Available online at http://www.journalcra.com

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

International Journal of Current Research
Vol. 9, Issue, 10, pp.58521-58529, October, 2017

ISSN: 0975-833X

RESEARCH ARTICLE

CANONIZATION OF THE HYPERSURFACES OF THE FIRST AND THE SECOND DEGREE AND APPLICATION FOR THE MAXIMAL ABSOLUTE AND RELATIVE INACCURACIES

[1]Kiril Kolikov, [2,*]Radka Koleva and [1]Yordan Epitropov

[1]Plovdiv University "Paisii Hilendarski", Tzar Assen str. 24, Plovdiv, Bulgaria
[2]University of Food Technologies, 26 Maritsa Blvd., Plovdiv, Bulgaria

ARTICLE INFO

Article History:
Received 06th July, 2017
Received in revised form
23rd August, 2017
Accepted 28th September, 2017
Published online 17th October, 2017

Key words:

Linear Homogeneous form of *n* Unknowns, Orthogonal Transformation of the Unknowns, Canonical forms of the Surfaces of the first and of the second degree, Affine Euclidean space, Maximal Absolute and Relative Inaccuracies.

ABSTRACT

In this paper we reduce a linear homogeneous form of n unknowns with an orthogonal transformation of the unknowns in a linear homogeneous form of one unknown with an exactly definite positive coefficient. As an application of this result we find the canonical form of an arbitrary hyperplane in the real n-dimensional affine Euclidean space E_n. This method for the obtaining of the canonical forms of the hyperplanes is new, since until nowa canonical form of hyperplanes is not defined and is not considered.Besides we find an effective canonical form of the surfaces of the second degree in E_n. Our methodfor the canonization of the surfaces of the second degree in E_n is effective since it gives the exact coefficients of the canonical form of the surface in a dependent ofthe coefficients of the givensurface equation. As an application we give a canonization of the hypersurfaces of the maximal absolute and relative inaccuracies (errors). Besides this method is different from the known approach in the case of the obtaining of a cylinder.

Citation: Kiril Kolikov, Radka Koleva and Yordan Epitropov. 2017. "Canonization of the hypersurfaces of the first and the second degree and application for the maximal absolute and relative inaccuracies", *International Journal of Current Research*, 9, (10), 58521-58529.

INTRODUCTION

The canonization of the surfaces of the second degree in the real n-dimensional affine Euclidean space E_n is well known ((Efimov, 2005; Konstantinov, 2000), and (Shafarevich, 2013)).In the known canonizations geometric interpretations are made while in our paper we offer an algebraic approach. In this paper we reduce with orthogonal transformation of the unknowns a real linear non-zero form $f = a_1x_1 + ... + a_nx_n$ of n unknowns $x_1, ..., x_n$ in a linear homogenous form $g = d_nu_n$ of a unknown u_n with a positive coefficient $d_n = \sqrt{a_1^2 + ... + a_n^2}$. Giving a definition of a canonical form of a planes in E_n, we apply the above result and we find the canonical form g of a surface of these hyperplanes. Until now a canonical form of hyperplanes is not defined and it is not considered. We obtain a effective canonical forms of the surfaces of the second degree in E_n. Our approach for an obtaining of the canonical form g in E_n of a surface f of the second degree is different from the known methods in a case of a cylinder. Besides this method is effective since it gives the exact coefficients of the canonical form g in a dependent of the coefficients of f. As an application of the obtained results we give the canonical forms of the maximal absolute and relative inaccuracies (errors).

1.Reduction of a linear homogenous form in a linear homogenous form of one unknown

In the following result with an orthogonal transformation of the unknowns we reduce a linear homogenous form of n unknownsin a linear homogenous formof one unknown.

***Corresponding author:* Radka Koleva,**
University of Food Technologies, 26 Maritsa Blvd., Plovdiv, Bulgaria.

Theorem1. If $f = a_1x_1 + ... + a_nx_n$ is a non-zeroreal linear homogenous formof n unknowns $x_1,..., x_n$, then thereexists an orthogonaltransformation of the unknownswhich reduces f ina linear homogenous form $g = d_nu_n$,where $d_n = \sqrt{a_1^2 + a_2^2 + ... + a_n^2}$ is a positive real number.

