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RESEARCH ARTICLE

LYAPUNOV EXPONENTS AND PREDICTABILITY OF CYCLONIC DISTURBANCES OVER THE NORTH INDIAN OCEAN

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ABSTRACT

The annual frequency of tropical cyclonic disturbances ($\leq$ 63 knots) and the annual frequency of cyclonic and severe cyclonic storms (34 to 63 knots) over the north Indian Ocean (comprising Bay of Bengal and Arabian Sea) is analyzed using the Fractal Construction Technique. Recognizing and quantifying chaos in time series represents an important step towards understanding not only the natural behavior of the system but also for the improvement of short term forecasts. An attempt is being made to find the fractal geometry of climate and to predict its periodicity on different temporal scales. Lyapunov exponent, Maximum Lyapunov characteristic exponent and Lyapunov time for the cyclonic disturbances including depression, cyclonic storm and severe cyclonic storm over the north Indian Ocean has been estimated.

INTRODUCTION

Tropical cyclones are the violent tropical maritime storms, often of an approximately circular shape when fully developed, with low central sea-level pressures, and wind speeds in excess of 34 knots. McBride (1995) stated that about 80 tropical cyclones with wind speeds equal to or greater than 34 knots form in the world's waters every year. The number of storms which form over the north Indian Ocean (comprising Bay of Bengal and Arabian Sea) is about 6.5% of the total number over the globe (Neumann, 1993). Tropical cyclones are regions of intensely low pressure but they are very different in character and intensity from mid-latitude cyclones or depressions. They are strictly oceanic phenomena, and tend to die out over land. A global observational study of atmospheric conditions associated with tropical disturbance and storm development is presented (Gray, 1968).

Non linear dynamical systems which often exhibit chaos are sensitive dependence on primary conditions. Lyapunov exponent is one of the tools which had been used to determine this sensitivity in a quantitative way. Wolf *et al*., (1985), Eckmann *et al*., (1986) and Rosenstein *et al*., (1993) proposed the methodology to estimate the largest Lyapunov exponent from an observed single time series data. Zeng *et al*., (1991) estimated the Lyapunov exponents for short noisy time series. The largest Lyapunov exponent is referred as the Maximal Lyapunov exponent (MLE) from which the notion of predictability for any dynamical system is always determined. A positive value of Maximal Lyapunov exponent is an indication that the system is chaotic. This paper analyses the fractal behavior of annual frequency of cyclonic disturbances ($\leq$ 63 knots) and the annual frequency of cyclonic and severe cyclonic storms (34 to 63 knots) over the north Indian Ocean for a period of 104 years. Various vital parameters like Lyapunov exponent, Maximum Lyapunov characteristic exponent and Lyapunov time are estimated and thereby the system is studied.

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MATERIALS AND METHODS

Cyclonic disturbances data over the north Indian Ocean are obtained from IMD Cyclone E- Atlas. The difficulty in obtaining a significant level of skill suggests the possibility that cyclones are highly irregular dynamic systems with sensitive dependence on initial conditions. Fraedrich and Leslie (1989) applied a recent technique of chaos theory to tropical cyclones in the Australian region and also measured the predictability of such systems. We focused on the statistical problem of quantifying chaos and non linearity via Lyapunov exponents. Chaos denotes a state of disorder. Chaos has inescapably become the part of modern science. According to Lorentz (1963), lack of periodicity is quiet common in natural systems and also any slight change of initial conditions might randomly bring a huge change in the future climate. Consider two points in a space, X_0 and $X_0 + \Delta x_0$, each of which will generate an orbit in that space using some equation or system of equations (fig. 1). These orbits can be thought of as parametric functions of a variable that is something like time. If we use one of the orbits a reference orbit, then the separation between the two orbits will also be a function of time. Because sensitive dependence can arise only in some portions of a system (like the logistic equation), this separation is also a function of the location of the initial value and has the form $\Delta x(X_0, t)$ (Glenn Elert, 1995). In a system with attracting fixed points or attracting periodic points, $\Delta x(X_0, t)$ diminishes asymptotically with time. If a system is unstable, like pins balanced on their points, then the orbits diverge exponentially for a while, but eventually settle down. For chaotic points, the function $\Delta x(X_0, t)$ will behave erratically. It is thus useful to study the mean exponential rate of divergence of two initially close orbits using the formula

$$\lambda = \lim_{\substack{t\to\infty \\ |\Delta x_0|\to 0}} (1/t)\log_2 \frac{|\Delta x\,(X_0,\, t)|}{|\Delta x_0|}$$

