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RESEARCH ARTICLE

VARIATIONAL PRINCIPLE FOR DETERMINING THE STRESS-STRAIN STATE OF POROUS BODIES

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ABSTRACT

We know that porosity is a dimensionless quantity, it does not depend on the size of the particles that make up the porous medium. More precisely, if we imagine two porous bodies that are geometrically similar to each other at the micro level and differ only in the particle size, then their porosity will be the same. The bottom line is that in the coarse-grained material the pores are larger, but their number per unit volume of the medium is smaller and these effects just compensate each other. When calculating the stability of thin-walled constructions, it is necessary to take into account geometric nonlinearity. Accounting for geometric nonlinearity in turn leads to great mathematical difficulties in solving stability problems. To avoid mathematical difficulties, an approximate method of mathematics is usually used. One of the effective approximate methods of mathematics is the variational method. In this paper we propose a functional whose Euler equations are equilibrium equations with allowance for geometric nonlinearity, physical relationships for porous bodies, and also boundary conditions with allowance for geometric nonlinearity.

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INTRODUCTION

Many natural bodies are porous: soils, rocks, wood, leather, bone, soft tissues of animals, as well as artificial materials: building (concrete, brick), food (bread), artificial leather, ceramics, metal parts obtained by powder metallurgy , etc. Porous is the soil, the upper layer of soil, which serves as the basis of agriculture. This simple enumeration already shows the enormous role played by porous wednesdays in people's lives. A characteristic feature of all these materials is the ability to accumulate a fluid and allow it to move under the action of external forces (Gasanov *et al*., 2016). The most important quantitative characteristic of porous bodies is their porosity m, defined as the fraction of body volume per pore, or the pore volume per unit volume of material. The porosity of most materials is in the range 0.1-0.4.Taking the value typical for many rocks for m = 0.25, we find that in 1 m^3 of the rock the pore volume is ~ 0.25 m^3 = 250 liters. When it comes to oil and gas reservoirs or layers saturated with fresh water in desert areas, porosity is the main parameter, because it determines the reserves of the deposit, that is, the amount of fluid in the reservoir. Porosity is a dimensionless quantity, it does not depend on the size of the particles that make up the porous medium. More precisely, if we imagine two porous bodies that are geometrically similar to each other at the micro level and differ only in the particle size, then their porosity will be the same. The bottom line is that in the coarse-grained material the pores are larger, but their number per unit volume of the medium is smaller and these effects just compensate each other. When calculating the stability of thin-walled constructions, it is necessary to take into account geometric nonlinearity. Accounting for geometric nonlinearity in turn leads to great mathematical difficulties in solving stability problems. To avoid mathematical difficulties, an approximate method of mathematics is usually used. One of the effective approximate methods of mathematics is the variational method. In this paper we propose a functional whose Euler equations are equilibrium equations with allowance for geometric nonlinearity, physical relationships for porous bodies, and also boundary conditions with allowance for geometric nonlinearity.

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Formulation of the problem

Consider a porous body, the pores of which are filled with liquid. Such a wednesday is considered two-phase. As is known, the instantaneous modulus of elasticity of such a porous body is determined by the following formula (Khasanov *et al*., 2011):

$$E_{ci} == \frac{m_s E_s + m_l \cdot E_l}{m} \tag{1}$$

Where E_{ci}-is the reduced elastic modulus, m_s-is the mass of the skeleton, m-is the mass of the porous body saturated with liquid, m_l-is the mass of the liquid in the pores, E_s- is the Young's modulus of the skeleton, and E_l-is the Young's modulus of the liquid upon compression. When the porous body is under the influence of external influences, it is deformed and the liquid filling the pores begins to move from the pores with higher pressures to the pores with low pressures, i.e. In the direction of the fluid pressure gradient in the pores. Consequently, the mass of the liquid in individual pores changes its value, i.e. E_{ci} is a function of time. In the book (Gulgazli, 2012) the following basic law of plasticity is obtained: *With uniaxial tension, the true residual stresses are directly proportional to the residual deformations.* Proceeding from this law, the following state equations for porous bodies were obtained in (Gasanov *et al*., 2016):

$$\varepsilon_{ij} = \frac{1-k_0}{E_{\text{пр}}}[(1+\nu)\sigma_{ij} - \nu I_1 \delta_{ij}] \tag{2}$$

Where

$$k_0 = \sqrt{\frac{\sigma_m^2}{I_1^2 + 2(1+\nu)I_2} - 1} \tag{3}$$

σ_m - the maximum value of the normal stress, which can always be determined from the experiment by uniaxial tension. ε_{ij}, σ_{ij} –Components of deformation and stress tensors, δ_{ij} –Kronecker symbols, I_1, I_2 –Respectively, the first and second invariants of the stress tensor.