Proof. Let $n = 1$, i.e. $f = a_1x_1$. If $a_1 > 0$, thenobviously $f = \sqrt{a_1^2}x_1 = d_1x_1$, $d_1 = \sqrt{a_1^2} > 0$and the theorem is proved. If $a_1 < 0$, then we apply the orthogonal transformation $x_1 = -u_1$and we obtain $f = (-a_1)u_1 = \sqrt{a_1^2}u_1 = d_1u_1$, $d_1 = \sqrt{a_1^2} > 0$. Therefore,the theorem holds for $n = 1$.

Suppose that $n \geq 2$. Let, for a determination, it holds $a_1 \neq 0$.

At first we reduce the form $f = a_1x_1 + a_2x_2 + ... + a_nx_n$ withan orthogonal transformation of $x_1, x_2,..., x_n$ in the linear form $f = d_2y_2 + a_3y_3 + ... + a_ny_n$, where $y_2,..., y_n$are unknowns, i.e. we exceptthe unknown x_1. Namely, we use the normed vector $\left(\frac{a_1}{d_2}, \frac{a_2}{d_2}\right)$, $d_2 = \sqrt{a_1^2 + a_2^2}$

and we form thelinear transformation

$$\ddot{\ }_1 : x_1 = \frac{a_2}{d_2}y_1 + \frac{a_1}{d_2}y_2, \quad x_2 = -\frac{a_1}{d_2}y_1 + \frac{a_2}{d_2}y_2, \quad x_i = y_i, \ i = 3,...,n. \tag{1}$$

It is easily to verify, thatthis transformation is orthogonal and that $f_1 = d_2y_2 + a_3y_3 + ... + a_ny_n$,i.e. the indicatedorthogonal transformation excepts theunknown x_1 in f.
Suppose, that bythe orthogonal transformation $\ddot{\ }_{\kappa-1}$from the form (1), where $1 \leq k - 1 \leq n - 2$,it is obtained the linear form

$$f_{k-1} = d_kz_k + a_{k+1}z_{k+1} + ... + a_nz_n,$$

where $z_1,..., z_n$ are unknowns and $d_k = \sqrt{a_1^2 + ... + a_k^2}$.

In the linear form f_{k-1} we exceptthe unknown z_k with theorthogonal transformation

$$\ddot{\ }_k : z_k = \frac{a_{k+1}}{d_{k+1}}t_{k-1} + \frac{d_k}{d_{k+1}}t_k, \ z_{k+1} = -\frac{d_k}{d_{k+1}}t_{k-1} + \frac{a_{k+1}}{d_{k+1}}t_k, \ z_i = t_i,$$

$i = 1,..., k - 1, k + 2,..., n$, where $d_{k+1} = \sqrt{d_k^2 + a_{k+1}^2}$, т.е. $d_{k+1} = \sqrt{a_1^2 + a_2^2 + ... + a_{k+1}^2}$ .In this way we obtain the linear form

$$f_k = d_{k+1}t_{k+1} + a_{k+2}t_{k+2} + ... + a_nt_n,$$

where $t_1,..., t_n$ are new unknowns.

The induction is ended. Finally we obtainthe linear formof the kind $g = f_{n-1} = d_nu_n$,where u_n is anunknownand $d_n = \sqrt{a_1^2 + ... + a_n^2}$ is a positive real number. The linear form g is obtained with a sequential realizationof the transforms $\ddot{\ }_1, \ddot{\ }_2,..., \ddot{\ }_{n-1}$, i.e. with the product $\ddot{\ }_1 \ddot{\ }_2 ... \ddot{\ }_{n-1}$. However it is well known, that the product of orthogonal transformations of the unknowns is an orthogonal transformation of the original unknowns. Therefore, $\ddot{\ }_1 \ddot{\ }_2 ... \ddot{\ }_{n-1}$ is an orthogonal transformation of $x_1,..., x_n$, whichreduces f in $g = d_nu_n$, where $d_n = \sqrt{a_1^2 + ... + a_n^2}$.
The theorem is proved.□

2.Canonical forms of the hypersurfaces of the first degree

The surfaces of the first degree in the real affine Euclidean space $E_n$are given bythe equation

$$f_1 = \sum_{i=1}^{n} a_i x_i + a = 0 \qquad (2)$$

where $a_i, a \in$ and at least $a_i \neq 0$.

