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RESEARCH ARTICLE

MULTIAXIAL FATIGUE CRITERIA BASED ON AN INTEGRAL APPROACH: JUSTIFICATION OF THE SUPERIORITY OF THIS APPROACH OVER THE CRITICAL PLANE APPROACH

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ABSTRACT

In the context of a study on multiaxial criteria for aluminum alloy welded structures 6000, biaxial tests are carried out on equivalent structures (simulated ZAT) in the field of fatigue with a large number of cycles. Critical plan criteria and global criteria are verified using the Castem 2000 code. The predictions compared with the experimental results show that some criteria work better for certain types of loading. Global criteria seem to give better predictions than critical plan criteria.

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INTRODUCTION

It is rare for a loading in the engineering field to be static or quasi-static in nature. In fact, it is well known that almost 95% of structures break because of the mechanism of fatigue. The current trend is to propose a lifetime prediction by minimizing the number of tests to be performed, and to simulate the fatigue behavior under any loading structures or parts. In the context of a study of welded aluminum 6082 assemblies in the transport field, the metallurgical structure of the Heat Affected Zone (ZAT) was simulated by controlling the local hardness and micro structure. Multiaxial criteria are studied using these results. A fatigue criterion defines a function parameter of the stress tensor path and the mechanical characteristics of the material and makes it possible to determine whether the part or the structure will break or not. Indeed, when the selected damage parameter (noted E) reaches a value greater than a threshold (Ec), we say that the structure will break.

$$E \geq E_C \rightarrow \textit{rupture}$$
$$E \prec E_C \rightarrow \textit{non} - \textit{rupture}$$

For the sake of simplicity, the value of E is normalized; the critical value of E is equal to 1 by dividing the formulas of E by Ec. Endurance limits in fatiguedness are often used as constants in function E.

$$E\left([\sigma_{ij}(t)]_T, \sigma_{-1}, f_{-1}, \sigma_0, f_0, \tau_{-1}, ...\right) = 1$$

Where $\left[\sigma_{ij}\left(t\right)\right]_T$ are terms of the multiaxial cycle of constraints defined over the period T (average values, maximum values, amplitudes...)in recent decades, multiaxial fatigue has been widely studied. Researchers have come up with their own criteria. Depending on the assumptions and methods used, different families of criteria can be distinguished: Criteria; Global Criteria ("effective stress approaches"); Energy criterion; etc. In this article, critical plan criteria and global criteria are particularly studied.

Criteria for critical intégrales approches

The global criteria are often expressed as a function of the invariants J_2 and I_1 of the constraint tensor. The criteria of integral approaches form the last family of criteria. Whether based on invariants of the stress tensor or deviator, on the root mean square of an indicator of damage or on an energy concept, they all have a global character by their formulation. Some criteria that use only the first stress tensor invariant and the second deviation tensor invariant can be considered as an intermediate approach between the critical plane and the global

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approach criteria because these two stress terms are proportional to normal stress and tangential that act on the octahedral plane constituting a particular plane. This family includes 17 modelizations; the oldest of which dates from 1955 and the most recent was formulated in 1994. This article will treat the criteria of Sines (1981), Crossland (1970), Marin (1956), Deitman and Isseler (1974), of Kinasushvili (1987) and of Fogué (1957).

Nomenclature

The majority of the criteria based on the concept of global approach use the first invariant $I_1(t)$ of the stress tensor $[\sigma_{ij}(t)]$ or the second invariant $J_2(t)$ of its deviator $[s_{ij}(t)]$. Used by their average value or their amplitude, these invariants represent the totality of the constraints by their definition. Indeed, the first invariant of the stress tensor $I_1(t)$ (or the triple of the hydrostatic pressure $P_H(t)$) is proportional to the average of the normal stresses $\sigma_{hh}(t)$ of all the material planes passing by a point for a state of constraints given at this point. Similarly, the second invariant $J_2(t)$ of the stress deviator (or the equivalent constraint $\sigma_{eq.VM}(t)$ in the sense of Von Misès) represents the quadratic mean of the tangential stresses $\tau_h(t)$ acting on all the possible physical planes at the point considered. The following list summarizes the terms used for these two invariants:

$I_{1\max} / I_{1\min}$: Maximum / minimum value (during the cycle) of the first invariant of the stress tensor,

$I_{1m} = \dfrac{I_{1\max} + I_{1\min}}{2}$: Average value of the first invariant of the stress tensor,

$I_{1a} = \dfrac{I_{1\max} - I_{1\min}}{2}$: Amplitude of the first invariant of the stress tensor,

P_{Hm} : Average hydrostatic pressure ($P_{Hm} = \dfrac{I_{1m}}{3}$),

P_{Ha} : Amplitude of the hydrostatic pressure ($P_{Ha} = \dfrac{I_{1a}}{3}$)

I_2 : Second invariant of the stress tensor,

J_{2a} :Amplitude of the second invariant of the constraint deviator. It generally corresponds to the maximum value of the second invariant of the deviator of the alternating stresses, where $J_{2a} = \underset{t}{Max}\left(\sqrt{J_{2a}(t)}\right)$ or $J_{2a}(t) = \left(s_{iia}^2(t) + 2s_{ija}^2(t)\right)/2$

J_{2m} : Value of the second deviator invariant of the average constraints.

