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RESEARCH ARTICLE

DETERMINATION OF STRAIN-DEFORMATION STATEIN THE MEDUIIM OF THE INITIAL STRAIN CONDITION

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ABSTRACT

It has been determined that when mass forces at plane deformation state aren't considered strain situation at any one-communication area of initial strain viscous –elastic medium depending on the given boundary conditions and its form doesn't depend in mechanical properties of the material. Plane strain situation of viscous-elastic medium has been determined when boundary conditions obviously depend on time.

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INTRODUCTION

Estimation of side pressure influencing casings in complicated rheological condition in big depths forms the basis of efficiency in reliable consolidation of the well walls and preparing of pipelines. It is known that before the drilling mine rocks are always at initial strain state. As mine rocks are always under the pressure when determining their strain deformation state their initial strain state has to be considered. When mass forces are not considered in plane deformation state, then strain state in any one bond area of viscous –elastic medium being dependent on boundary conditions and its form does not depend on mechanical properties of the material. Let's study strain –deformation state of the viscous-elastic medium under initial stress by applying Levi theorem let's think that under initial stress, viscous-elastic body is subjected to plane deformation on *OXY* plane. Then

$$u_1 = u_1(x, y, t)$$
$$u_2 = u_2(x, y, t) \qquad \text{.........................(1)}$$
$$u_3 = 0$$

Here x,y-are plane coordinates; t- is time, u_1, u_2, u_3- are correspondingly displacements on x,y, z coordinates.

When deformation is small and mass forces are not considered equilibrium equations are as following:

$$\frac{\partial \sigma_{11}}{\partial x} + \frac{\partial \sigma_{12}}{\partial y} = 0;$$
$$\qquad \text{.........................(2)}$$
$$\frac{\partial \sigma_{21}}{\partial x} + \frac{\partial \sigma_{22}}{\partial y} = 0.$$

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Here $\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}$ are components of strain tensor components. If to denote initial strain tensor components as $\sigma_{11}^0, \sigma_{12}^0, \sigma_{21}^0, \sigma_{22}^0$ and to consider that the object under the initial strain

Is balanced then

$$\frac{\partial \sigma_{11}^0}{\partial x} + \frac{\partial \sigma_{12}^0}{\partial y} = 0; \qquad \frac{\partial \sigma_{21}^0}{\partial x} + \frac{\partial \sigma_{22}^0}{\partial y} = 0. \quad \text{..................................(3)}$$

If to consider (3) in (2)

$$\frac{\partial \overline{\sigma}_{11}^0}{\partial x} + \frac{\partial \overline{\sigma}_{12}^0}{\partial y} = 0; \qquad \frac{\partial \overline{\sigma}_{21}^0}{\partial x} + \frac{\partial \overline{\sigma}_{22}^0}{\partial y} = 0. \quad \text{.................................(4)}$$

$$\overline{\sigma}_{11} = \sigma_{11} - \sigma_{11}^0; \;\; \overline{\sigma}_{12} = \sigma_{12} - \sigma_{12}^0;$$
$$\overline{\sigma}_{21} = \sigma_{21} - \sigma_{21}^0; \;\; \overline{\sigma}_{22} = \sigma_{22} - \sigma_{22}^0.$$

When medium is under initial strain the deformation is formed at the expense of stress change. That's why physical relations are as following [2]

$$\left.\begin{aligned} \varepsilon_{11} &= \frac{1}{E}\left[\overline{\sigma}_{11} - \mu\left(\overline{\sigma}_{22} + \overline{\sigma}_{33}\right)\right] + \frac{1}{E}\int_0^t H(t,\tau)\left[\overline{\sigma}_{11} - \mu\left(\overline{\sigma}_{22} + \overline{\sigma}_{33}\right)\right]d\tau \\ \varepsilon_{22} &= \frac{1}{E}\left[\overline{\sigma}_{22} - \mu\left(\overline{\sigma}_{11} + \overline{\sigma}_{33}\right)\right] + \frac{1}{E}\int_0^t H(t,\tau)\left[\overline{\sigma}_{22} - \mu\left(\overline{\sigma}_{11} + \overline{\sigma}_{33}\right)\right]d\tau \\ \varepsilon_{12} &= \frac{1+\mu}{E}\overline{\sigma}_{12} + \frac{1+\mu}{E}\int_0^t H(t,\tau)\overline{\sigma}_{12}d\tau \end{aligned}\right\} \quad \text{..................................(5)}$$

Here E – is Young's modulus; μ – Puasson coefficient. $H(t,\tau)$ – is nucleus.