Definition1. We say, that a plain has a canonical form (canonical equation) if this plain has the form $z = 0$, where z is an unknown. Let in E_n an orthogonal coordinate system $Ox_1x_2...x_n$ is given. We notethat the equation $x_n = 0$ in E_n is theplain defined by the coordinate axes $Ox_1,...,Ox_{n-1}$, since every point $(x_1, x_2,..., x_{n-1}, 0)$ of E_n satisfies this equation.

In the following result we obtain the canonical form of the plain f_1 given by (2).

Theorem 2. In the many-dimensional real affine Euclidean space E_n the following cases hold for the plain f_1.

1)If at least one coefficient a_i is non-zero, then the canonical form of f_1 is given by the equation $z_n = 0$, i.e. this canonical form the plain, defined by the coordinate axes $O_1z_1,...,O_1z_{n-1}$ of some coordinate system $O_1z_1z_2...z_n$ of E_n.
2)If $a_1 = ... = a_n = a = 0$, then f_1 is the space E_n.
3)If $a_1 = ... = a_n = 0$ и $a \neq 0$, then f_1 is the empty set.

Proof. Statements 2) and 3) of the theorem are trivial. We shall prove the statement 1). We represent the surface f_1 in the form $f_1 = f + a$, where $f = a_1x_1 + ... + a_nx_n$.Theorem1 impliesthat there exists an orthogonal transformationwhich reduces $f = a_1x_1 + ... + a_nx_n$ in $f = d_ny_n$, $d_n = \sqrt{a_1^2 + ... + a_n^2}$.In the last form of f we replace $x_1,..., x_n$ with the unknowns $y_1,..., y_n$. Consequently f_1 obtains the form $d_ny_n + a = 0$. We make the transformation $z_n = y_n + \frac{a}{d_n}$ which wesupplementwith $y_1 = z_1,..., y_{n-1} = z_{n-1}$, where $z_1,..., z_n$ are unknowns.In this way we obtain a translation in E_n and f_1 obtains theform $d_nz_n = 0$, i.e. $z_n = 0$.

The theorem is proved.□

4. Canonical forms of the hypersurfaces of the second degree

In the many-dimensional real affine Euclidean space E_n the surfaces of the second degree are given bythe equation

$$f(x_1,..., x_n) = \sum_{i,j=1}^{n} a_{ij}x_ix_j + 2\sum_{i=1}^{n} a_ix_i + a = 0,\ a_{ji} = a_{ij},\ i, j = 1, 2,..., n, \qquad (3)$$

where $a_{ij}, a_i, a \in$ and at least $a_j \neq 0$ and $x_1,..., x_n$ are unknowns.
The forms

$$f_1 = 2\sum_{i=1}^{n} a_ix_i + a,\ f_2 = \sum_{i,j=1}^{n} a_{ij}x_ix_j,\quad a_{ji} = a_{ij},\ i, j = 1, 2,..., n,$$

are called a linear and a quadratic part of f, respectively. Denote by $r(f_2)$ the rang of the quadratic part f_2 of f. If $n = 1$, i.e. in a space E_1, the canonical form of $f = a_1x^2 + 2a_1x + a$ is well known and absolutely trivial.

If $n = 2$ and $n = 3$ the canonical form of f is also well known. We shall consider $n \geq 2$, in order to do an analogy.We put

$$X = \begin{pmatrix} x_1 \\ ... \\ x_n \end{pmatrix} \text{ and } Y = \begin{pmatrix} y_1 \\ ... \\ y_n \end{pmatrix},$$

where $x_1,..., x_n$ and $y_1,..., y_n$ are systems of unknowns.