Other criteria use either a damage indicator per plane in order to realize a quadratic average over all the possible material planes, or quantities relating to deformation energies. For these two types of formulations, a specific nomenclature is given during their presentation.

The Sines Criterion (Sines and Ohgi, 1981)

Established first in 1955, this criterion was modified by its author in 1981 in order to express it according to the invariants of the tensor and its deviator and no longer according to the two terms of the constraints relative to the octahedral plane (normal octahedral constraints and tangential).

The formulation of the criterion is a function of and of:

$$E_{SI} = \frac{\sqrt{J_{2a}} + \alpha I_{1m}}{A} \tag{1}$$

At the fatigue limit, the fatigue function of the criterion is equal to unity. This is valid in particular for the two fatigue limits chosen to calibrate the criterion. With fatigue limit in repeated tension, and symmetrical alternating torsion, the constants and A are expressed as follows:

$$A = \tau_{-1} \qquad \alpha = 2\frac{\tau_{-1}}{\sigma_0} - \frac{1}{\sqrt{3}} \tag{2}$$

The validity of the criterion is ensured by the condition, i.e. by:

$$\frac{\tau_{-1}}{\sigma_0} > \frac{1}{2\sqrt{3}}.$$

The Crossland Criterion (Crossland, 1970)

Formulated in 1956, the Crossland criterion is very similar to that of Sines. Crossland proposes to use the maximum value of the first constraint invariant $I_{1\max}$ instead of taking only the mean. The fatigue function of the criterion is written as follows:

$$E_{CR} = \frac{\sqrt{J_{2a}} + BI_{1\max}}{A} \tag{3}$$

The two constants A and B, obtained by calibrating the criterion on the two limits of fatigue σ_{-1} and τ_{-1}, are expressed according to:

$$A = \tau_{-1} \qquad B = \frac{\tau_{-1}}{\sigma_{-1}} - \frac{1}{\sqrt{3}} \tag{4}$$

The criterion is valid when the $\dfrac{\tau_{-1}}{\sigma_{-1}}$ ratio is greater than $\dfrac{1}{\sqrt{3}}$.

The Marin Criterion (Marin, 1956)

Marin also proposes in 1956 a global approach criterion in which he compares $\sqrt{3}\sqrt{J_{2a}}$ (equivalent stress of the alternating parts of the constraints in the sense of Von Misès) σ_{-1} (to fatigue limit in symmetrical alternating traction) and $\sqrt{3}\sqrt{J_{2m}}$ (equivalent stress of the average stresses) to R_m (resistance maximum traction).

$$E_{MA} = \left(\frac{\sqrt{3}\sqrt{J_{2a}}}{\sigma_{-1}}\right)^2 + \left(\frac{\sqrt{3}\sqrt{J_{2m}}}{R_m}\right)^2 \tag{5}$$

The Deitman & Issler Criterion (Deitman and Issler, 1974)

In 1974, Deitman & Issler proposed three models of fatigue behavior. The first of these, the Deitman & Issler criterion, is inspired by the modeling of the Haigh diagram by Gerber's parabola:

$$E_{DI1} = \left(\frac{\sqrt{3}\sqrt{J_{2a}}}{f_{-1}}\right)^2 + \frac{3P_{Hm}}{R_m} \qquad (6)$$

The Kinasoshvili Criterion (Fogue, 1987)

The author models the fatigue behavior using the second invariant of the middle J_{2m} and J_{2a} alternate parts of the stress deviator. It combines linearly these two quantities with coefficients depending on the fatigue limits obtained in repeated traction (σ_0) and alternating symmetric (σ_{-1}). The expression of the criterion established in 1976 is as follows:

$$E_{KS} = \frac{\sqrt{3}\sqrt{J_{2a}}}{\sigma_{-1}} + \frac{\sigma_{-1} - \sigma_0}{\sigma_{-1}\sigma_0}\sqrt{J_{2m}} \qquad (7)$$

The Fogue Criterion (Findley, 1957)

