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RESEARCH ARTICLE

THEORETICAL EXPLANATION AND CALCULATION FOR KEPLERIAN ORBITAL ELEMENTS IN LOW EARTH ORBIT AND GEOSTATIONARY ORBIT

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ABSTRACT

After the detailed study about orbits, GEO is chosen based on developments to be done in the inclination and eccentricity. Hohmann transfer is needed to attain the GEO, so delta-V calculation is carried out for four different cases and proved the lower delta-V total. To extend further, calculations for orbital elements and perturbations to be done, to analysis the satellite's position in its respective orbit. By determining these values, they can be entered in the software and orbit design will be optimized. In this paper, theoretical calculation for orbital elements, brief explanation of orbital man oeuvres and how to determine it will be studied. And calculation for six keplerian orbital elements for both LEO and GEO will also be carried out. The importance of epoch both in practical and software cases are explained in a detailed manner. The three dimensional trajectories which will be useful to locate the satellite's position and direction are described in the manner how the software works on it.

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INTRODUCTION

The motion trajectory of the centroid of an operating spacecraft mainly refers to its orbits. Orbits can be generally classified into three categories based on different flight missions i.e. the artificial Earth satellite trajectory, Earth-to-Moon flight trajectory and the interplanetary flight trajectory. Which satisfy the fundamental motion equation of the restricted two body problem are usually keplerian ones (for the typical obits). The orbits can be precisely predicted by the fundamental motion equations and the Kepler equations. These orbits are called typical orbit equations. Methods of spacecraft orbit and their design theories have been developing rapidly since the successful launch of the first artificial Earth satellite in the 1950s. At that time the primary missions of a spacecraft are to provide communication and navigation for the army, the navy and the air force, and to conduct reconnaissance and surveillance of hotspot areas. Earlier the design of a spacecraft is mainly focused on the absolute orbit design methods in which it takes the earth as the reference and the relative motion between the spacecraft and the ground area. Between special space orbits and the typical orbits there are major distinctions in their theories and design methods. Therefore different space orbits are required for different missions which results in the discrepancy of the theories, principles and methods while designing these special orbits.

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For example, a type of relative orbits such as hover orbit that regards the target spacecraft as a reference and the spiral cursing orbit that takes the target orbit as a reference. Hence, in combination with the specific mission requirements and orbit types it is necessary to introduce the theories and design methods of special space orbits. To describe the orbit, we need to calculate the six orbital parameters because this system has six degrees of freedom. These are mainly corresponded to the three spatial dimensions (x, y, z in a Cartesian coordinate system) which define the position and velocity of these dimensions. But this is an inconvenient way to represent an orbit, and they are described as orbital state vectors. Instead of orbital state vectors, six keplerian orbital elements are used. Basically three of the parameters are used to describe the shape of the plane and the position of the satellite in its required orbit. Another three parameters are used to describe how the plane is oriented in the celestial inertial reference frame and also where the satellite is located in that plane. These orbital parameters are known as keplerian orbital elements and it differs based on the mission. Rather than the part of the reference frame sometimes the epoch is considered to be a seventh orbital parameter.

MATERIALS AND METHODS

Orbital elements determination

Hohmann transfer is required to place a satellite into geostationary orbit. It is considered as an elliptical orbit in orbital mechanics and used to transfer from one circular orbit to another of different radii in the same plane. Hohmann transfer orbit uses two impulsive burns, to control the orbital manoeuvre.

Orbital maneuvers

Sometimes it is necessary to change the orbital parameters, to attain its specific mission or to correct the orbit for launch errors or perturbations. By means of a thruster on the satellite manoeuvre is been achieved with velocity increment. On the orbital elements, it is necessary to know the effect of an impulse for a planned mission. And to determine the appropriate point in the orbit at which the manoeuvre should be carried out.

Coplanar manoeuvres

When a satellite is launched, it should be placed into desired orbit through:

- Directly from launch
- A booster at particular point to transfer into another orbit

Based on the fundamental principle an orbit is determined by position and velocity at any point. By changing the velocity vector at any point, the trajectory instantly transforms which correspond to the new velocity vector. As a result the orbit of a satellite will be changed. The changes in semi-major axis and eccentricity of the orbit is been made by coplanar manoeuvres without changing the orbital plane. Four kinds of coplanar manoeuvres are introduced, they are:

- Tangential-orbit manoeuvre.
- Non-tangential orbit manoeuvre.
- Hohmann transfer.
- Bi-elliptic orbit transfer.