σ_m

In the special case, when the loading is dynamic $k_0 = 0$, and (2) takes the form::

$$\varepsilon_{ij} = \frac{1}{E_{\text{пр}}}[(1+\nu)\sigma_{ij} - \nu I_1 \delta_{ij}] \tag{4}$$

The system (4) is Hooke's law for linearly elastic bodies.

Thus, if the process of deformation in porous bodies is quasistatic, then system (2) should be used as the equation of state, if the process is dynamic, then system (4) should be used. E_{ci}- entering in (2) and (4) is defined by the equality (1). Let the porous body of volume V be under the action of external forces. Surface forces are given on the surface part S_σ, and displacements are given on the rest of S_u surface Then the equation of equilibrium, taking into account the geometric nonlinearity, the connection between the components of the strain tensor and stresses and the boundary conditions in the Cartesian coordinate system have the following form (Amenezade, 1976)

$$\left[\sigma_{ij}(u_{\alpha,i} + \delta_{\alpha i})\right]_{,j} = 0 \tag{5}$$

$$\varepsilon_{ij} = (1-k_0)\left(\frac{1+\nu}{E_{\text{пр}}}\sigma_{ij} - \frac{\nu I_1}{E_{\text{пр}}}\delta_{ij}\right) \tag{6}$$

$$\begin{cases} \sigma_{ij}(\delta_{\alpha i} + u_{\alpha,i}) \cdot n_j = \bar{N}_\alpha \text{на} S_\sigma \\ u_i = \bar{u}_i \text{на} S_u \end{cases} \tag{7}$$

Where commas means differentiation with respect to the coordinate with the index that follows after commas, u_α –components of the displacement vector, n_j –components of the normal vector, $\bar{N}_i$-Components of the surface force vector on S_σ, $\bar{u}_i$-Surface movements S_u,For repeated indices, summation is from 1 to 3.We rewrite system (6) as follows:

$$\varepsilon_{ij} = \left(\frac{1-k_0}{E_{\text{пр}}}\right)[(1+\nu)\sigma_{ij} - \nu \cdot I_1 \delta_{ij}] \tag{8}$$

We introduce the following notation:

$$K=\frac{1-k_0}{E_{\text{пр}}} \tag{9}$$

Taking into account (9) in (8) we have:

$$\varepsilon_{ij}=\mathrm{K}[(1+\nu)\sigma_{ij}-\nu I_1 g_{ij}] \tag{10}$$

The Poisson coefficient -ν varies insignificantly, so we assume that ν= $const.$

The proposed functional has the following form (Alizade *et al*., 1979):

$$J=\int_V\left\{\dot\sigma_{ij}\dot\varepsilon_{ij}+\frac{1}{2}\sigma_{ij}\dot u_{\alpha,j}\dot u_{\alpha,i}-\dot\sigma_{ij}[\dot K\left((1+\nu)\sigma_{ij}-\nu I_1\delta_{ij}\right)+K((1+\nu)\dot\sigma_{ij}-\nu\dot I_1\delta_{ij})]-\int_{s_\sigma}\dot{\bar N}_i\dot u_i ds+\right.$$
$$\int_{s_u}(\dot u_i-\dot{\bar u}_i)\dot N_i ds \tag{11}$$

Where the dot over the values denotes the time derivative. Let us prove that the Euler equations of the proposed functional yield the system (5), (6), (7). We calculate the first variation of the functional (11) We assume that only the rates of displacements and stresses variation.

$$\delta J=\int_V\{\dot\sigma_{ij}\delta\dot\varepsilon_{ij}+\dot\varepsilon_{ij}\delta\dot\sigma_{ij}+\sigma_{ij}\dot u_{\alpha,j}\delta\dot u_{\alpha,i}-[\dot K((1+\nu)\sigma_{ij}-\nu I_1\delta_{ij})+K((1+\nu)\dot\sigma_{ij}-\nu\dot I_1\delta_{ij})]\delta\dot\sigma_{ij}-\dot\sigma_{ij}[((1+$$
$$\nu)\sigma_{ij}-\nu I_1\delta_{ij})\cdot\delta\dot K+((1+\nu)\delta\dot\sigma_{ij}--\nu\delta_{ij}\delta_{kl}\delta\dot\sigma_{kl})]\}dV-\int_{s_\sigma}\dot{\bar N}_i\delta\dot u_i\,ds+\int_{s_u}(\dot u_i-\dot{\bar u}_i)\delta\dot N_i ds. \tag{12}$$

