab+ba)a) = f_n(aab + aba + aba+ baa)
= f_n (aab + baa) + 2f_n(aba)

$$= \sum_{i=1}^{n} f_i(a)t_{i-1}(a)t_{i-1}(b) + f_i(b)t_{i-1}(a)t_{i-1}(a) + 2f_n(aba) \quad \ldots(2)$$

Now, compare (1) and (2) we get

$$2f_n(aba) = 2\sum_{i=1}^{n} f_i(a)t_{i-1}(b)t_{i-1}(a)$$

Since R is 2-torsion free we obtain that F is Jordan generalized triple higher left centralizer of R.

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