Inspired by the works of Grübisic & Simbürger, Fogue proposes in 1987 a criterion based on the quadratic mean of a damage indicator E_h. It introduces in this indicator the relative influences of the amplitude and the average value of the normal stress in the plane as well as the amplitude of the shear: (10)

$$E_h = \frac{a\tau_{ha} + b\sigma_{hha} + d\sigma_{hhm}}{\sigma_{-1}} \qquad (8)$$

The criterion then realizes the quadratic mean of the indicators of all the physical planes by using the sphere of unit radius (of area S = 4) defined by Grübisic & Simbürger. The fatigue function of the criterion is written as follows:

$$E_{FG} = \sqrt{\frac{1}{S}\int_S E_h^2 dS} \qquad (9)$$

Calibration of the criterion, performed with the three fatigue limits τ_{-1}, σ_{-1} and, σ_0 leads to the following constants and validity domains:

$$b = \sqrt{\frac{15 - \sqrt{9\left(25 - 8\left[\left(\frac{\sigma_{-1}}{\tau_{-1}}\right)^2 - 3\right]^2\right)}}{2}}$$
$$a = \sqrt{\frac{12\left(\frac{\sigma_{-1}}{\tau_{-1}}\right)^2 - 21 + b^2}{2}} \qquad (10)$$
$$d = \frac{1}{3}\left\{-(3b + 2a) + \sqrt{(3b + 2a)^2 + 45\left(4\left(\frac{\sigma_{-1}}{\tau_{-1}}\right)^2 - 1\right)}\right\}$$

With: $\frac{1}{\sqrt{3}} < \frac{\tau_{-1}}{\sigma_{-1}} < \frac{\sqrt{3}}{2}$ and $\frac{1}{2} < \frac{\sigma_{-1}}{\sigma_0} < 1$

The global approach criteria propose numerous modeling of fatigue behavior. The formulations are for some of them a rather complex implementation: root mean square of a damage indicator per plan (Fogue), calculation based on energies (Froustey & Lasserre and Palin-Luc) or effective values. The family of global approach criteria brings together a variety of models, the justifications of which are based on various and original theories. We can, however, reverse the remark given in conclusion of the critical plane criteria: if the global approach criteria are appropriate when a large number of physical planes passing through the studied point are equal-damaged, what about their validity when only some plans are activated. The comparison of all the criteria with the test database will make it possible to compare the two approaches (critical plan and global approach) and, within each family, the criteria relative to each other as to their capacity. Intrinsic to predict the fatigue behavior of materials.

Criteria for critical plan approaches

Experiments in the field of multiaxial fatigue show that there are critical directions for certain loads (in-phases). The plankitic type criteria look for the plane where the damage caused by fatigue is maximum. Thus, they determine the lifespan of the structure and the critical direction of crack initiation. These criteria often use a combination of the shear stress and the normal stress on a given plane. In this paper, the smallest circumscribed method is used to determine the terms used in the criteria studied. Different criteria use different definitions of damage on a given plane: Findley's criterion (McDiarmid, 1973), MacDiarmid's criterion. (Dang Van *et al*., 1984), criterion of Dang Van. (Matake, 1980), Matake's criterion. (Robert *et al*., 1994) and J.-L. Robert (Robert *et al*., 1994).

Nomenclature

The stress terms involved in the critical plane fatigue criteria are mainly the normal and tangential components of the stresses acting on a physical plane of normal h during the multiaxial cycle under consideration. These are obtained by projection of the stress states on the plane and on its normal. At time t, the stress tensor $[\sigma(t)]$ acts on the plane of normal h through its constraint vector $\phi_h(t)$ defined by:

$$\phi_h(t) = [\sigma(t)]h \qquad (11)$$

The stress constraint $\phi_h(t)$, decomposed to normal stress $\sigma_{hh}(t)$ and tangential $\tau_h(t)$:

$$\sigma_{hh}(t) = h[\sigma(t)]h$$
$$\tau_h(t) = \sqrt{\sigma_{uh}^2(t) + \sigma_{vh}^2(t)} \qquad (12)$$

With:

$$\sigma_{uh}(t) = u[\sigma(t)]h$$
$$\sigma_{vh}(t) = v[\sigma(t)]h \qquad (13)$$

Where (u, v, h) is the reference linked to the physical plane. The figure gives a representation of the decomposition of the constrained vector on the physical plane (π) of normal h passing through the point Pat time t. The reference linked to the material and in which the constraints are expressed is (1, 2, 3).

