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International Journal of Current Research
Vol. 10, Issue, 02, pp.64926-64930, February, 2018

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

STUDY OF THE NATURAL BENDING OSCILLATIONS

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ARTICLE INFO

Article History:
Received 10th November, 2017
Received in revised form
25th December, 2017
Accepted 19th January, 2018
Published online 18th February, 2018

Key words:
The downhole packer system packing element, Oscillations, Frequencies, Oscillation waveform, Bending oscillations, Potential energy, Lagrange, Wellbore vertical and Horizontal interval.

ABSTRACT

The article describes the study of natural bending oscillations in packer systems used on strings in offshore production wells. To do so the Lagrange's variational method has been applied. This study is performed on the first approximation using some simplifying assumptions to analyze elastic bending deformations of the packing elements in the down hole packer systems (which isolate production strings with two packers set on a vertical section of the string and horizontal section of the string) and to study natural bending oscillations in the packer systems operated in directional offshore production wells.

Citation: Mammadov, V. T. and Abdulaliyev, A. M. 2018. "Study of the natural bending oscillations", *International Journal of Current Research*, 10, (02), 64926-64930.

INTRODUCTION

Oscillations shall be assumed of low elastic mode, subject to a linear Hooke's law (Leybenzon, 1951-1955; Babanly *et al*., 2016; Timoshenko, 1967). Due to this classical linear elastic theory will be applied. The packer elements configuration shall be set in the wellbore interval of the vertical and horizontal string (Fig. 1).

Presentation of the problem

The packer systems (downhole packers) are widely used to isolate the wellbore interval in directional wells of offshore fields. Finding solutions for the problem of packer system oscillations is becoming more and more critical for packer designs in the offshore applications. The subject of our study is to analyze natural oscillations of packing elements in packers set in directional (curved profile) wellbore (Fig. 1).

Objective of the study

In our analytical study of the natural oscillations in the packer system packing elements under consideration we have applied Lagrange's variational equation as it provides a convenient method for determination of the oscillation frequencies and waveforms (Ponovka, 1976). The plane to be assumed as mean plane of the analyzed packing elements: Z axis, directed perpendicular to the drawing plane (Fig. 1)

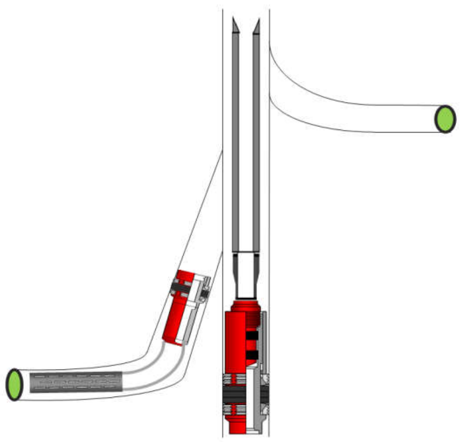

Fig.1. Well Schematic

In our case the stress on the surface of the packing elements shall be as follows (Leybenzon, 1951-1955)

$$\int_S -h\rho \cdot \frac{\partial^2 W}{\partial t^2} \cdot \delta W_1 dxdy - \delta U = 0 \qquad (1)$$

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Here the U-potential energy of elastic bending in the packer (packing element) shall be determined by a known Kirchhoff formula (Leybenzon, 1951-1955)

$$\cup = \frac{D_r}{2}\int_s \left\{(\Delta W_1)^2 - 2(1-\mu)\left[\frac{\partial^2 W_1}{\partial x^2}\cdot\frac{\partial^2 W_1}{\partial y^2} - \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right]\right\} \text{dxdy},$$

where $D_r = \frac{Eh^2}{12(1-\sigma)}$ – cylindrical stiffness;

h - packing element thickness;
E- elasticity model of the packing material;
$\mu -$ Poisson ratio;
$\Delta - \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ –two-dimensional Laplace operator (Mammadov and Gurbanov, 2015);
ρ- material density;
$\frac{\sigma^2 W}{dt^2}$– packing element point acceleration towards Z axis
S - area of the packing element's median plane.