At first we shall obtain the canonical form of f reducing the quadratic part f_2 in a canonical form by an orthogonal transformation

$$X = QY, \quad Q = \begin{pmatrix} q_{11} & \dots & q_{1n} \\ \dots & \dots & \dots \\ q_{n1} & \dots & q_{nn} \end{pmatrix}, \; q_{ij} = q_{ji}, \; q_{ij} \in \quad . \tag{4}$$

We replace every x_i by $x_i = \sum_{j=1}^{n} q_{ij} y_j$. Thenthe linear part $f_1$of f obtain the form

$$f_1 = 2\sum_{j=1}^{n} b_j y_j + a, \; b_j = \sum_{i=1}^{n} a_i q_{ij}, \;\; j = 1,\dots,n. \tag{5}$$

In view of Theorem2,we shall suppose, that at least one coefficient $a_{ij} \neq 0$, i.e. $r = r(f_2) > 0$.Let the characteristic roots $\mathsf{a}_1,\dots,\mathsf{a}_r$ of the quadratic part f_2 of f aredifferent from zero and $\mathsf{a}_{r+1},\dots,\mathsf{a}_n = 0$. Then we set

$$c = a - \frac{b_1^2}{\mathsf{a}_1} - \dots - \frac{b_r^2}{\mathsf{a}_r}. \tag{6}$$

In the following result we shall obtain the canonical form of the surface f of the second degree in E_n by orthogonal transformations of the unknowns and by translations. We shall suppose that f is given by equation (3).

Theorem3. Let $r > 0$ be the rang of the quadratic part f_2 of the surface f of the second degree in the real affine Euclidean space E_n, $n \geq 2$.Then the following cases hold, in which the numbers b_j and c, $j = 1,\dots,n$, are defined by (5)and (6), respectively.
1)Let $r = n$.Then the surface f is with a canonical equation

$$\mathsf{a}_1 z_1^2 + \dots + \mathsf{a}_n z_n^2 = -c, \tag{7}$$

where c is defined by (6) for $r = n$. Besides the following subcases hold.

1.1) If $\mathsf{a}_1,\dots,\mathsf{a}_n$ have the same sign and $c \neq 0$, then f is an *ellipsoid*(an *ellipse*,if $n = 2$).
1.2) Letat least two of the signs of $\mathsf{a}_1,\dots,\mathsf{a}_n$ are different and $c \neq 0$. Then f is *ahyperboloid* (*a hyperbola*, if $n = 2$).
1.3) If $c = 0$,then f is a*cone*. Besides, if $\mathsf{a}_1,\dots,\mathsf{a}_n$ have the same signs, then f is an imaginary cone with only one real point (0,0,…,0). (If $n = 2$ and the signs of a_1 and a_2 are different, then f is *a pair intersecting straight lines*.)
2)Let $r = n - 1$.We have two subcases.
2.1) Let $b_n \neq 0$. Then thesurface f is *aparaboloid*(a *parabola*, if $n = 2$) with a canonical equation

$$\mathsf{a}_1 z_1^2 + \dots + \mathsf{a}_{n-1} z_{n-1}^2 = -2b_n z_n. \tag{8}$$

Besides if $\mathsf{a}_1,\dots,\mathsf{a}_{n-1}$are with the same signs, then f is *an elliptical paraboloid*andif at least two of signs of $\mathsf{a}_1,\dots,\mathsf{a}_{n-1}$ are different, then f is *ahyperbolic paraboloid*.

2.2) Let $b_n = 0$ .Thenthesurface f is *acylinder*with a canonical equation

$$\mathsf{a}_1 z_1^2 + \dots + \mathsf{a}_{n-1} z_{n-1}^2 = -c. \tag{9}$$

3)Let $1 \leq r \leq n - 2$ .Then the surface f is a*cylinder*.Besides the following subcases hold.

3.1) If at least one of the coefficients b_i is non-zero, $i = r+1,\dots,n$, then f is a *parabolic cylinder* and f has a canonical equation

$$\mathsf{a}_1 z_1^2 + \dots + \mathsf{a}_r z_r^2 = -s z_n, \qquad s = \sqrt{b_{r+1}^2 + b_{r+2}^2 + \dots + b_n^2}. \tag{10}$$

3.2) If $b_{r+1} = ... = b_n = 0$,then the canonical form of f is

$$\mathrm{a}_1 z_1^2 + ... + \mathrm{a}_r z_r^2 = -c \,. \tag{11}$$

Proof. Afterthereductionof f_2 inacanonicalformby orthogonal transformation (4), equation (2), in view of (5), obtains the form

$$\mathrm{a}_1 y_1^2 + ... + \mathrm{a}_r y_r^2 + 2\sum_{j=1}^{n} b_j y_j + a = 0 \,.$$