Tangential-orbit manoeuvre

When the velocity vector of a spacecraft is tangent to its position vector, then this orbit manoeuvre occurs. It typically happens at a perigee point.

Non-tangential orbit manoeuvre

Orbital manoeuvres cannot be limited only at apogee and a perigee point. If condition is permitted, then the satellite can be able to perform the manoeuvres at any point.

Hohmann transfer

It is first proposed by Walters Hohmann in 1925, which is used to transfer a satellite between two non-intersecting orbits. It is considered as a simple manoeuvre, which employs intermediate elliptic orbit which is tangent to both initial and final orbits at their apside. Two burns are needed to complete the transfer. When the spacecraft coasts from periapsis to apoapsis, first burn will get access to insert the vehicle into the transfer orbit. At apoapsis, the second burn is used to place the vehicle in its final orbit. Tangential delta-V1 is applied to the circular orbit velocity. The requirement of the apogee radius of the resulting transfer orbit must equals, the radius of the final circular orbit which can determine the magnitude of delta-V1. Another delta-V must be added to the satellite when it reaches the apogee of the transfer, if not the satellite will remain in the transfer ellipse itself. It is nothing but the difference between the apogee velocity in the transfer orbit and the circular orbit velocity in the final orbit. If the satellite reaches its final orbit, delta-V2 is been applied and the complete hohmann transfer is attained G. Maral & M. Bousquet. (2002).

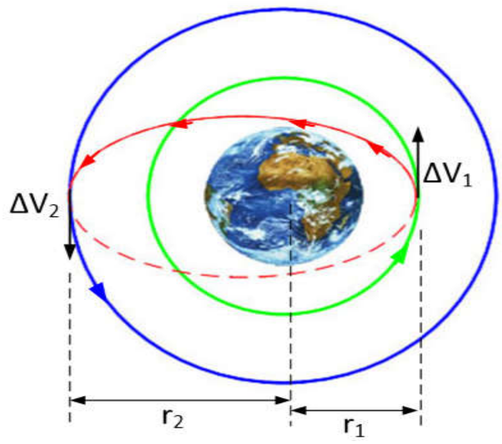

Figure 1. Demonstration of Hohmann Transfer for GEO

Classical orbital elements

Artificial satellites revolving around the earth will be defined by the six orbital elements, which referred to as the Keplerian element set. From six elements, semi-major axis and the eccentricity gives the amount of ellipse of an orbit. Third, the true anomaly determines the position of the satellite in its respective orbit at a reference time known as Epoch. In the Earth's equatorial plane the rotation of the orbit's perigee point relative to the orbit's line of nodes, is determined by the argument of perigee. These parameters are required to identify a set of mathematical parameters that enables us to describe the satellite's motion and its orbit.

Purpose of orbital elements

- Discriminate one satellite orbit from another one.
- Determine the amount and direction of manoeuvre.
- Predicts where a satellite will be in its future and where it has been in past.

A real orbit (and its elements) changes over time due to various perturbations.

Epoch time

Celestial coordinates or orbital elements are specified by this epoch because it is a specific moment in time. From which the other orbital parametric are calculated in order to predict future position. This applied tools of the mathematical disciplines of these orbital mechanics used to generate an ephemeris data. It is a table of values at a given time which gives the position of astronomical objects in the sky. It is also used to calculate the given proper time from the epoch. These calculations mainly results in an elliptical path on a plane. It is been defined by the two foci of the ellipse or random point in the orbit. It creates shifts in three dimensions of the spherical trigonometry which is used to calculate the relative positions while viewing from another orbiting body from its own trace and orbit. To be a useful predictive tool to predict future location of the object ephemeris data need to specify only one set of equations because the dynamics in three dimensions are also elliptical. The epoch factors to be recalculated from time to time because in exactitudes and other errors accumulate more and greater errors to be predicated and that requires a new epoch to be defined. Nowadays the epochs are generally defined in an international agreement due to speedy communications.

Epoch in softwares

In a computer system, the time is kept internally which expressed as the number of time units that have elapsed. It is always specified as midnight Universal Time on some particular date, since it is a specified epoch. Hugh variation in the time units happens in this software timekeeping system when compared to epoch in astronomy. Some may use this time units as large as a day and others may use nanoseconds. Like for an example, an epoch date of midnight UTC (00:00) on January 1, 1900 and the time of the midnight (24:00) between January 1 and 2, 1900 is represented by the number 86400 (number of seconds in one day). It is common to use the same system when times prior to the epoch but with negative numbers. These representations are mainly for internal use. The software will convert this internal number into a date and time representation, if an end user interaction with date and time is required. It is comprehensible to humans.