Assuming the packing element points execute harmonic oscillations we will express the oscillation frequency as ρ and can record them (Malsev, 1978)

$$W_1(x,y;t) = W(x,y)\cos\rho t \qquad (3)$$

Taking into account (3), the Lagrange's variational equation (1) shall be as follows

$$\delta(T_{max} - \omega_{\max}) = 0$$

Where the maximum kinetic energy of the packer element shall be

$$T_{max} = \frac{h\rho h^2}{2}\int_s W^2 dxdy \qquad (5)$$

Maximum potential energy (Babanly *et al.*, 2016; Kocoshvili, 1978)

$$\omega_{max} = \frac{\partial}{2}\int\{(\Delta W)^2 - 2(1-\mu^m)\left[\frac{\sigma^2 W}{\sigma x^2}\cdot\frac{\sigma^2 W}{\sigma y^2} - \left(\frac{\sigma^2 W}{\sigma x \sigma y}\right)^2\right)\}\text{dx}\cdot\text{dy} \qquad (6)$$

Function $W(x,y)$ - the packer element deflection, shall be preliminary selected in such manner as to satisfy the conditions of its setting.

The specific solution of the problem in this article is found based on analytical study of the natural bending oscillations (with above assumptions) in the packer element which constitutes a simply connected area with the contour, formed by two elliptical segments, horizontal and vertical, which lie in the ellipsis axes in the following setting options:

1) The packer element (packer 1) is compressed in the horizontal interval of the contour (fig. 2)
2) The packer element (packer 2) is compressed in the vertical interval of the contour (fig. 3)
3) For the first setting option we shall have the following boundary conditions

$$W=0,\ \frac{\partial W}{\partial Y}=0 \quad \text{at} \quad Y=0$$

The function $W(x,y)$ which satisfies these conditions may be selected as follows

$$W(x,y) = (\alpha_1 x + \alpha_2)y^2 \qquad (7)$$

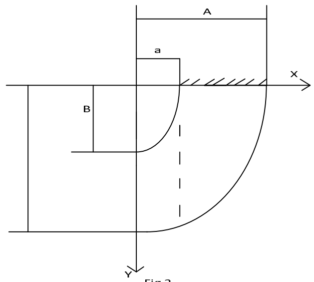

Fig.2

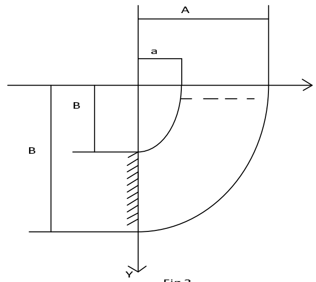

Fig.3

where α_1, α_2 - constant, unknown for now independent of one another coefficients, where the ration between them may be determined any time.

For the second setting option we shall have the following

$$W=0,\ \frac{\partial W}{\partial X}=0 \quad \text{at} \quad X=0.$$

Then $W(x,y) = (\alpha_1 x + \alpha_2)x^2$ (8)

If we insert (7) and (8) in (5) and (6) of the conversion field for maximum values of kinetic energy and potential energy we shall obtain the following

$$T^{|}_{max} = \frac{h\rho p^2}{2}\int_s(\alpha_1 xy^2 + \alpha^2 y^2)^2 ds$$

$$U^{|}_{max}(4) \frac{D_x}{2}\int_s(\{\{(4(\alpha_1^2 x^2 + 2\mu_1\alpha_2\cdot x + \alpha_2^2)) + 2(1-\mu)(4\alpha_1^2 y^2)\}$$ ds

$$T^{|}_{max} = \frac{h\rho p^2}{2}\int(\alpha_1^2 x^4 y^2 + 2\alpha_1\alpha_2 x^4 y + \alpha_2^2 x^4)ds \qquad (10)$$

$$U^{|}_{max} = \frac{D_x}{2}\int_s 4(\alpha_1^2 y_1^2 + 2\alpha_1\alpha_2 y + d_2^2) + 8(1-\mu)\alpha_1^2 x^2)ds$$

Expression $ds=dxdy$ - element of area in rectangular coordinates for convenience of calculating the integral in expressions (7) and (8) by formulas

$$x = r\cos\theta,\ y = r\sin\theta$$

Now we will proceed from Cartesian coordinates to polar coordinate system.