In this equation we make the following transformation

$$\mathrm{a}_i y_i^2 + 2b_i y_i = \mathrm{a}_i z_i^2 - \frac{b_i^2}{\mathrm{a}_i}, \quad z_i = y_i + \frac{b_i}{\mathrm{a}_i}, \quad i = 1,...,r \,. \tag{12}$$

We add the last equalities with $z_{r+1} = y_{r+1},..., z_n = y_n$ and we obtain a translation of the surface (12). In this way (12) obtains the form

$$\mathrm{a}_1 z_1^2 + ... + \mathrm{a}_r z_r^2 + 2\sum_{i=r+1}^{n} b_i z_i + c = 0 , \tag{13}$$

where $c = a - \frac{b_1^2}{\mathrm{a}_1} - ... - \frac{b_r^2}{\mathrm{a}_r}$.

We shall consider the different cases of the theorem.

1)Let $r = n$. Then (13) obtains the form (7) andcase 1 of the theorem is fulfilled together with the consider subcases.
2)Let $r = n-1$. Equation (13) obtains the form

$$\mathrm{a}_1 z_1^2 + ... + \mathrm{a}_{n-1} z_{n-1}^2 = -2b_n z_n - c \,. \tag{14}$$

We consider the following subcases.

2.1) Let $b_n \neq 0$. Since

$$2b_n z_n + c = 2b_n \left(z_n + \frac{c}{2b_n} \right),$$

then by making the translation $t_n = z_n + \frac{c}{2b_n}$, $t_1 = z_1,..., t_{n-1} = z_{n-1}$, (14) obtains form (8). Besides the subcaseshold for elliptical and hyperbolic paraboloids, i.e. statement 2.1 of the theorem is fulfilled.

2.2) Let $b_n = 0$. Then (14) obtains form (9), i.e. f is a cylinder and condition 2.2of the theorem is fulfilled.
3) Let $1 \le r \le n-2$. We consider the following subcases.
3.1) Let at leastone coefficient b_i in (13) is non-zero, $i = r+1,...,n$.Then, byTheorem1, wetransformthelinearhomogenouspartof(13) inalinearhomogenousform of one unknown t_n, applying an orthogonal transformation of the unknowns $z_{r+1},..., z_n$, expressed by $t_{r+1},...,t_n$ and supplemented with $z_1 = t_1,..., z_r = t_r$. In this way we obtain the equation

$$\mathrm{a}_1 t_1^2 + ... + \mathrm{a}_r t_r^2 + s t_n + c = 0, \quad s = \sqrt{b_{r+1}^2 + ... + b_n^2} > 0 \,.$$

For this equation we make the translation $u_1 = t_1,..., u_{n-1} = t_{n-1}$, $u_n = t_n + \frac{c}{s}$.Then (13) Obtain form (10),i.e. f is a cylinder and subcase 3.1 of the theorem holds.

3.2) Let $b_{r+1} = ... = b_n = 0$. Then f is also a cylinder and (13) obtain form (11), i.e. subcase 3.2 of the theorem is fulfilled.
The theorem is proved.□

We note that $r < n$, then the surface f belong to aparabolicclass.

5. Hypersurfaces of the maximal absolute and relative inaccuracies

In this section instead of an error of a physical experiment we shall use the concept an inaccuracy ((Kolikov *et al*., 2010; Kolikov *et al*., 2010; Kolikov *et al*., 2015)).

Let Y be an indirectly measurable variable depending on the directly measurable variables $X_1, X_2, ..., X_n$. Denote by f the real function of arguments $X_i \left(i = 1, 2, ..., n\right)$ such that $Y = f\left(X_1, X_2, ..., X_n\right)$. Let k observations of $x_{i1}, x_{i2}, ..., x_{ik}$ of $X_i \left(i = 1, 2, ..., n\right)$ are made in an experimental investigation.

