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RESEARCH ARTICLE

ARC-SEQUENCE IN COMPLETE AND REGULAR FUZZY GRAPHS

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ABSTRACT

In a fuzzy graph, the arcs are mainly classified in to α, β and δ. In this paper, some arc sequences in fuzzy graphs are introduced, whose concept are based on the classification of arcs. Besides complete in fuzzy graphs, regular in fuzzy graph are obtained. It is shown that α -arc sequence of a complete is a zero-one sequence, δ-arc sequence of a regular fuzzy graph is a zero sequence.

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INTRODUCTION

Graph theory has now become a major branch of applied mathematics due to its large variety of applications and effectiveness. Graph theory is a widely used tool for solving combinatorial problems in different areas such as geometry, algebra, number theory, topology, optimization and computer science. In models, when we have an uncertainty about either the set of vertices or the set of edges or both, the models becomes a fuzzy graph. Currently, the theory of fuzzy graphs is an intense area of research. Fuzzy graphs differ from the classical ones in several ways, among them the most prominent one is connectivity. Distance and central concepts also play important roles in applications related with fuzzy graphs. Rosenfeld (1975) gave a mathematical definition for a fuzzy graph in 1975. Bhattacharya (1987) had established some connectivity concepts regarding fuzzy cutnodes and fuzzy bridges. Bhutani (1989) had studied automorphisms on fuzzy graphs and certain properties of complete fuzzy graphs. Pathinathan and Jesintha Rosline (2014) defined relationship between different types of arcs in both regular and totally regular fuzzy graph. Sunil Mathew and Sunitha (Bhutani, 2003; Mathew, 2009; Sunitha, 1999; Sunitha, 2002; Sunitha, 2005) introduced many connectivity concepts in fuzzy graphs. Kalaiarasi (2011) defined Optimization of fuzzy integrated vendor-buyer inventory models.

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In this article, the concept of arc sequence in fuzzy graphs are discussed. These concepts are derived from the notion of connectivity in fuzzy graphs. Also a comparative study is made between regular and totally regular fuzzy graphs with reference to different types of arc sequence in fuzzy graphs. Also a necessary condition for a graph to be regular or totally regular is formulated in terms of arc-sequence

2. Preliminaries

Definition 2.1

A fuzzy graph G is a pair of function $G:(\sigma,\mu)$ where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^*:(V,E)$ where $E\subseteq V\times V$.

Definition 2.2

A fuzzy graph G is complete if $\mu(uv)=\sigma(u)\wedge\sigma(v)$ for all $u,v\in V$, where uv denotes the edge between u and v

Definition 2.3

The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x,y)$.

Definition 2.4

An fuzzy graph $G:(\sigma,\mu)$ is connected if for every x, y in σ^*, $CONN_G(x,y)>0$.

Definition 2.5

An arc (u,v) is a fuzzy bridge of $G:(\sigma,\mu)$ if the deletion of (u,v) reduces the strength of connectedness between some pair of nodes. Equivalently (u,v) is a fuzzy bridge if and only if there are nodes x, y such that (u,v) is a arc of every strongest $x-y$ path.

Definition 2.6

A node is a fuzzy cutnode of $G:(\sigma,\mu)$ if removal of it reduces the strength of connectedness between some other pair of nodes. Equivalently w is a fuzzy cutnode if and only if there exist u,v distinct from w such that w is an every strongest $u-v$ path.

Definition 2.7

Let $G:(\sigma,\mu)$ be a fuzzy graph. The degree of a vertex u is $d_G(u)=\sum_{u\neq v}\mu(u,v)$.

Definition 2.8

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. If $d_G(v)=k$ for all $v\in V$, that is if each vertex has same degree k, then G is said to be a regular fuzzy graph of degree k or a k-regular fuzzy graph.

Definition 2.9

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The total degree of a vertex $u\in V$ is defined by $td_G(u)=\sum_{u\neq v}\mu(u,v)+\sigma(u)=d_G(u)+\sigma(u)$. If each vertex of G has the same degree k, then G is said to be a totally regular fuzzy graph of total degree k or k-totally regular fuzzy graph.

Definition 2.10

Let $G:(\sigma,\mu)$ be a connected or regular fuzzy graph with $\sigma^*=\{v_1,v_2,....,v_q\}$ in some order. Then a finite sequence $\alpha_{arc(s)}(G)=(n_1,n_2,....,n_q)$ is called the α- arc sequence of G if n_k= number of α-strong edges incident on v_k and equal to zero, if no α-strong edges incident on v_k.