The formulas (9) and (10) will be as follows:

Where

$$T^{|}_{max} = \frac{h\rho p^2}{2}\int_0^{\frac{\pi}{2}}\int_{r_{1(\theta)}}^{r_2(\theta)}(\alpha_1^2 r^6 \sin^4\theta\cos^2\theta + 2\alpha_1\alpha_2\cos^2\theta + 2\alpha_1\alpha_2 r^5 \sin^4\cos\theta + \alpha_2^2 r^4 \times\times \sin^4\theta)\, rdrd\theta$$

$$\cup^{|}_{max} = D_k\int_0^{\frac{\pi}{2}}\int_{r_1}^{r_{2(\theta)}}\{\alpha_1^2(r^2\cos^2\theta + 2\alpha_1\alpha_2 r\cos\theta + \alpha_2^2)) +$$

$$T_{max} + \frac{h\rho p^2}{2}\int_0^{\frac{\pi}{2}}\int_{r_{1(\theta)}}^{r_2(\theta)}(\alpha_1^2 r^6\sin^4\theta\cos^2\theta + 2\alpha_1\alpha_2\cos^2\theta + 2\alpha_1\alpha_2 r^5\sin^4\cos\theta + \alpha_2^2 r^4\cdot\sin^4\theta)\, rdrd\theta \qquad (11)$$

$$U^{||}_{max} = 2D\int_0^{\frac{\pi}{2}}\int_{r_1(\theta)}^{r_{2(\theta)}}((\alpha_1^2 r^3\sin^2\theta + 2\alpha_1\alpha_2 r^2\sin\theta + \alpha_2^2 r) + +2(1-\mu)\alpha_1^2 r^3\cos^2\theta)rdrd\theta \qquad (12)$$

in its turn, A, B and a, $в$ are ellipsis half-axes respectively.

After calculating the integrals in these expressions and a number of conversions we shall finally obtain the following:

$$T^{|}_{max}=\frac{h\rho p^2}{2}\{\alpha_1^2\left[\frac{(A^8-a^8)}{8}J_1+\frac{(A^6B^2-a^6b^2)}{2}J_4+\frac{3(A^4B^4-a^4b^4)}{4}J_4+\frac{(A^2B^6-a^2b^6)}{2}J_5++\frac{(B^8-b^8)}{8}J_3\right]+\frac{2\alpha_1\alpha_2}{T}[J_{10}-J_{10}^1)+\alpha_2^2(\frac{(A^6-a^6)}{6}J_6+\frac{(A^4B^2-a^4b^2)}{2}J_7+\frac{(A^2B^4-a^2b^4)}{2}J_8++\frac{(B^6-b^6)}{6}J)\} \quad (13)$$

$$U^{|}=\frac{D}{2}\{\alpha_1^2(A^4-a^4)J_{15}+\left[2\alpha_1^2(A^2B^2-a^2b^2)+(1-\mu)(A^4-a^4))J_{16}++(J_1^2(B^4-b^4)+4(1-\mu)\cdot(A^2B^2-a^2b^2))J_{17}+\frac{8\alpha_1\alpha_2}{3}\cdot(J_{21}-J_{21}^1)+(2\alpha_1^2(1--\mu)(B^4-b^4))J_{18}+(2\alpha_2^2(A^2-a^2)J_{19}+[2\alpha_2^2(B^2-b^2)]J_{20}\} \quad (14)$$