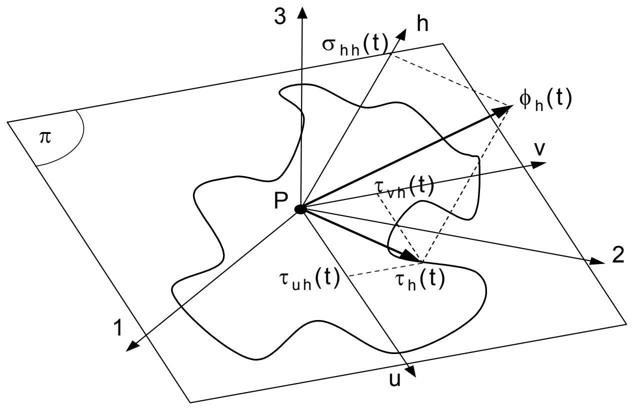

Figure 1. Decomposition of the constrained vector $\phi_h(t)$ in the reference (u, v, h) linked to the physical plane of normal h.

a) Terms relating to normal stresses

The different criteria use several quantities resulting from the evolution of the normal stress at the physical plane during the cycle:

$\sigma_{hh\min}$: Minimum normal stress

$\sigma_{hh\max}$: Maximum normal stress

σ_{hha} : Amplitude of the normal stress

$\sigma_{hha}(t)$: Alternating part of the normal stress at time t

σ_{hhm} : Average normal stress

These different quantities are linked by the following relations:

$$\sigma_{hhm} = \frac{\sigma_{hh\max} + \sigma_{hh\min}}{2} \quad (14)$$

$$\sigma_{hha} = \frac{\sigma_{hh\max} - \sigma_{hh\min}}{2} \quad (15)$$

$$\sigma_{hha}(t) = \sigma_{hh}(t) - \sigma_{hhm} \quad (16)$$

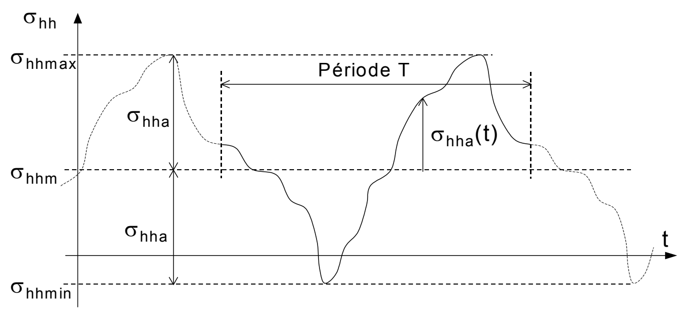

Figure 2. Definition of the different terms related to the normal stress

b) Terms relating to tangential constraints

The terms associated with the evolution of the tangential stress during the cycle are similar in their definition principle to those of the normal stress. However, their determination is more complex because of the two-dimensional nature of the tangential constraint. The determination of the amplitude, the alternating part and the average value of the tangential stress requires the construction of the smallest circle circumscribing the load path (Robert *et al*., 1994), which is constituted by the end of the vector shear stress $\tau_h(t)$ on the physical plane (Figure 3). For convenience of writing, all the notations of the different terms of the tangential constraints represent vectors in Figure 3and their norm when they are used in the formulations of the criteria.

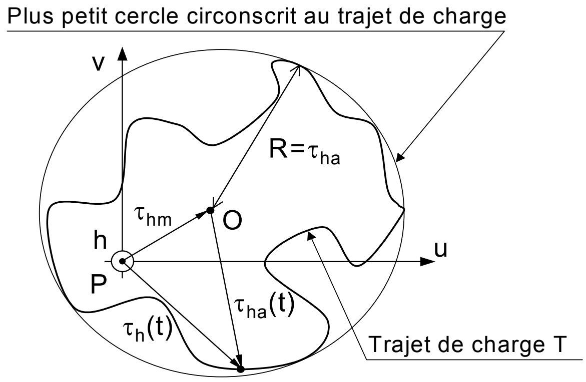

Figure 3. Definitions of different stress tangential relative terms $\tau_h(t)$

The three terms resulting from the tangential constraint are defined by:

τ_{ha} : Amplitude of the tangential stress (radius of the smallest circle circumscribed to the load path $\tau_{ha}(t) = \tau_h(t) - \tau_{hm}$)

$\tau_{ha}(t)$: alternating part of the tangential constraint at time t (defined vectorially by)

τ_{hm} : mean tangential stress (vector PO in Figure 3)

The Findley Criterion (McDiarmid, 1973)

Findley's 1957 criterion is similar to that of Stulen & Cummings. It differs in the choice of its critical plan, obtained by looking for the plane where the linear combination " $\tau_{ha} + \alpha\sigma_{hh\max}$ " is maximal. The criterion is written:

$$E_{FD} = \frac{\tau_{ha} + \alpha\sigma_{hh\max}}{\beta} \quad (17)$$

The McDiarmid Criterion (Dang Van *et al*., 1984)