Definition 2.11

Let $G:(\sigma,\mu)$ be a connected or regular fuzzy graph with $\sigma^*=\{v_1,v_2,....,v_q\}$ in some order. Then a finite sequence $\beta_{arc(s)}(G)=(n_1,n_2,....,n_q)$ is called the β- arc sequence of G if n_k= number of β-strong edges incident on v_k and equal to zero, if no β-strong edges incident on v_k.

Definition 2.12

Let $G:(\sigma,\mu)$ be a connected or regular fuzzy graph with $\sigma^*=\{v_1,v_2,....,v_q\}$ in some order. Then a finite sequence $\delta_{arc(s)}(G)=(n_1,n_2,....,n_q)$ is called the δ - arc sequence of G if n_k= number of δ -arc incident on v_k and equal to zero, if no δ -arc incident on v_k.

Definition 2.13

Let $G:(\sigma,\mu)$ be a connected or regular fuzzy graph with $\sigma^*=\{v_1,v_2,....,v_q\}$ in some order. Then a finite sequence $S_{arc(s)}(G)=(n_1,n_2,....,n_q)$ is called the strong- arc sequence of G if n_k= number of α or β strong edges incident on v_k and equal to zero, if no α and β incident on v_k.

Example 2.1

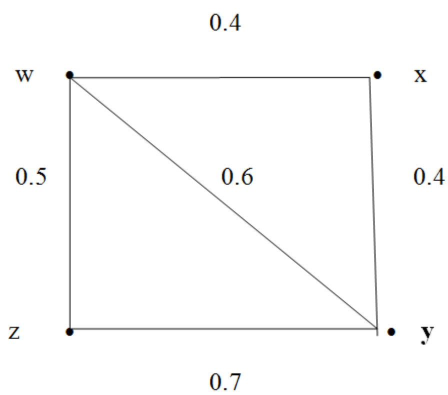

Fig .1 An Connected fuzzy graph with all types of arcs
$\alpha_{arc(s)}(G)=(0,0,1,1)$, $\beta_{arc(s)}(G)=(1,2,1,0)$, $\delta_{arc(s)}(G)=(2,0,1,1)$, $S_{arc(s)}(G)=(1,2,2,1)$

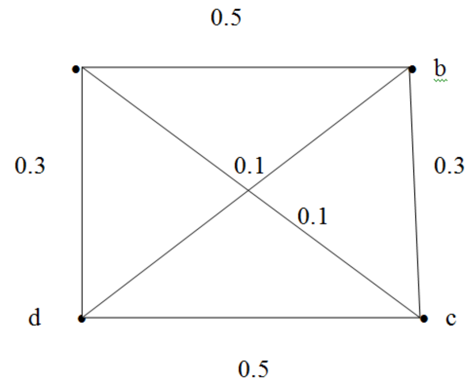

Fig .2 An Regular fuzzy graph with all types of arcs
$\alpha_{arc(s)}(G)=(1,1,1,1)$, $\beta_{arc(s)}(G)=(1,1,1,1)$, $\delta_{arc(s)}(G)=(1,1,1,1)$, $S_{arc(s)}(G)=(2,2,2,2)$

3. Complete in Fuzzy Graphs

A complete fuzzy graph has no fuzzy cutnodes. In this section, we present a necessary condition which must be satisfied by a complete in fuzzy graphs. Also two necessary and sufficient conditions are included.

Definition 3.1 A sequence of integers is called a zero-one sequence if it contains only zero's and one's.

Theorem 3.1

If a connected fuzzy graph $G:(\sigma,\mu)$ is complete, then $\alpha_{arc(s)}(G)$ is a zero-one sequence.

Proof: Suppose that $G:(\sigma,\mu)$ is complete. We have to prove that $\alpha_{arc(s)}(G)$ is zero-one sequence. This means we need to prove that $\alpha_{arc(s)}(G)$ contains only zeros and ones. If possible, suppose that contrary. Suppose that there exist an entry which is at least two in $\alpha_{arc(s)}(G)$. Let $n_k = 2$. That is, there are two different α-strong edges incident on the node x_k. Now an edge in μ^* of an fuzzy graph G is an fuzzy bridge if and only if it is an α-strong edge (21). Also if a node is common to more than one fuzzy bridge, then it is an fuzzy cut node (13). Therefore we see that x_k is an fuzzy cut node of G then by the proposition 2.2 (15) which is a contradiction to our assumption that the fuzzy graph G has no fuzzy cut nodes since it is complete. So our assumption is wrong. Hence $n_k < 2$. That is $n_k =$ zero or one. Thus $\alpha_{arc(s)}(G)$ is zero-one sequence.