If to consider. $u_{33} = 0$ then for ε_{33} we'll get

$$\varepsilon_{33} = \frac{\partial u_3}{\partial x_3} = 0$$

From physical relations we get

$$\varepsilon_{33} = \frac{1}{E}\left[\overline{\sigma}_{33} - \mu\left(\overline{\sigma}_{11} + \overline{\sigma}_{22}\right)\right] + \frac{1}{E}\int_0^t H(t,\tau)\left[\overline{\sigma}_{33} - \mu\left(\overline{\sigma}_{11} + \overline{\sigma}_{22}\right)\right]d\tau \quad \text{.................................(6)}$$

If to accept the following

$$\overline{y}_1(x,y,t) = \overline{\sigma}_{33} - \mu\left(\overline{\sigma}_{11} + \overline{\sigma}_{22}\right)$$

then equality (6) is as following

$$\overline{y}_1 + \int_0^t H(t,\tau)\overline{y}d\tau = 0 \quad \text{.................................(7)}$$

When $t = 0$

as $\bar{y}_1(x,y,0)=0$

Solution of homogenous equality is $\bar{y}_1=0$; that is

$$\bar{\sigma}_{33}=\mu(\bar{\sigma}_{11}+\bar{\sigma}_{22})$$

Then solution of homogenous integral equation (6) is:

$$\bar{\sigma}_{33}-\mu(\bar{\sigma}_{11}+\bar{\sigma}_{22})=0 \qquad \text{.....(8)}$$

If to consider (8) in (5), time equations have the following form;

$$\left.\begin{aligned}\varepsilon_{11}&=\frac{1+\mu}{E}[\bar{\sigma}_{11}-\mu(\bar{\sigma}_{22}+\bar{\sigma}_{22})]+\int_0^t H(t,\tau)[\bar{\sigma}_{11}-\mu(\bar{\sigma}_{11}+\bar{\sigma}_{22})]d\tau\\ \varepsilon_{22}&=\frac{1+\mu}{E}[\bar{\sigma}_{22}-\mu(\bar{\sigma}_{11}+\bar{\sigma}_{22})]+\int_0^t H(t,\tau)[\bar{\sigma}_{22}-\mu(\bar{\sigma}_{11}+\bar{\sigma}_{22})]d\tau\\ \varepsilon_{12}&=\frac{1+\mu}{E}\left[\bar{\sigma}_{12}+\int_0^t H(t,\tau)\bar{\sigma}_{12}d\tau\right]\end{aligned}\right\} \qquad \text{.....(9)}$$

Only one of the mutuality equations of deformation remains

$$\frac{\partial^2\varepsilon_{11}}{\partial y^2}+\frac{\partial^2\varepsilon_{22}}{\partial x^2}=2\frac{\partial^2\varepsilon_{12}}{\partial x\partial y} \qquad \text{.....(10)}$$

From equation (4)

$$\frac{\partial^2\bar{\sigma}_{11}}{\partial x^2}+\frac{\partial^2\sigma_{22}}{\partial y^2}=-2\frac{\partial^2\sigma_{12}}{\partial x\partial y} \qquad \text{.....(11)}$$

If to consider (9) in (10);

$$\begin{aligned}&\frac{\partial^2}{\partial y^2}\left\{[\bar{\sigma}_{11}-\mu(\bar{\sigma}_{11}+\bar{\sigma}_{22})]+\int_0^t H(t,\tau)[\bar{\sigma}_{11}-\mu(\bar{\sigma}_{11}+\bar{\sigma}_{22})]d\tau\right\}+\\ &+\frac{\partial^2}{\partial x^2}\left\{[\bar{\sigma}_{22}-\mu(\bar{\sigma}_{11}+\bar{\sigma}_{22})]+\int_0^t H(t,\tau)[\bar{\sigma}_{22}-\mu(\bar{\sigma}_{11}+\bar{\sigma}_{22})]d\tau\right\}=\\ &=2\frac{\partial^2}{\partial x\partial y}\left[\bar{\sigma}_{12}+\int_0^t H(t,\tau)\bar{\sigma}_{12}d\tau\right]\end{aligned} \qquad \text{.....(12)}$$

or if to consider (11)

$$\Delta(\bar{\sigma}_{11}+\bar{\sigma}_{22})+\int_0^t H(t,\tau)\Delta(\bar{\sigma}_{11}+\bar{\sigma}_{22})d\tau=0 \qquad \text{.....(13)}$$