RESULTS AND DISCUSSIONS

Keplerian orbital elements

In celestial mechanics, these elements describes about the various motion of the orbit (ellipse, conic, parabolic and hyperbolic) and forms a two-dimensional orbital plane in a three-dimensional space by neglecting the perturbations caused due to the solar radiation pressure, atmospheric drag and a non-spherical body of the earth. The flow chart that represents the six keplerian orbital elements and GEO orbit is chosen based on Eccentricity and Inclination:

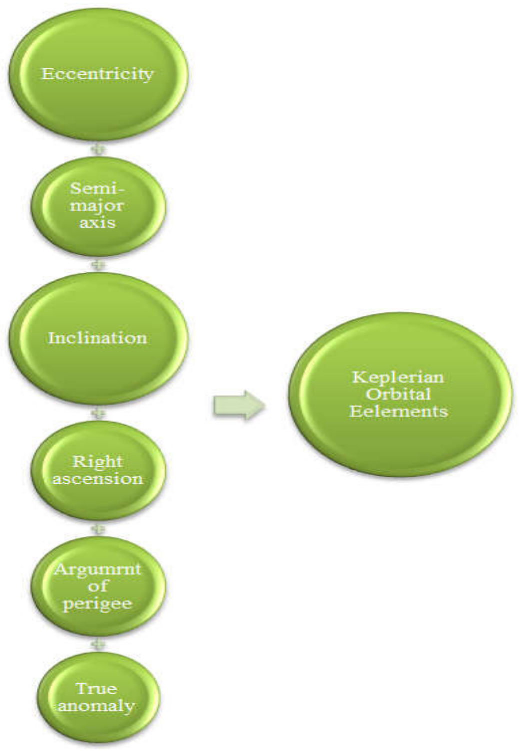

Figure 2. This flow chart represents the six keplerian orbital elements for GEO based on Eccentricity and Inclination

- **Eccentricity (e):** Which describes the shape of an ellipse on how much it is elongated when compared to a circular one.
- **Semi major axis (a):** Describes how much big or small the orbit is, by determining the sum of the periapsis and apoapsis distances divided by two-size of the orbit.
- **Inclination (i):** It is nothing but the vertical tilt of an orbit (ellipse) with respect to its reference plane, measured at the ascending node which goes from west to north. The greater latitude from north or south reached by the sub-satellite point is equal to the inclination.
- **Right ascension (Ω):** With respect to its reference frame's vernal point, it horizontally orients the ascending node of the ellipse.
- **Argument of perigee (ω):** In the orbital plane it defines the orientation of the ellipse.
- **True anomaly (ν, θ or *f*):** At a specific epoch, it defines the position of the orbiting body along the ellipse.

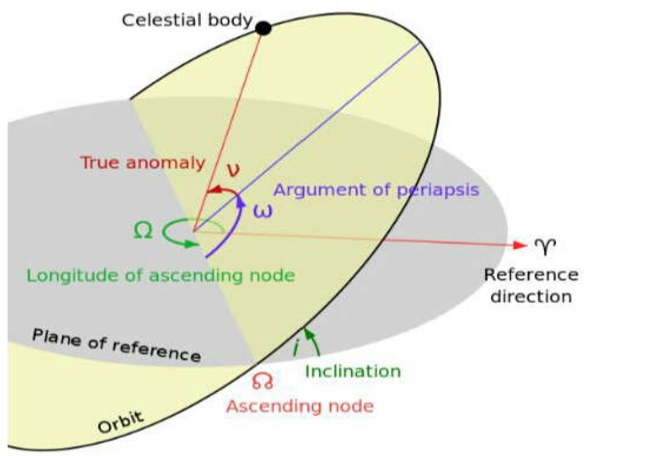

Figure 3. Diagram illustrating and explaining various terms in relation to Orbits of Celestial bodies. [Source: Author-Lasunncty, published date: 10 October 2007]