The number λ (lambda) is called the Lyapunov exponent value which explains that on average how fast the predictability in system is being missed. The sum of all the Lyapunov exponents gives an estimate of the rate of the disorder production τ (k). Fig.1

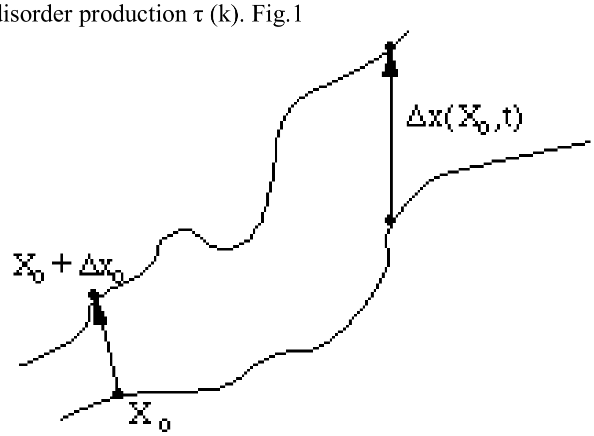

Fig. 1: Measuring chaos

RESULTS AND DISCUSSION

The Lyapunov exponents for a period from 1901 to 2004 are calculated for the annual frequency of cyclonic disturbances and cyclonic and severe cyclonic storms data over the north Indian Ocean separately. An understanding of the total spectrum of Lyapunov exponents will give a measure of the total error growth in all possible modes. For detecting chaos, estimating the dominant Lyapunov exponent is most important. The local Lyapunov exponents, which are defined as the finite time average divergence rates, may provide more relevant measure of predictability.

(i) $\lambda < 0$: The negative value of λ shows a constant point or a stable periodic cycle. These systems exhibit asymptotic stability.

(ii) $\lambda = 0$: The system only oscillates around a constant point. In this state, each chosen primary point oscillates around a stable limited cycle. This system is called Lyapunov stability.

(iii) $\lambda > 0$: The positive sign of Lyapunov exponent in a dynamic bounded system predicts the occurrence of chaos. There is no constant point or periodic stable cycle in fact the points are unstable. The system is bounded and chaotic.

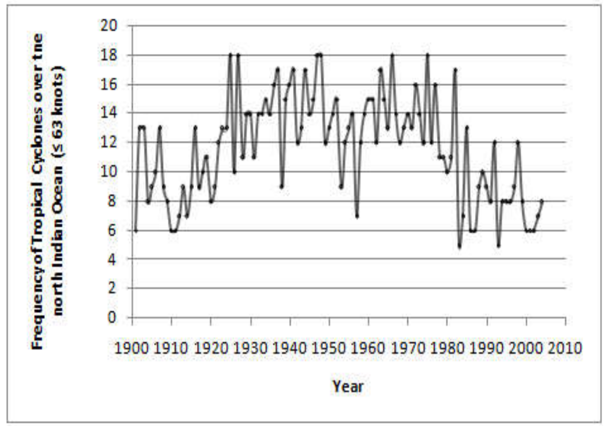

Fig. 2: Plot of the north Indian Ocean annual cyclonic disturbances

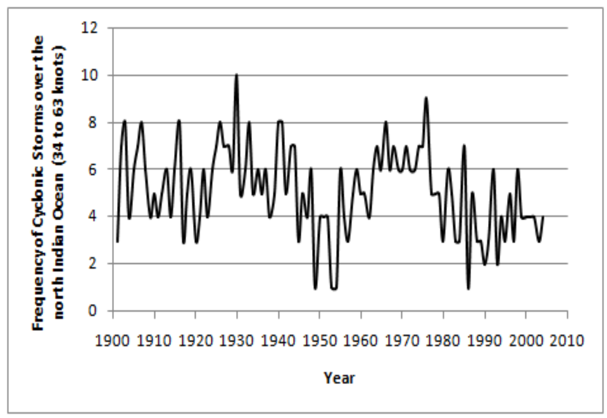

Fig. 3: Plot of the north Indian Ocean annual cyclonic and severe cyclonic storms.