The maximal absolute inaccuracy by the method of (Kolikov *et al*., 2010) is

$$\Delta^1 Y = \sum_{i=1}^{n} A_i \left|\Delta X_i\right|, \tag{15}$$

where

$$A_i = \frac{1}{k}\sum_{m=1}^{k}\left|\frac{\partial f}{\partial X_i}\left(x_{1m}, ..., x_{im}, ..., x_{nm}\right)\right|, \; i = 1, ..., n \tag{16}$$

and

$$\left|\Delta X_i\right| = \frac{1}{k}\sum_{j=1}^{k}\left|\Delta x_{ij}\right|, \; i = 1, 2, ..., n, \tag{17}$$

where Δx_{ij} are the maximal absolute inaccuracies of the directly measurable variables.

The maximal relative inaccuracy $\frac{\Delta^1 Y}{Y}$ of Y, according to (Kolikov *et al*., 2012), is

$$\frac{\Delta^1 Y}{Y} = \sum_{i=1}^{n} B_i \left|\frac{\Delta X_i}{X_i}\right|, \tag{18}$$

where

$$B_i = \frac{1}{k}\sum_{m=1}^{k}\left|\frac{x_{im}}{f\left(x_{1m}, ..., x_{nm}\right)}\frac{\partial f}{\partial X_i}\left(x_{1m}, ..., x_{nm}\right)\right|, \; i = 1, ..., n \tag{19}$$

and

$$\left|\frac{\Delta X_i}{X_i}\right| = \frac{1}{k}\sum_{j=1}^{k}\left|\frac{\Delta x_{ij}}{x_{ij}}\right|, \; i = 1, 2, ..., n, \tag{20}$$

where $\frac{\Delta x_{ij}}{x_{ij}}$ are the maximal relative inaccuracies of the directly measurable variables.

We note, that $\frac{\partial f}{\partial X_i}\left(x_{1m}, ..., x_{nm}\right)$ and $\frac{x_{im}}{f\left(x_{1m}, ..., x_{nm}\right)}\frac{\partial f}{\partial X_i}\left(x_{1m}, ..., x_{nm}\right)$ in (16) and (19) are the values of $\frac{\partial f}{\partial X_i}$ and $\frac{X_i}{f}\frac{\partial f}{\partial X_i}$, respectively, calculated in the m^{th} observation and A_i and B_i are the arithmetic mean of these values for $m = 1, 2, ..., k$.

The maximal absolute inaccuracy $\Delta^2 Y$ and the maximal relative inaccuracy $\frac{\Delta^2 Y}{Y}$ of the second order of $Y = f\left(X_1, X_2, ..., X_n\right)$ according to (Kolikov *et al*., 2015) are

$$\Delta^2 Y=\sum_{i,j=1}^{n} A_{ij}\left|\Delta X_i\right|\left|\Delta X_j\right| \text{ и } \frac{\Delta^2 Y}{Y}=\sum_{i,j=1}^{n} A_{ij}\left|\frac{\Delta X_i}{X_i}\right|\left|\frac{\Delta X_j}{X_j}\right|, \quad (21)$$

respectively, where A_{ij} of $\Delta^2 Y$ and of $\frac{\Delta^2 Y}{Y}$ are given by the equalities:

$$A_{ij}=\frac{1}{k}\sum_{m=1}^{k}\left|\frac{\partial^2 f}{\partial X_i \partial X_j}\left(x_{1m},...,x_{nm}\right)\right|,\ i,j=1,2,...,n \quad (22)$$

and

$$A_{ij}=\frac{1}{k}\sum_{m=1}^{k}\left|\frac{x_{im}x_{jm}}{f\left(x_{1m},...,x_{nm}\right)}\frac{\partial^2 f}{\partial X_i \partial X_j}\left(x_{1m},...,x_{nm}\right)\right|,\ i,j=1,2,...,n \quad (23)$$

respectively.

In (22) and (23) $\frac{\partial^2 f}{\partial X_i \partial X_j}\left(x_{1m},...,x_{nm}\right)$ and $\frac{x_{im}x_{jm}}{f\left(x_{1m},...,x_{nm}\right)}\frac{\partial^2 f}{\partial X_i \partial X_j}\left(x_{1m},...,x_{nm}\right)$ arethe values of $\frac{\partial^2 f}{\partial X_i \partial X_j}$ and $\frac{X_i X_j}{f}\frac{\partial^2 f}{\partial X_i \partial X_j}$, respectively, calculatedin the*m*-thobservationand A_{ij} is the arithmetic mean of these values for $m=1,2,...,k$.