Example 3.1

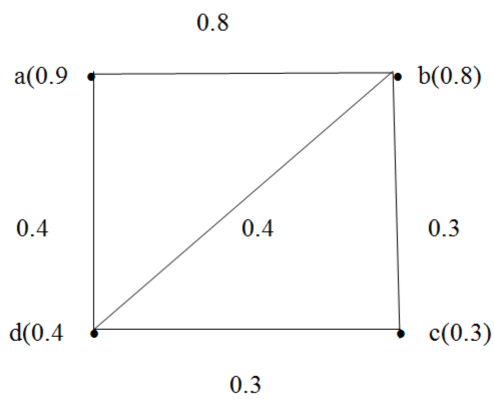

Fig. 3. An Complete fuzzy graph $\alpha_{arc(s)}$ is a zero-one sequence

$$\alpha_{arc(s)} = (1,1,0,0)$$

Consider $G:(\sigma,\mu)$ where

$\sigma(a)=0.9, \sigma(b)=0.8, \sigma(c)=0.3, \sigma(d)=0.4, \mu(a,b)=0.8, \mu(b,c)=0.3, \mu(c,d)=0.3, \mu(d,a)=0.4 and \mu(d,b)=0.4$

Here G is a complete fuzzy graph. Also α-arc sequence of G is a zero-one sequence. That is $\alpha_{arc(s)} = (1,1,0,0)$

Remark 3.1

The above theorem is only necessary. It is not sufficient as seen from the following example. In the above example $\alpha_{arc(s)}(G) = (1,1,0,0,0)$. It is a zero-one sequence. But the fuzzy graph is not complete. Since the node x is a cut node and hence an fuzzy cut node.

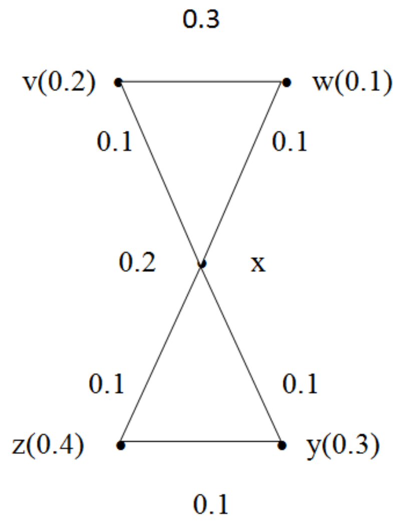

Fig 4. Fuzzy graph with a cut node

In the next one we restrict the underlying graph G^* of G to be a complete. That means G^* has no cut nodes.

Theorem 3.2

Let $G:(\sigma,\mu)$ be a connected fuzzy graph such that underlying graph G^* is complete. Then G is complete if and only if $\alpha_{arc(s)}(G)$ is zero-one sequence.

Proof: Let $G:(\sigma,\mu)$ be a connected fuzzy graph such that underlying graph G^* is complete. If G is complete by theorem 3.1 $\alpha_{arc(s)}(G)$ is zero-one sequence.

Conversely suppose that, $\alpha_{arc(s)}(G)$ is zero-one sequence. We have to prove that G is complete. That is we have to prove that G has no fuzzy cut nodes. If possible, let G have an fuzzy cut node x_k. Then there exist two nodes w and y in G such that $w \neq x_k \neq y$ and $CONN_{G-x_k}(w,y) < CONN_G(w,y)$. Since G^* is complete, it has no cut node, and hence x_k is not a cut node. Therefore we can consider many possible $w-y$ path not passing through the node x_k. Now from the above inequality, clearly the weights of all edges in the $w-x_k-y$ path are strictly greater than weights of all edges in the possible $w-y$ paths which are not passing through the node x_k. This means all edges in the $w-x_k-y$ path are α-strong, and hence x_k is incident with at least two different α-strong edges. Therefore $n_k = 2$ which shows that $\alpha_{arc(s)}(G)$ is not wrong. Hence G is complete.