Figure 2 & 3 illustrates the trend in the annual frequencies of cyclonic disturbances and cyclonic storms over the north Indian Ocean. Table 1 illustrates the values of the Lyapunov exponent of cyclonic disturbances and cyclonic storms over the north Indian Ocean for a period of 104 years. From the table it is inferred that the Lyapunov exponent values for both these cases (almost all the values of λ) are negative and one or two with a positive values which are too small. The maximum value of Lyapunov exponent for the above system is 0.2. Neumann (1993), Lander and Guard (1998) pointed out that there is no appreciable long-term variation in the frequency of the tropical cyclones observed in the north Indian, South-west Indian and south-west Pacific Oceans about 160°E. The Lyapunov time is defined as the inverse of the maximum Lyapunov exponent. The Lyapunov time for the above system is 5 that indicate the time after which the system starts to behave in a chaotic manner. Thorough analysis shows the stable periodic cycle of cyclonic disturbances over the north Indian Ocean with a very low dimensional chaos.

Table 1: Lyapunov exponent of frequency of cyclonic disturbances and frequency of cyclonic storms over the north Indian Ocean for the years 1901-2004

S.No.	TC's (≤ 63 knots)	error	λ	τ (k)	TC's (34 to 63 knots)	error	λ	τ (k)
1	6	-5.64423	-2.05766	-2.05766	3	-2.13462	-0.1945	-0.1945
2	13	1.35577	-1.02883	-3.08649	7	1.865385	0.212378	0.017878
3	13	1.35577	-0.21039	-3.29688	8	2.865385	-0.30392	-0.28605
4	8	-3.64423	-0.27348	-3.57036	4	-1.13462	-0.32564	-0.61169
5	9	-2.64423	-0.35587	-3.92623	6	0.865385	-0.0389	-0.65059
6	10	-1.64423	-0.34294	-4.26918	7	1.865385	0.070793	-0.57979
7	13	1.35577	-0.15628	-4.42545	8	2.865385	-0.18608	-0.76587
8	9	-2.64423	-0.0789	-4.50435	6	0.865385	-0.11397	-0.87984
9	8	-3.64423	1.11E-16	-4.50435	4	-1.13462	-0.44301	-1.32285
10	6	-5.64423	9.99E-17	-4.50435	5	-0.13462	-0.09118	-1.41403
11	6	-5.64423	-0.02558	-4.52993	4	-1.13462	-0.36246	-1.77649
12	7	-4.64423	-0.09116	-4.62109	5	-0.13462	-0.10855	-1.88504
13	9	-2.64423	-0.02164	-4.64273	6	0.865385	-0.07014	-1.95517
14	7	-4.64423	-0.07814	-4.72087	4	-1.13462	-0.09304	-2.04821
15	9	-2.64423	-0.13718	-4.85804	6	0.865385	0.028317	-2.01989
16	13	1.35577	-0.06837	-4.92641	8	2.865385	1.54E-07	-2.01989
17	9	-2.64423	-0.10467	-5.03108	3	-2.13462	-0.23453	-2.25443
18	10	-1.64423	-0.17395	-5.20503	5	-0.13462	-0.07236	-2.32679
19	11	-0.64423	-0.03322	-5.23825	6	0.865385	1.30E-07	-2.32679
20	8	-3.64423	-0.0547	-5.29295	3	-2.13462	-0.04559	-2.37238
21	9	-2.64423	-0.18989	-5.48284	4	-1.13462	-0.06203	-2.43441
22	12	0.35577	-0.09353	-5.57637	6	0.865385	-0.04144	-2.47585
23	13	1.35577	-0.08946	-5.66584	4	-1.13462	-0.05663	-2.53248
24	13	1.35577	0.007137	-5.6587	6	0.865385	-0.0081	-2.54059
25	18	6.35577	-0.07117	-5.72987	7	1.865385	0.01699	-2.5236
26	10	-1.64423	0.006588	-5.72329	8	2.865385	-0.00748	-2.53108