If to consider $\Delta(\bar{\sigma}_{11}+\bar{\sigma}_{22})=0$ .....(14)

at $t=0$ moment then homogenous equation (13) can be as following, in (13) and (14) $\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ – is Laplace's operator, let's accept $\bar{y}_2=\Delta(\bar{\sigma}_{11}+\bar{\sigma}_{22})$ then we get;

If to consider that at the initial moment the material elasticity is subjected to elastic deformation and Levi relations are compensated, then

$$\bar{y}_2 + \int_0^t H(t,\tau)\bar{y}_2 d\tau = 0 \qquad \text{.........................(15)}$$

$$\begin{aligned}\bar{y}_2(0) &= \Delta(\bar{\sigma}_{11} + \bar{\sigma}_{22}) = \Delta(\sigma_{11} + P_\infty + \sigma_{22} + P_\infty) = \\ &= \Delta(\sigma_{11} + \sigma_{22}) + 2\Delta P_\infty = \Delta(\sigma_{11} + \sigma_{22}) = 0\end{aligned} \qquad \text{.........................(16)}$$

Equation (16) coincides with Levi equation. If to include Eri stress functions

$$\bar{\sigma}_{11} = \frac{\partial^2\Phi}{\partial y^2}; \quad \bar{\sigma}_{12} = -\frac{\partial^2\Phi}{\partial x \partial y}; \quad \bar{\sigma}_{33} = \frac{\partial^2\Phi}{\partial x^2}.$$

then $\Delta\Delta\Phi = 0$.

Thus, if not to consider mass-forces at plane deformation state, though strain situation at any one communication area of initial strain viscous-elastic medium depends on the given boundary conditions and boundary form it doesn't depend on mechanical properties of the material. That is when homogenous initial strain sitiation not strains themselves, is considered but their changes compensate Lame equations.

Let's determine stress-deformation state of viscous–elastic rocks in the initial stress by the proved theorem.

When gravitation force isn't considered balance equations for rocks under the initial stresses will be as following in polar coordinates for the case symmetric to axis:

$$\frac{d(\sigma_r^{(3)} + P_\infty)}{d\tau} + \frac{\sigma_r^{(3)} - \sigma_t^{(3)}}{\tau} = 0 \qquad \text{..............................(17)}$$

$\sigma_r^{(3)}, \sigma_t^{(3)} -$ are correspondingly radial and tangential stresses. P_∞ is infinite pressure of rocks. as the rocks are accepted viscous-elastic physical relations can be as following:

$$\begin{aligned}\varepsilon_r^{(3)} &= \frac{1}{E}\left[\sigma_t^{(3)} + P_\infty - \mu(\sigma_t^{(3)} + 2P_\infty + \sigma_z^{(3)})\right] + \frac{1}{E}\int_0^t H^{(3)}(t-\tau) \times \\ &\times \left[\sigma_t^{(3)} + P_\infty - \mu(\sigma_t^{(3)} + \sigma_z^{(3)} + 2P_\infty)\right] d\tau \\ \varepsilon_t^{(3)} &= \frac{1}{E}\left[\sigma_t^{(3)} + P_\infty - \mu(\sigma_t^{(3)} + \sigma_z^{(3)} + 2P_\infty)\right] + \frac{1}{E}\int_0^t H^{(3)}(t-\tau) \times \\ &\times \left[\sigma_t^{(3)} + P_\infty - \mu(\sigma_t^{(3)} + \sigma_z^{(3)} + 2P_\infty)\right] d\tau\end{aligned} \qquad \text{..............................(18)}$$

$\varepsilon_r^{(3)}, \varepsilon_t^{(3)} -$ are correspondingly radial and tangential deformations in the rocks. E is Young's modulus for rocks.

$\mu -$ isPuasson coefficient for rocks;

$H^{(3)}(t-\tau)$ - is nuclear.

As the task is symmetric:

$$U_\varphi^{(3)} = 0; \; U_z^{(3)} = 0; \; U_\tau^{(3)} = U_r^{(3)}(r,t) \qquad \text{.................................(19)}$$

geometric relations are as following:

$$\varepsilon_r^{(3)} = \frac{du_r^{(3)}}{dr}, \; \varepsilon_t^{(3)} = \frac{u_r^{(3)}}{r} \qquad \text{..................................(20)}$$

$U_\varphi^{(3)}, U_z^{(3)}, U_r^{(3)} -$ correspondingly are tangential, axial and radial directions displacements.