The first version of the McDiarmid criterion dates from 1973. The critical plane is defined as the one where the amplitude of the shear is maximal, ie by the quantity « $\underset{h}{Max}(\tau_{ha})$ ». In this respect, the fatigue function of the criterion is given by

$$E_{MD1} = \frac{\tau_{ha} + B(\sigma_{hha})^{3/2}}{A} \quad (18)$$

The two constants A and B are determined by checking the criterion ($E_{MD1} = 1$) for two fatigue limits of the symmetrical alternating tensile and torsional material (σ_{-1} and τ_{-1}). We obtain:

$$A = \tau_{-1}$$

$$B = \frac{\tau_{-1} - \frac{\sigma_{-1}}{2}}{\left(\frac{\sigma_{-1}}{2}\right)^{3/2}} \quad (19)$$

The criterion is valid under the condition: $\frac{\tau_{-1}}{\sigma_{-1}} > \frac{1}{2}$.

It should be noted that this criterion does not take into account the average constraints of the cycle of constraints whereas the two previous criteria treat on an equal plane the average and alternate parts of the normal stress.

The criterion McDiarmid (Dang Van *et al.*, 1984)

McDiarmid gives a second version of his criterion in 1974 in which he introduces the influence of the average normal stress by a factor of the shear amplitude of the critical plane. The definition of the critical plan remains unchanged (search for). The fatigue function of the second criterion of McDiarmid is written:

$$E_{MD2} = \frac{\left(1 - \frac{2\sigma_{hm}}{R_m}\right)^{-1/2} \tau_{ha} + B(\sigma_{hha})^{3/2}}{A} \quad (20)$$

With R_m is the maximum tensile strength of the material.

The constants A and B of the criterion are identical to those of the first version of the criterion. The validity of the criterion imposes two conditions to be met: $\frac{\tau_{-1}}{\sigma_{-1}} > \frac{1}{2}$ and $\frac{\sigma_{hhm}}{R_m} > \frac{1}{2}$

The criterion of Dang Van (Matake, 1980)

The criterion of Dang Van, in its first version, is the first criterion of fatigue to have been introduced in the French industry (PSA). Justified by considerations established at the microscopic scale, the criterion uses macroscopic magnitudes of the stresses which are the alternating part of the shear and the hydrostatic pressure. As with the Yokobori criterion, the fatigue function is a maximization of a damage indicator per plane defined by:

$$E_h = \underset{t}{Max}\left\{\frac{\tau_{ha}(t) + \alpha P_H(t)}{\beta}\right\} \quad (21)$$

With:

$$P_H(t) = \frac{\sigma_{11}(t) + \sigma_{22}(t) + \sigma_{33}(t)}{3} \quad (22)$$

The fatigue function of the criterion is then written:

$$E_{DV1} = \underset{h}{Max}(E_h) \quad (23)$$

By definition, the fatigue function of the criterion is equal to the unit at the fatigue limit of the material, and more particularly for the symmetrical alternating tensile and torsional fatigue limits serving as calibration of the criterion. The constants α and β are thus given by:

$$\alpha = 3\left(\frac{\tau_{-1}}{\sigma_{-1}} - \frac{1}{2}\right) \quad (24)$$

$$\beta = \tau_{-1}$$

The condition of validity of the criterion of Dang Van 1 is realized when $\frac{\tau_{-1}}{\sigma_{-1}} > \frac{1}{2}$.

Matake criterion (Robert *et al.*, 1994)

In 1977, Matake formulates a similar criterion to those of Stulen & Cummings and Findley. It differs only in the choice of the critical plane, the one where the amplitude of the shear is maximal, that is to say: $\underset{h}{Max}(\tau_{ha})$. The fatigue function on this plan is written:

$$E_{MT} = \frac{\tau_{ha} + \alpha\sigma_{hh\max}}{\beta} \quad (25)$$

The constants α and β condition of validity of the criterion are identical to those of Findley.

Robert's criterion (Robert *et al.*, 1994)

Robert's criterion is established in 1992. The author defines a damage indicator by plane E_h in which he dissociates the respective influences of the middle and alternate parts of the normal stress and combines them with the alternating part of the tangential stress. The damage indicator is maximization on the cycle of these quantities:

$$E_h = \underset{t}{Max}\left\{\frac{\tau_{ha}(t) + \alpha\sigma_{hha}(t) + \beta\sigma_{hhm}}{\theta}\right\} \quad (26)$$