27	18	6.35577	-0.11597	-5.83925	7	1.865385	-0.0072	-2.53828
28	11	-0.64423	-0.04502	-5.88427	7	1.865385	-0.04652	-2.5848
29	14	2.35577	-0.04347	-5.92774	6	0.865385	0.040986	-2.54382
30	14	2.35577	-0.10437	-6.03211	10	4.865385	-0.1329	-2.67672
31	11	-0.64423	-0.04066	-6.07278	5	-0.13462	-0.04202	-2.71874
32	14	2.35577	-0.03939	-6.11217	6	0.865385	0.013274	-2.70546
33	14	2.35577	-0.02273	-6.1349	8	2.865385	-0.12082	-2.82628
34	15	3.35577	-0.03708	-6.17198	5	-0.13462	-0.03831	-2.86459
35	14	2.35577	-0.01068	-6.18266	6	0.865385	-0.11392	-2.97851
36	16	4.35577	-0.0021	-6.18476	5	-0.13462	-0.03618	-3.01469
37	17	5.35577	-0.02957	-6.21433	6	0.865385	-0.02464	-3.03933
38	9	-2.64423	-0.01974	-6.23407	4	-1.13462	-0.10492	-3.14426
39	15	3.35577	-0.00959	-6.24365	5	-0.13462	0.010891	-3.13337
40	16	4.35577	-0.00189	-6.24555	8	2.865385	0.010619	-3.12275
41	17	5.35577	-0.09726	-6.34281	8	2.865385	-0.09725	-3.21999
42	12	0.35577	-0.04899	-6.3918	5	-0.13462	-0.00463	-3.22462
43	13	1.35577	-0.00176	-6.39356	7	1.865385	-0.00452	-3.22915
44	17	5.35577	-0.02865	-6.42221	7	1.865385	5.61E-08	-3.22915
45	14	2.35577	-0.01667	-6.43888	3	-2.13462	-0.0886	-3.31775
46	15	3.35577	0.003724	-6.43516	5	-0.13462	-0.01982	-3.33757
47	18	6.35577	0.003644	-6.43151	4	-1.13462	-0.02771	-3.36528
48	18	6.35577	-0.08308	-6.51459	6	0.865385	0.01987	-3.34541
49	12	0.35577	-0.04199	-6.55658	1	-4.13462	-0.01861	-3.36402
50	13	1.35577	-0.02521	-6.58179	4	-1.13462	-0.01824	-3.38226
51	14	2.35577	-0.01471	-6.5965	4	-1.13462	-0.01788	-3.40013
52	15	3.35577	-0.02104	-6.61754	4	-1.13462	0.018342	-3.38179
53	9	-2.64423	-0.07524	-6.69278	1	-4.13462	0.017996	-3.3638
54	12	0.35577	-0.0381	-6.73089	1	-4.13462	-0.02412	-3.38792
55	13	1.35577	-0.02292	-6.7538	6	0.865385	-0.01658	-3.40449
56	14	2.35577	-0.00502	-6.75883	4	-1.13462	4.40E-08	-3.40449
57	7	-4.64423	-0.06996	-6.82879	3	-2.13462	-0.06995	-3.47444
58	12	0.35577	-0.02173	-6.85052	5	-0.13462	-0.02246	-3.4969
59	14	2.35577	-0.01271	-6.86324	6	0.865385	-0.06758	-3.56448
60	15	3.35577	-0.0125	-6.87574	5	-0.13462	-0.06645	-3.63093
61	15	3.35577	-0.06537	-6.94111	5	-0.13462	-0.01495	-3.64588
62	12	0.35577	-0.00122	-6.94233	4	-1.13462	-0.02101	-3.66689
63	17	5.35577	-0.01191	-6.95424	6	0.865385	-0.00309	-3.66997
64	15	3.35577	-0.03215	-6.98639	7	1.865385	-0.02035	-3.69033
65	13	1.35577	0.002635	-6.98376	6	0.865385	0.006535	-3.68379
66	18	6.35577	-0.0191	-7.00286	8	2.865385	-0.01974	-3.70353
67	14	2.35577	-0.05952	-7.06237	6	0.865385	-0.0029	-3.70643
68	12	0.35577	-0.03026	-7.09263	7	1.865385	-0.01916	-3.72558
69	13	1.35577	-0.01827	-7.1109	6	0.865385	-0.01888	-3.74446
70	14	2.35577	-0.0294	-7.1403	6	0.865385	-0.00278	-3.74724
71	13	1.35577	-0.00527	-7.14556	7	1.865385	-0.01835	-3.76559
72	16	4.35577	-0.01751	-7.16307	6	0.865385	-0.01809	-3.78368
73	14	2.35577	-0.05463	-7.2177	6	0.865385	-0.00266	-3.78634
74	12	0.35577	0.002315	-7.21538	7	1.865385	-0.00263	-3.78897
75	18	6.35577	-0.05317	-7.26855	7	1.865385	0.011422	-3.77755