Here we took into account, that $\delta\dot N_i=0$ on the S_σ and $\delta\dot u_i=0$ on the S_u, because on the S_σ is given $\dot N_i$, but on the S_u is given $\dot u_i$, therefore, they cannot vary. Besides, $I_1=\sigma_{kl}\delta_{kl}$. We transform the integrands. It is known that (Amenezade, 1976)

$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}+u_{\alpha,i}\cdot u_{\alpha,j})$$

Then:

$$\dot\varepsilon_{ij}=\frac{1}{2}(\dot u_{i,j}+\dot u_{j,i}+\dot u_{\alpha,i}u_{\alpha,j}+u_{\alpha,i}\dot u_{\alpha,j})$$

$$\delta\dot\varepsilon_{ij}=\frac{1}{2}(\delta\dot u_{i,j}+\delta\dot u_{j,i}+u_{\alpha,j}\delta\dot u_{\alpha,i}+u_{\alpha,i}\,\delta\dot u_{\alpha,j})=$$
$$=\frac{1}{2}(\delta\dot u_{i,j}+u_{\alpha,i}\delta\dot u_{\alpha,j})+\frac{1}{2}(\delta\dot u_{j,i}+u_{\alpha,j}\delta\dot u_{\alpha,i}) \tag{13}$$

Given that $\delta\dot u_{i,j}=\delta_{\alpha i}\delta\dot u_{\alpha,j}$ и $\delta\dot u_{j,i}=\delta_{\alpha j}\delta\dot u_{\alpha,i}$, from (13) we have:

$$\delta\dot\varepsilon_{ij}=\frac{1}{2}\left(\delta_{\alpha i}+u_{\alpha,i}\right)\delta\dot u_{\alpha,j}+\frac{1}{2}\left(\delta_{\alpha j}+u_{\alpha,j}\right)\delta\dot u_{\alpha,i} \tag{14}$$

We multiply both sides of (14) by $\dot\sigma_{ij}$ иWe sum over i, j from 1 to 3, then

$$\dot\sigma_{ij}\cdot\delta\dot\varepsilon_{ij}=\frac{1}{2}\dot\sigma_{ij}\cdot(\delta_{\alpha i}+u_{\alpha,i})\delta\dot u_{\alpha,j}+\frac{1}{2}\dot\sigma_{ij}\cdot(\delta_{\alpha j}+u_{\alpha,j})\delta\dot u_{\alpha,i}$$

If we take into account the summation over the indices i, j, then in the second part of the right side of the last equality, replacing i by j, and j by i, we obtain;

$$\dot\sigma_{ij}\cdot\delta\dot\varepsilon_{ij}=\frac{1}{2}\dot\sigma_{ij}\cdot(\delta_{\alpha i}+u_{\alpha,i})\delta\dot u_{\alpha,j}+\frac{1}{2}\dot\sigma_{ji}\cdot(\delta_{\alpha i}+u_{\alpha,i})\delta\dot u_{\alpha,j}$$

If we bear in mind the symmetry of the components of the stress tensor with respect to the indices, i.e. $\dot\sigma_{ij}=\dot\sigma_{ji}$ then the last expression will be:

$$\dot\sigma_{ij}\delta\dot\varepsilon_{ij}=\frac{1}{2}\dot\sigma_{ij}(\delta_{\alpha i}+u_{\alpha,i})\delta\dot u_{\alpha,j}+\frac{1}{2}\dot\sigma_{ij}(\delta_{\alpha i}+u_{\alpha,i})\delta\dot u_{\alpha,j}=$$
$$=\dot\sigma_{ij}(\delta_{\alpha i}+u_{\alpha,i})\delta\dot u_{\alpha,j}$$