The fatigue function is based on the search for the critical physical plane, the one whose indicator of damage is maximal:

$$E_{RB} = \underset{h}{Max}(E_h) \quad (27)$$

The criterion has three constants α, β and θ, which are determined by verifying the formulation using three fatigue limits: the fatigue limit in symmetrical σ_{-1}, alternating traction σ_0, that in repeated traction and that in symmetrical alternating torsion τ_{-1}. The expressions of the three constants are:

$$\alpha = \frac{\frac{2\tau_{-1}}{\sigma_{-1}} - 1}{\sqrt{\frac{2\tau_{-1}}{\sigma_{-1}}\left(2 - \frac{2\tau_{-1}}{\sigma_{-1}}\right)}}$$

$$\theta = \tau_{-1}\sqrt{\alpha^2 + 1} \quad (28)$$

$$\beta = \frac{2\theta}{\sigma_0} - \frac{\sigma_0}{8\theta} - \alpha$$

The validity domain of the criterion is given by: $\frac{1}{2} < \frac{\tau_{-1}}{\sigma_{-1}} < 1$ and $\frac{1}{2} < \frac{\sigma_{-1}}{\sigma_0} < 1$.

The critical plane fatigue criteria model the fatigue behavior of the multiaxial stress cycle material by assuming that damage on the critical plane alone controls the fatigue behavior of the material. The definitions of the critical plan as well as the expression of the fatigue function are specific to each criterion. As such, two remarks can be made. The first of these concerns the determination of the critical plan. Some authors define a priori the critical plane (according to a certain parameter of the constraints) on which they calculate the value of the function of fatigue generally different and more complex than the expression which makes it possible to determine the critical plane. This choice is ambiguous in that there may be a physical plane other than the critical plane and for which the value of the fatigue function is greater. On the other hand, the criteria that adopt either the maximization of a damage indicator per plan, or which search for maximum stress terms in such a way that there is no ambiguity as to the maximum value of the function of fatigue, appear more logical in their approach. For these last criteria, the critical plan is the one that lives the maximum damage in the sense that it is defined by these. There is consistency between the amount that describes the fatigue damage and the plan selected as the most requested. The second remark relates to the very nature of critical plane fatigue criteria. Indeed, when the solicitations undergone by the matter make that there is more uniqueness of the critical plane but on the contrary that there exist a large number of equal-damaged planes, the description of the phenomenon of fatigue on a single physical plane is it sufficient to correctly account for the actual fatigue behavior of the material. The global approach criteria, described below, provide some evidence of a negative response to this question, particularly for the global approach criteria which also use a damage indicator per plan.

4. Characterization of material

Uniaxial tensile and fatigue tests were performed (Table 1). Whöler's triforbours (SN) are obtained using Bastenaire's method (Bastenaire, 1982). This information is needed to calculate the coefficients in the criteria and for the predictions of the lifetime.

Table 1. Comparison between simulated ZAT and Metal of Basecomparisonbetweensimulated HAZ & parent metal

	$R_m(MP_a)$	$R_e(MP_a)$	A%	H_v
ZAT simulée	274	230	16	80
Métal de base	355	330	135	110

5. Results of the experimental tests

The cruciform test pieces are designed so that the crack starts in the middle. Indeed, the thickness at the center is only 1mm, but 8mm at the edge. Inspired by the test pieces described in the literature (Johan Singh *et al*., 2003), the geometry of this specimen is given in figure.5. Two groups, in-phase and out-of-phase, of loadings are applied on these specimens. The classical phase differences applied are conventional, 30 °; 45 °; 60 ° and 90 °. The X and Y axes are identified in Figure 4. There are three non-ruptured (NR) and ten ruptured (R) tests (Table 2).