Example 3.2

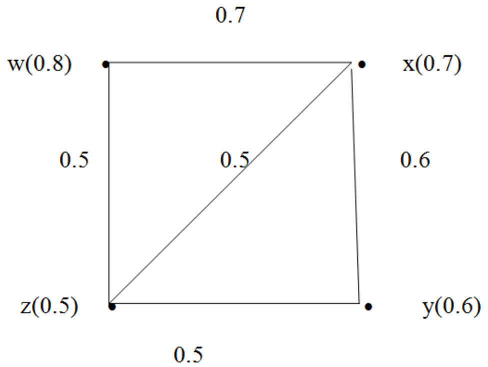

Fig. 5 Complete Fuzzy Graph $\alpha_{arc(s)}$ is not a zero-one sequence

$$\alpha_{arc(s)} = (1,2,1,0)$$

Consider $G:(\sigma,\mu)$ where
$\sigma(w)=0.8, \sigma(x)=0.7, \sigma(y)=0.6, \sigma(z)=0.5, \mu(w,x)=0.7, \mu(x,y)=0.6, \mu($

Here G is a complete fuzzy graph. Also α-arc sequence of G is not a zero-one sequence. That is $\alpha_{arc(s)} = (1,2,1,0)$

4. Regular fuzzy graph with some special arc-sequence

Theorem 4.1

"A fuzzy graph G whose crisp graph is an odd cycle is regular if and only if μ is a constant function".(24)

Theorem 4.2

"A fuzzy graph G whose crisp graph is an even cycle is regular if and only if μ is a constant function or alternative edges will have same values".(24)

Definition 4.1

A zero sequence is a real sequence containing all the entries 0. It is denoted by (0).

Theorem 4.3

Let $G:(\sigma,\mu)$ be a regular fuzzy graph such that crisp graph G^* is a odd cycle. Then G is a regular fuzzy graph if and only if $\alpha_{arc(s)}(G)=(0)$ and $\delta_{arc(s)}(G)=(0)$.

Proof: Suppose that $\alpha_{arc(s)}(G)=(0)$ and $\delta_{arc(s)}(G)=(0)$. This means G contains only $\beta_{arc(s)}(G)$. Then by the definition (2.11), we have $\mu(x,y)=CONN_{G-(x,y)}(x,y)$. Thus all the arcs in G will have the same membership value. Then by the theorem (4.1), we get G as a regular fuzzy graph. Conversely, suppose that G be a regular fuzzy graph. Then by the theorem (4.1). The membership value is a constant function. Thus the deletions of any arc in G will not affect the strength of $\mu(x,y)=CONN_{G-(x,y)}(x,y)\forall(x,y)\in G$. That is G contains only $\beta_{arc(s)}(G)$. Thus $\alpha_{arc(s)}(G)=(0)$ and $\delta_{arc(s)}(G)=(0)$.

Remark 4.1

The above condition does not hold for totally regular fuzzy graph.

Example 4.1

Consider $G:(\sigma,\mu)$ where
$\sigma(a)=0.6, \sigma(b)=0.7, \sigma(c)=0.5, \mu(a,b)=0.4, \mu(b,c)=0.4 and \mu(c,a)=0.6$

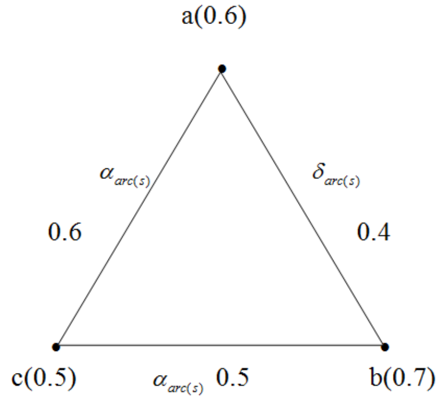

Fig. 6. Totally regular fuzzy graph with $\alpha_{arc(s)}(G)$ and $\delta_{arc(s)}(G)$ Here G is totally regular fuzzy graph. But it contains $\alpha_{arc(s)}(G)$ and $\delta_{arc(s)}(G)$.

Theorem 4.4

A regular fuzzy graph $G:(\sigma,\mu)$ whose even cycle is the crisp graph $G^*:(V,E)$ contains $\alpha_{arc(s)}(G)$ and $\beta_{arc(s)}(G)$. Also $\delta_{arc(s)}(G)=(0)$.

Proof:

Assume $\delta_{arc(s)}(G)$ is a zero sequence. That is $\delta_{arc(s)}(G)=(0)$. Then by the definition 2.10 & 2.11 we have $\mu(x,y)\geq CONN_{G-(x,y)}(x,y)$ which implies that membership value μ has either constant values or alternative arcs will have same values. Then by the theorem (4.2) we get G as a regular fuzzy graph. Conversely, Let G be a regular fuzzy graph then by the theorem (4.2) the membership value μ is either constant or alternative arcs will have same value. That is $\mu(x,y)\geq CONN_{G-(x,y)}(x,y)$. This implies $\delta_{arc(s)}(G)$ is a zero sequence. That is $\delta_{arc(s)}(G)=(0)$.

Remark 4.2 The above condition does not hold for totally regular fuzzy graph.