As $\varepsilon_z^{(3)} = 0$, let's accept the direction:

$$\frac{1}{E}\left[\sigma_z^{(3)}+P_\infty-\mu\left(\sigma_r^{(3)}+\sigma_t^{(3)}+2P_\infty\right)\right]+\frac{1}{E}\int_0^t H^{(3)}(t-\tau)\times$$
$$\times\left[\sigma_z^{(3)}+P_\infty-\mu\left(\sigma_r^{(3)}+\sigma_t^{(3)}+2P_\infty\right)\right]d\tau=0 \quad \text{.....(21)}$$
$$y_1(r,t)=\sigma_r^{(3)}+P_\infty-\mu\left(\sigma_z^{(3)}+\sigma_t^{(3)}+2P_\infty\right).$$

Then equality (20) will be

$$y_1+\int_0^t H^{(3)}(t-\tau)y_1 d\tau=0 \quad \text{.....(22)}$$

As $y_1(r,0)=0$ the solving of homogeneous equality (22) is $y_1=0$. Then for viscous-elastic rocks

$$\sigma_z^{(3)}+P_\infty=\mu\left(\sigma_r^{(3)}+\sigma_t^{(3)}+2P_\infty\right) \quad \text{.....(23)}$$

If to consider (23) in (18) equations will be as following:

$$\varepsilon_r^{(3)}=\frac{1+\mu}{E}\left\{\left[\sigma_r^{(3)}+P_\infty-\mu\left(\sigma_r^{(3)}+\sigma_t^{(3)}+2P_\infty\right)\right]+\int_0^t H^{(3)}(t-\tau)\times\right.$$
$$\left.\times\left[\sigma_r^{(3)}+P_\infty-\mu\left(\sigma_r^{(3)}+\sigma_t^{(3)}+2P_\infty\right)\right]d\tau\right\} \quad \text{.....(24)}$$
$$\varepsilon_t^{(3)}=\frac{1+\mu}{E}\left\{\left[\sigma_t^{(3)}+P_\infty-\mu\left(\sigma_r^{(3)}+\sigma_t^{(3)}+2P_\infty\right)\right]+\int_0^t H^{(3)}(t-\tau)\times\right.$$
$$\left.\times\left[\sigma_t^{(3)}+P_\infty-\mu\left(\sigma_r^{(3)}+\sigma_t^{(3)}+2P_\infty\right)\right]d\tau\right\}$$

Considering initial stress state of rocks in viscous-elastic environment then not stresses themselves, but their difference will enter boundary conditions.

$$r=r_3, \overline{\sigma}_r(r_3)=\sigma_r(r_3)+P_\infty, \quad \sigma_r(r_3)=-P_3$$

As $\overline{\sigma}_r(r_3)=P_\infty-P_3$

We get $r=\infty,\ \overline{\sigma}_r(\infty)=\overline{\sigma}_2(\infty)+P_\infty,\ \sigma_r(\infty)=-P_\infty$

then $\overline{\sigma}_r(\infty)=P_\infty-P_\infty=0$

When stresses formed in the rocks are determined, not stresses themselves but their (displacements) shifts correspond to solution of Lame task /1/

$$\sigma_r=A\frac{1}{r^2}+B(1+2\ln r)+2C$$

$$\sigma_t=-A\frac{1}{r^2}+B(1+2\ln r)+2C$$

As $\sigma_r=-P_\infty;\ \sigma_t=-P_3$

We get $\overline{\sigma}_{r,t}^{(3)}=\mp\frac{r_3^2}{r^2}(P_3-P_\infty)$

But radical shift is

$$u_r^{(3)}(r_3)=\frac{1+\mu}{E}\left(P_3-P_\infty\right)\cdot r_3+\frac{1+\mu}{E}\int_0^t H^{(3)}(t-\tau)\cdot\left(P_c-P_\infty\right)r_3 d\tau \quad \text{.....(25)}$$

By the help of equation (25) it is possible to determine contact pressure formed in the boundary.

Conclusion

When boundary conditions are obviously dependent on time, that's when volter principles don't justify themselves, for one-communication areas the method determining plane strain situation of viscous-elastic mediums has been given.Contact pressure in the boundary has been determined by the help of this proved method.

REFERENCES

Amenzade Y.A. Elasticity theory M.Vishayashkola, 1976

Yaqubov N.I., Gulgazli A.S., Abbasov S.G. 1994. About the application of Moris Levi's theorem to the tasks of viscous elasticity. Oil and gas, 1994, N2, p.87-90

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