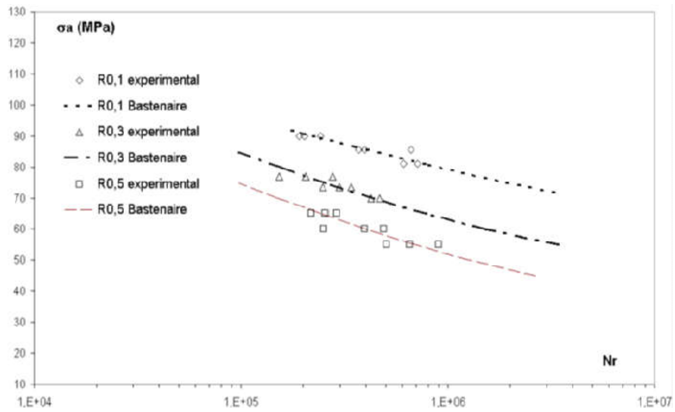

Figure 4. Uniaxial test results

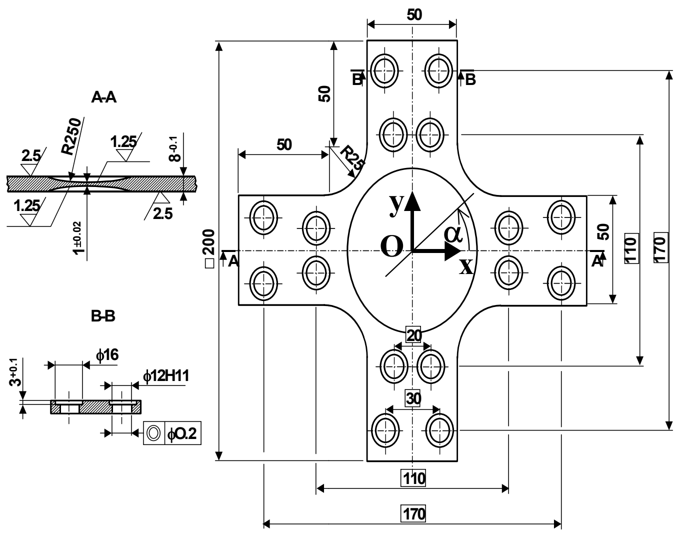

Figure 5. Geometry of the cruciform test piece

5.1. Prediction of behavior of criteria using experimental results

a) Predictions of critical plan criteria

Five critical plan criteria were verified in our cases (Figure 6). These are Dang Van's Criterion (Matake, 1980) MacDiarmid's Criterion. (8) Findley Criterion (7), Matake Criterion. (Robert *et al*., 1994) and Robert J.L., Fogue M (1994). (XX_EN are the points corresponding to the loadings in phases, and the pointsXX_HORS correspond to the loads out of phases.)

$$\begin{cases} P_0 = \dfrac{N_{prédiction}}{N_{rtest}} \\ P = P_0, P_0 > 1 \\ P = \dfrac{1}{P_0}, P_0 < 1 \end{cases}$$

Thus, P represents the ratio of the number of cycles and the number of cycles measured experimentally.

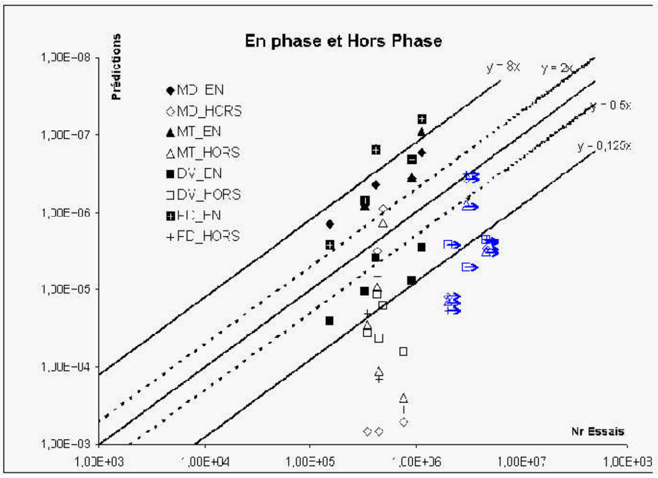

Figure 6. Predictions of critical plan type criteria

For the five cases in case the specimens are broken and the loadings are in phases, the proportions P of the criteria are: Dang Van: 3.0 ~ 7.0 with P0 <1 in all cases; Findley: 3.48 ~ 15.5 with P0> 1 in all cases; Matake: 3.48 ~ 15.7 with P0> 1 in all cases; McDiarmid: 3.93 ~ 5.48 with P0> 1 in all cases. And for the five out-of-phase loads, the predictions are much more dispersed than in the case of phases. Dang Van: 5.0 ~ 19.0 with P0 <1 in all cases, is also the best. Nans are very different on crack propagation. Only in this criterion of McDiarmid, the two terms are treated differently. This is probably why this criterion works better. On the other hand, predictions on off-phase loads are poor, confirming that critical plan criteria do not work well when loads are out of phase□*nnm*et□*nna*sont, for all these simulations, the McDiarmid, Findley, and Matake criteria lead to non-conservative forecasts for phase loading and very conservative for off-phase loading. The criterion of Dang Van, K. leads to conservative predictions, since the instantaneous amplitude value of the shear is used in its criterion. The McDiarmid criterion gives the closest predictions of the experimental results in the case of phases. As each type of loading is applied only once, the set of results gives a statistical idea that this criterion can be used for this alloy. The effects of according to the literature, critical plane criteria work best for phase loading, but less well for off-phase loading. Here, for the five out-of-phase loads, none of the verified criteria can make correct predictions. It can be seen that predictions are often too conservative with respect to experimental results. The best criterion for these loads is that of Dang Van, K. But his predictions are also too conservative; we even find a ratio of 48.5 for the case 'P90Test'. Based on these results, the critical plane type criteria as they are not valid for off-phase loading.

Critical Directions

Critical directions were measured on cracks observed in broken test pieces and then compared with predictions. But the difference between the criteria is not meaningful. For each criterion, 83.33% of the γ predictions correspond well to the measured directions (difference less than 10°). However, for φ, only 50%.