The maximal absolute inaccuracy ΔY of Y of the second approximation we call the function

$$\Delta Y=\Delta^1 Y+\frac{1}{2}\Delta^2 Y. \quad (24)$$

The maximal relatively inaccuracy $\frac{\Delta Y}{Y}$ of Y of the second approximation we call the function

$$\frac{\Delta Y}{|Y|}=\frac{\Delta^1 Y}{|Y|}+\frac{1}{2}\frac{\Delta^2 Y}{|Y|}. \quad (25)$$

In (Kolikov *et al*., 2010; Kolikov, 2012; Kolikov, 2015) we assume that ΔX_i and $\frac{\Delta X_i}{X_i}$ $\left(i=1,2,...,n\right)$ in (15) and (18) are unknown values with constant coefficients. Then (15) and (21) imply, that (24) has the form

$$\Delta Y=\sum_{i=1}^{n} A_i.\left|\Delta X_i\right|+\frac{1}{2}\sum_{i,j=1}^{n} A_{ij}.\left|\Delta X_i\right|.\left|\Delta X_j\right|. \quad (26)$$

From (18) and (21) weobtainanalogously, that(25) has the form

$$\frac{\Delta Y}{Y}=\sum_{i=1}^{n} B_i.\left|\frac{\Delta X_i}{X_i}\right|+\frac{1}{2}\sum_{i,j=1}^{n} B_{ij}.\left|\frac{\Delta X_i}{X_i}\right|.\left|\frac{\Delta X_j}{X_j}\right|. \quad (27)$$

Wechange ΔY with y_{n+1} and ΔX_i with x_i in (26) and, analogously, $\frac{\Delta Y}{Y}$ with y_{n+1} and $\frac{\Delta X_i}{X_i}$ with x_i in (27) $\left(i=1,2,...,n\right)$.

Thenwe obtain, that the maximal absolute and relative inaccuracies of Y of the second approximation in $\left(n+1\right)$-dimensional affine Euclidean space E_{n+1} have the general kind

$$y_{n+1}=\sum_{i,j=1}^{n} a_{ij}x_i x_j+2\sum_{i=1}^{n} a_i x_i, \quad (28)$$

where a_{ij} and a_i are non-negative constants, $a_{ij}=a_{ji}$ and at least one of the coefficients is distinct from zero ($i,j=1,2,...,n$).

We shall make an algebraic classification of (28), i.e. a classification of the surfaces of the maximal inaccuracies of Y of the second approximationasa corollary from the canonization of the surfaces in the many-dimensional affine Euclidean space E_{n+1}, i.e. as acorollary of Theorems 2 and 3.

Theorem 4. Let r be the rang of the quadratic part of f_2 of the surface f ofthe maximal absolute (relative) inaccuracy of the second approximation in the real affine Euclidean space E_{n+1}, $n \geq 1$. Then f is from parabolic typeand the following cases hold.
(i)Let $r = n$. Then the canonical equation of f is

$$\alpha_1 z_1^2 + \ldots + \alpha_n z_n^2 = z_{n+1}, \tag{29}$$

i.e. f is a paraboloid. Besides, ifthecharacteristicroots $\alpha_1, \ldots, \alpha_n$ of f_2 arewithsamesigns, then f isan*ellipticparaboloid*and if at least twocharacteristicrootsof f_2 are with opposite signs, then f is a *hyperbolicparaboloid*.
(ii) Let $1 \leq r \leq n-1$. Then the canonical equation of f is

$$\alpha_1 z_1^2 + \ldots + \alpha_r z_r^2 = -S z_n, \quad S = \sqrt{b_{r+1}^2 + \ldots + b_n^2 + \frac{1}{4}}, \tag{30}$$

where b_i aredefinedby (5), i.e. f is ac*ylinder*.
(iii) If $r = 0$, then the canonical form of f is $z_{n+1} = 0$, i.e. f is a *hyperplane*, defined from thecoordinate axes $O_1 z_1, \ldots, O_1 z_n$ of same coordinate system $O_1 z_1 z_2 \ldots z_{n+1}$ of E_{n+1}.