Taking this equality into account, the first term of (12) is transformed as follows

$$\int_V \dot{\sigma}_{ij}\delta\dot{\varepsilon}_{ij}dv = \int_V \dot{\sigma}_{ij}(\delta_{\alpha i}+u_{\alpha,i})\delta\dot{u}_{\alpha,j}dv =$$
$$\int_V [\dot{\sigma}_{ij}(\delta_{\alpha i}+u_{\alpha,i})\delta\dot{u}_\alpha]_{,j}dv - -\int_V [\dot{\sigma}_{ij}(\delta_{\alpha i}+u_{\alpha,i})]_{,j}\delta\dot{u}_\alpha dv \quad (15)$$

When obtaining (15), we took into account the symmetry of the tensor σij. Applying the Gauss-Ostrogradsky theorem, we obtain from (15):

$$\int_V \dot{\sigma}_{ij}\delta\dot{\varepsilon}_{ij}dv = \int_{s_\sigma+s_u} \dot{\sigma}_{ij}(\delta_{\alpha i}+u_{\alpha,i})\delta\dot{u}_{\alpha,j}n_j ds -$$
$$-\int_V [\dot{\sigma}_{ij}(\delta_{\alpha i}+u_{\alpha,i})]_{,j}\delta\dot{u}_\alpha dv \quad (16)$$

where n_j –components of the normal to the surface S. Applying the Gauss-Ostrogradsky theorem, we transform the third term in (12) as follows

$$\int_V \sigma_{ij}\dot{u}_{\alpha,i}\delta\dot{u}_{\alpha,j}dv = \int_V [\sigma_{ij}\dot{u}_{\alpha,i}\delta\dot{u}_\alpha]\,dv -$$
$$-\int_V [\sigma_{ij}\dot{u}_{\alpha,i}]_{,j}\delta\dot{u}_\alpha dv = \int_{S_\sigma+S_u} \sigma_{ij}\dot{u}_{\alpha,i}\delta\dot{u}_\alpha n_j ds -$$
$$-\int_V [\sigma_{ij}\dot{u}_{\alpha,i}]_{,j}\delta\dot{u}_\alpha dv \quad (17)$$

Thus, for the first variation, taking into account the transformations (16) and (17), we obtain:

$$\delta J = \int_V [\sigma_{ij}(u_{\alpha,i}+\delta_{\alpha i})]^{\cdot}_{,j}\delta\dot{u}_\alpha dv +$$
$$+\int_V [(\dot{\varepsilon}_{ij} - [\dot{K}((1+\nu)\sigma_{ij} - \nu I_1\delta_{ij}) + K((1+\nu)\dot{\sigma}_{ij} - \nu\dot{I}_1\delta_{ij})])\delta\dot{\sigma}_{ij}]dv$$
$$+\int_{S_\sigma} [\sigma_{ij}(u_{\alpha,i}+\delta_{\alpha i})]^{\cdot}\, n_j\delta\dot{u}_\alpha dv - \int_{S_\sigma} \dot{\bar{N}}_i\delta\dot{u}_i\, ds + \int_{S_u} (\dot{u}_i - \dot{\bar{u}}_i)\delta\dot{N}_i ds$$

Equating to zero δJ and using the main lemma of the calculus of variations, we obtain

$$[\sigma_{ij}(u_{\alpha,i}+\delta_{\alpha i})]^{\cdot}_{,j} = 0 \quad (18)$$

$$\dot{\varepsilon}_{ij} = \frac{d}{dt}\{K[(1+\nu)\sigma_{ij} - \nu I_1\delta_{ij}]\} \quad (19)$$

$$[\sigma_{ij}(u_{\alpha,i}+\delta_{\alpha i})n_j]^{\cdot} = \dot{N}_\alpha \text{на} S_\sigma\,; \quad \dot{u}_i = \dot{\bar{u}}_i \text{на} S_u \quad (20)$$

Integrating (18), (19), (20) with respect to time and taking into account that there was no liquid transfer at the initial instant of time, we obtain the complete system (5), (6), (7). Thus, we showed that the stationary of the proposed functional is achieved on functions describing the stress-strain state of a porous body with allowance for geometric nonlinearity and the effect of porosity on the mechanical characteristics of materials. Get an analytical solution to complex problems, always associate with great mathematical difficulties. The advantage of the proposed functional (11) is that in the practical application of this functional, after taking approximations for displacements and integrating over spatial coordinates, we do not obtain a system of nonlinear algebraic equations, which happens in the case when the stresses and deformations themselves vary independently, and the system Linear ordinary differential equations, whose numerical integration is much easier than solving a system of nonlinear algebraic equations.

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