5.2 Predictions of global criteria

Five global type criteria were applied for the predictions (Figure 8). These are the criteria of Sines, Crossland, Marin, Deitman & Issler, and Kinasoshvili.

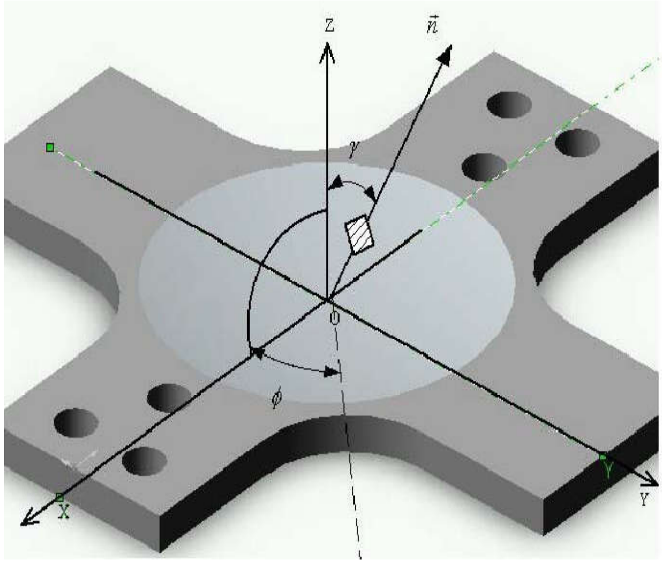

Figure 7. Definitions of critical direction

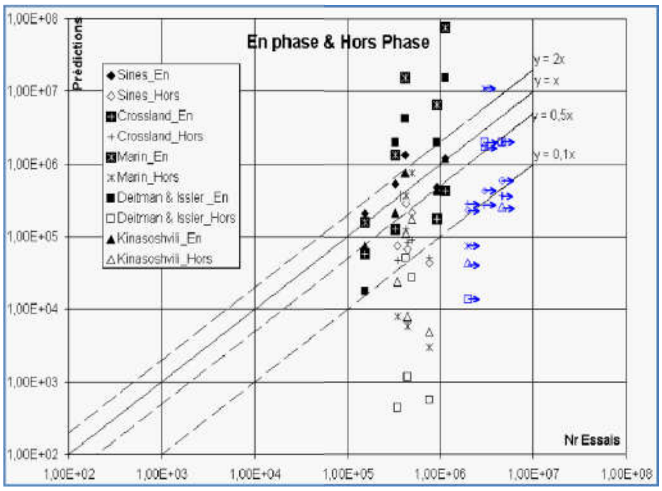

Figure 8. Predictions of global criteria

As shown in Figure 7, the predictions are highly dispersed for the Marin, Deitman & Issler criteria. The predictions of the Sines, Crossland, Kinasoshvili criteria are much less dispersed. Of the five criteria, Sines gives the best predictions. There are six points of the Sines criterion forecasts falling in the band of 0.5 ~ 2.0, and a point close to this band. So 77.78% of the predictions are good for this criterion. The Kinasoshvili criterion gives conservative predictions. There are six points in the band 0.1 ~ 0.5, or 60%. The Crossland criterion is conservative; the proportions between predictions and experimental results fall in the band of 0.1 to 0.5. Of the five criteria, Sines is the best, it is the one that gives 60% of correct predictions, which are in the band 0.5 ~ 2.0, and the difference between the two groups in-phases and out-phases is the smallest. Kinasoshvili is the second; Indeed, predictions for in-phase loadings still remain in the band of 0.5 ~ 2.0 (50% of cases). But for off-phase shipments the predictions seem too conservative. Crossland is the third, which still gives conservative predictions in the 0.1 ~ 0.5 band. But Crosland's predictions are much less dispersed than Kinasoshvili's predictions. For out-of-phase shipments, the Sines and Crossland criteria are more conservative than the in-phase loads. It is interesting that one finds the global type criteria are better in the phase cases than the out of phase cases for our simulated ZAT.

Conclusion

- Critical plan criteria are best when the load is in phase. For phase loadings, the predictions given by the four criteria checked are in the same precision band. That of Dang Van, K. is always conservative, the other three criteria are non-conservative. The criteria of Dang Van and McDiarmid seem to be better than the others by taking into account all the cases.
- For critical plan criteria to be better in out-of-phase cases, some modifications are needed.
- Among the global criteria, the Sines and Crossland criteria work best and give the least dispersed results. The Kinasochvili criterion gives very good predictions for phase cases, and its out-of-phase predictions are more dispersed than those of Sines and Crossland.

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