Orbit view determination for three-dimensional trajectories

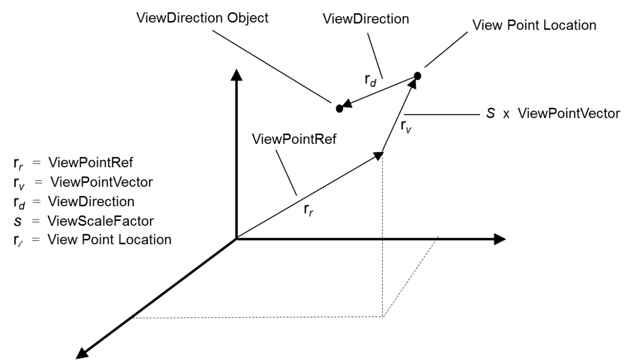

Figure 4. Three dimensional trajectories in orbital view

By using the View Point Reference, View Point Vector, View Direction, View up Coordinate System and View up Axis fields, we can specify the view location and direction. Either by giving a vector in the format [x y z] or by specifying an object name we can supply View Point Reference, View Point Vector and View Direction fields. If a vector is given for one of those quantities mentioned above, then we can use it in its appropriate place in the computation. We must determine the vector associated for a given object. View Point Reference field defines the point from which View Point Vector is measured. To determine which direction appears in the upper direction in an orbit view plot, View up Coordinate System and View up Axis fields are used. The coordinate system selected under the coordinate system will be the same under View up Coordinate System field. To define which axis of the View up Coordinate System field will appear as in the upper direction in an orbit plot will be allowed by View up Axis field.

Orbital elements for low earth parking orbit

The parking orbit which is circular and of radius 170 km is considered for the design of the orbit as from the determination of ΔV budget it is found to have the lowest ΔV value. Some values are obtained by the theoretical calculations and remaining values are obtained from simulation.

- Eccentricity, $e = \frac{A-P}{A+P} = 1.528080541 \times 10^{-4}$
- Semi major axis, $a = \frac{A+P}{2} = 6544.157675$ km
- Semi minor axis, $b = a\sqrt{1-e^2} = 6544157599km$
- Time period, $P = \sqrt{\frac{4\pi^2 a^3}{\mu}} = 5265.879543\ s$
- True anomaly, $\theta = \cos^{-1}\left(\frac{1-(1-e^2)\frac{a}{r}}{e}\right) = 90.00445806890213°$
- Inclination, $i = \cos^{-1}\left(\frac{h_z}{h}\right) = 21.7°$
- Right ascension of the ascending node, $\Omega = \cos^{-1}\left(\frac{n_x}{n}\right) = 306.6148021947984°$
- Argument of perigee, $\omega = \cos^{-1}\left[\frac{(n.e)}{(n.e)}\right] = 314.1905515370898°$
- *Specific angular momentum*, h is the satellites total angular momentum divided by its mass.
- It is the cross product of position and velocity vectors.

$$\boldsymbol{h} = \boldsymbol{r} \times \mathbf{v}$$

- *Nodal vector n* is the vector pointing towards the ascending node
- In the direction of ascending node

$$\boldsymbol{n} = \boldsymbol{z} \times \boldsymbol{h}$$

- We can calculate the *eccentricity vector* from the following equation

$$\boldsymbol{e} = \left(\frac{1}{\mu}\right)\left\{\left(V^2 - \frac{\mu}{r}\right)\boldsymbol{r} - (\boldsymbol{r}.\mathbf{v})\mathbf{v}\right\}$$

Orbital elements for final geo orbit

- Semi major axis, $a = 42164.164\ km$
- Eccentricity, $e = 0$ (circular equatorial orbit)
- Time period, $T = \sqrt{\frac{4\pi^2 a^3}{\mu}} = 86163.98054$ s
- Inclination, $i = 0°$
- Right ascension of the ascending node, $\Omega = not\ defined$ (circular equatorial orbit)
- True anomaly, $\theta = not\ defined$ (circular equatorial orbit)
- Argument of perigee, $\omega = not\ defined$ (circular equatorial orbit)

Conclusion

From the above calculated values for low earth orbit and geostationary orbit, we can create our own GEO orbit and the hohmann transfer in simulation will be. In correction, if there is a slight variation been made in these orbital elements and in the epoch time, they cause a major error in the script of software and not been able to attain our required geostationary orbit. Eventually keeping everything in scale, calculation is been done and obtained answers correctly. If the parameters values are correct then obviously, epoch will be in a perfect plot. Because here latitude and longitude at ISRO SrihariKota is been used for calculating the parameters, while performing the ground track plot simulation in software, the launching is made from ISRO SrihariKota.

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