76	12	0.35577	-0.00492	-7.27347	9	3.865385	-0.05246	-3.83001
77	16	4.35577	-0.04066	-7.31414	5	-0.13462	-0.05178	-3.88179
78	11	-0.64423	-0.04014	-7.35428	5	-0.13462	-0.05112	-3.93291
79	11	-0.64423	-0.02252	-7.3768	5	-0.13462	3.12E-08	-3.93291
80	10	-1.64423	-0.03914	-7.41594	3	-2.13462	-0.01628	-3.94919
81	11	-0.64423	-0.00093	-7.41688	6	0.865385	-0.04922	-3.99841
82	17	5.35577	0.00287	-7.41401	5	-0.13462	3.01E-08	-3.99841
83	5	-6.64423	-0.00339	-7.4174	3	-2.13462	2.97E-08	-3.99841
84	7	-4.64423	-0.0245	-7.44189	3	-2.13462	-0.00232	-4.00073
85	13	1.35577	0	-7.44189	7	1.865385	0.011221	-3.98951
86	6	-5.64423	0	-7.44189	1	-4.13462	-0.04636	-4.03587
87	6	-5.64423	-0.01257	-7.45447	5	-0.13462	2.84E-08	-4.03587
88	9	-2.64423	-0.02022	-7.47469	3	-2.13462	2.80E-08	-4.03587
89	10	-1.64423	-0.01229	-7.48698	3	-2.13462	0.006228	-4.02964
90	9	-2.64423	-0.00701	-7.49399	2	-3.13462	2.74E-08	-4.02964
91	8	-3.64423	-0.04382	-7.53781	3	-2.13462	-0.01431	-4.04395
92	12	0.35577	0.002558	-7.53526	6	0.865385	0.006025	-4.03793
93	5	-6.64423	-0.00679	-7.54204	2	-3.13462	-0.0098	-4.04773
94	8	-3.64423	-0.00671	-7.54876	4	-1.13462	2.62E-08	-4.04773
95	8	-3.64423	-0.00664	-7.5554	3	-2.13462	-0.04197	-4.0897
96	8	-3.64423	-0.0114	-7.5668	5	-0.13462	2.57E-08	-4.0897
97	9	-2.64423	-0.04111	-7.60791	3	-2.13462	-0.01343	-4.10313
98	12	0.35577	-0.00644	-7.61435	6	0.865385	-0.0093	-4.11243
99	8	-3.64423	0	-7.61435	4	-1.13462	-0.00921	-4.12164
100	6	-5.64423	0	-7.61435	4	-1.13462	-0.00912	-4.13076
101	6	-5.64423	0	-7.61435	4	-1.13462	-0.00903	-4.13979
102	6	-5.64423	-0.00276	-7.61711	4	-1.13462	2.42E-08	-4.13979
103	7	-4.64423	-0.00613	-7.62323	3	-2.13462	-0.00885	-4.14864
104	8	-3.64423			4	-1.13462		

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