Proof. For the obtaining of the result we shall applyTheorem 3 and 2. Under the use of these theoremswe have to change E_n by E_{n+1}, $n$by $n+1$ and b_n by $b_{n+1} = -\frac{1}{2}$. The equations (28) and (29) we shall obtain directlyfrom the cases 2.1 и 3.1 of the defined Theorem 3.

Really, if $r = n$, then the canonical equation (8) (of case 2.1) of Theorem 3 obtains the form (29),sinse $b_{n+1} = -\frac{1}{2} \neq 0$. The case(i) is completed.

Let $1 \leq r \leq n-1$. It holds case 3 only withsubcase 3.1 with equation (10) which obtains the form (30), i.e.case (ii) is completed. If $r = 0$, then case(iii) is obtained by case 1 of Theorem 2.

The proof is competed.□

A significant part of the proved Theorem 4 canbeobtainedfromtheknowncanonicalformsofthehyperspacesoftheseconddegree ((Efimov, 2010), (Konstantinov, 2000) and (Shafarevich, 2013)), but indirectly, by same non-trivial additional reasonings. Wenoteexplicitly, thatcase (iii) ofTheorem 4 cannotbeobtainedfromtheknowncanonizationsofthehypersurfaces ((Efimov, 2005), (Konstantinov, 2000) и (Shafarevich, 2013)), sincecase (iii)is obtained from our original Theorem 2 which gives the canonical form of an arbitraryhyperplane in E_n. Furthermore, we indicate in Theorem 4 the exact parameters of the canonical forms.

Corollary 5. If the rang r of the quadraticpart f_2 of the hypersur face of the maximalabsolute (relative) inaccuracyofthesecondapproximationin F_2 is 1, then the canonical equation of this surface is the parabola $y = \alpha_1 x^2$, where α is non-zero characteristic root of f_2.

Proof. It holds only case**(i)**of formulated Theorem4and the canonical equation $y = \alpha_1 x^2$ is obtained from (29) by the changes $z_1 = x$ and $z_2 = y$.□

Conclusion

Our approach for a canonization of the hypersurfaces of the second degree in the many-dimensional space stress more to the algebraic part of the question since the geometric interpretations are well known. This approach is effective since it gives the exact coefficients of the given equation of the surface. Namely this effectiveness together with Theorem 2, gives us a possibility to obtain the canonical equation of the hypersurfaces of the maximal inaccuracies (errors).

REFERENCES

Efimov, N. And Rosendorn, E. 2005. *Linear algebra and multidimensional geometry*, Fizmatlit. (in Russian)

Kolikov, K., Krastev, G., Epitropov, Y., Hristozov, D. 2010. *Analytically determining of the absolute inaccuracy (error) of indirectly measurable variable and dimensionless scale characterising the quality of the experiment*, Chemometr Intell Lab, 102, pp. 15-19 http://dx.doi.org/10.1016/j.chemolab.2010.03.001

Kolikov, K., Krastev, G., Epitropov, Y., Hristozov, D. 2010. *Method for analytical representation of the maximum inaccuracies of indirectly measurable variable (survey)*, Proceedings of the Anniversary International Conference “Research and Education in Mathematics, Informatics and their Applications” – REMIA 2010, Bulgaria, Plovdiv, 10-12 December, pp. 159-166, 2010. http://hdl.handle.net/10525/1449

Kolikov, K., Krastev, G., Epitropov, Y., Corlat A. 2012. *Analytically determining of the relative inaccuracy (error) of indirectly measurable variable and dimensionless scale characterising the quality of the experiment*, CSJM, vol. 20, no. 1, pp. 314-331.http://www.math.md/en/publications/csjm/issues/v20-n1/11035/

Kolikov, K., Epitropov, Y., Corlat, A., Krastev, G. 2015. *Maximum Inaccuracies of Second Order*, CSJM v.23, no.1 (67). http://www.math.md/en/publications/csjm/issues/v23-n1/11891/

Konstantinov, M., *Elements of analytical geometry: Curves and surfaces of the second degree*, Sofia, 2000. (in Bulgarian)

Shafarevich, I. & Remizov, A., *Linear algebra and geometry*, Springer, 2013

****