Example 4.2 Consider $G:(\sigma,\mu)$ where
$\sigma(w)=0.4,\sigma(x)=0.2,\sigma(y)=0.2,\sigma(z)=0.4,$
$\mu(w,x)=0.4,\mu(x,y)=0.4,\mu(y,z)=0.4 and \mu(z,w)=0.2$

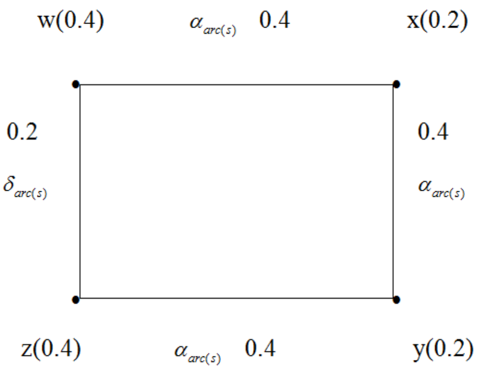

Fig 7. Totally regular fuzzy graph with $\delta_{arc(s)}(G)$

Here G is totally regular but it contains $\delta_{arc(s)}(G)$

Theorem 4.5

A regular fuzzy graph $G:(\sigma,\mu)$ with its crisp graph $G^*:(V,E)$ as even cycle is both regular and totally regular if $\delta_{arc(s)}(G)=(0)$.

Proof

Let $G:(\sigma,\mu)$ be a regular fuzzy graph. Then its crisp graph $G^*:(V,E)$ is a even cycle and G be both regular and totally regular fuzzy graph. There are two case arise.

Case (i)

Let G be both regular and totally regular fuzzy graph with constant values in σ and μ then by the definition (2.11) this means G contains only $\beta_{arc(s)}(G)$.

Example 4.3 Consider $G:(\sigma,\mu)$ where

$\sigma(m)=\sigma(n)=\sigma(o)=\sigma(p)=0.2$
$\mu(m,n)=\mu(n,o)=\mu(o,p)=\mu(p,m)=0.3$

Here G is a regular and totally regular fuzzy graph without $\delta_{arc(s)}(G)$.

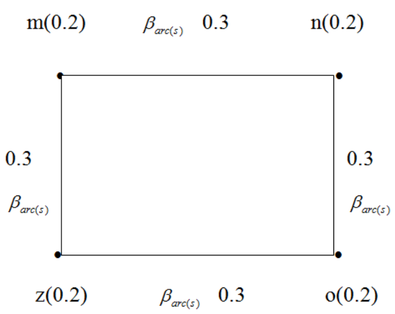

Fig 8 Regular and Totally regular fuzzy graph without $\delta_{arc(s)}(G)$

Case (ii)

Let G be both regular and totally regular fuzzy graph with constant values in σ and with same alternative values in μ then by the definitions 2.10 and 2.11 G contains only $\alpha_{arc(s)}(G)$ and $\beta_{arc(s)}(G)$. This means $\delta_{arc(s)}(G)=(0)$.

Example 4.4 Consider $G:(\sigma,\mu)$

where $\sigma(m)=\sigma(n)=\sigma(o)=\sigma(p)=0.1$
$\mu(m,n)=0.4,\mu(n,o)=0.3,\mu(o,p)=0.4 and \mu(p,m)=0.3$

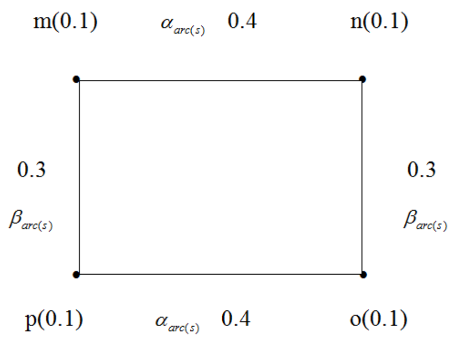

Fig. 9. Regular and Totally regular fuzzy graph without $\delta_{arc(s)}(G)$

Here G is a regular and totally regular fuzzy graph without $\delta_{arc(s)}(G)$.

Conclusion

Structural Properties of fuzzy graphs are introduced in this article. Different types of arc-sequences in fuzzy graphs are introduced. Besides, as reduction in strength between two nodes in more important than the total disconnection of the graph, the authors made use of the connectivity concepts in defining these arc-sequences. We used the characteristics of different types of arc-sequence to categorize regular and totally regular fuzzy graphs. Also α -arc sequence of a complete and δ -arc sequence of a regular fuzzy graph are discussed. Further analysis, may lead us to a better understanding of the nature of fuzzy graphs.

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