$$T^{||}_{max}=\frac{h\rho p^2}{2}\{\alpha_1^2\left[\frac{(A^8-a^8)}{8}J_{11}+\frac{3(A^4B^4-a^4b^4)}{4}J_4+\frac{(B^8-b^8)}{8}J_5+\frac{(A^6B^2-a^6b^2)}{2}J_1+\frac{(A^2B^6-a^2b^6)}{2}J_2+\left[\frac{2\alpha_1\alpha_2}{7}(J_{14}-J_{14}^1)\right]+\alpha_2^2\left[\frac{(A^6-a^6)}{6}J_{12}+\frac{(A^4B^2-a^4b^2)}{2}J_{13}+\frac{(A^2B^4-a^2b^4)}{2}J_6+\frac{(B^6-b^6)}{6}J_7)\} \quad (15)$$

$$U^{||}_{max}=2D\{\frac{\alpha_1^2}{4}[(A^4-a^4)J_{16}+2(A^2B^2-a^2b^2)J_{17}+(B^4-b^4)J_{18}]+\left[\frac{2\alpha_1\alpha_2}{3}-(J_{12}-J_{22}^1)\right]+\frac{J_2^2}{2}[(A^2-a^2)J_{19}+(B^2-b^2)]+\frac{(1-\mu)}{2}[(A^4-a^4)J_{21}+2(A^2B^2-a^2b^2)J_{16}+(B^4-b^4)J_{17}]\} \quad (16)$$

Where

$J_1=\int_0^{\frac{\pi}{2}}cos^{10}\theta sin^4\theta d\theta=\frac{9\pi}{2^{12}}$

$J_2=\int_0^{\frac{\pi}{2}}cos^6\theta sin^8\theta d\theta=\frac{5\pi}{2^{12}}$

$J_3=\int_0^{\frac{\pi}{2}}cos^2\theta sin^{12}\theta d\theta=\frac{33\pi}{2^{12}}$

$J_4=\int_0^{\frac{\pi}{2}}cos^8\theta sin^6\theta d\theta=\frac{5\pi}{2^{12}}$

$J_5=\int_0^{\frac{\pi}{2}}cos^4\theta sin^{10}\theta d\theta=\frac{9\pi}{2^{12}}$

$J_6=\int_0^{\frac{\pi}{2}}cos^6\theta sin^4\theta d\theta=\frac{3\pi}{2^9}$

$J_7=\int_0^{\frac{\pi}{2}}cos^4\theta sin^6\theta d\theta=\frac{3\pi}{2^{12}}$

$J_8=\int_0^{\frac{\pi}{2}}cos^2\theta sin\theta d\theta=\frac{7\pi}{2^9}$

$J_9=\int_0^{\frac{\pi}{2}}sin^{10}\theta d\theta=\frac{63\pi}{2^{12}}$

Note: The indexes above the symbols specify the problem number (according to the setting method)

$$J_{10}=\int_0^{\frac{\pi}{2}}\sqrt{(A^2cos^2\theta+B^2sin^2\theta)^7}\,sin^4\theta cos\theta d\theta=\frac{B^9}{12(B^2-A^2)}-\frac{A^2B^9}{40(B^2-A^2)^2}+\frac{A^4B^7}{320(B^2-A^2)}+\frac{7A^6B^5}{1920(B^2-A^2)^2}++\frac{7A^8B(3A^2+2B^2)}{3072(B^2-A^2)}+\frac{7A^{12}}{1024\sqrt{(B^2-A^2)^2}}*\ln\left|\frac{B+\sqrt{B^2-A^2}}{A}\right|$$

$J_{11}=\int_0^{\frac{\pi}{2}}cos^{12}\theta sin^2\theta d\theta=\frac{33\pi}{2^{12}}$

$J_{12}=\int_0^{\frac{\pi}{2}}cos^8\theta sin^2\theta d\theta=\frac{7\pi}{2^9}$

$J_{13}=\int_0^{\frac{\pi}{2}}cos^{10}\theta d\theta=\frac{63\pi}{2^9}$

$$J_{14}=\int_0^{\frac{\pi}{2}}\sqrt{(A^2cos^2\theta d\theta+Bsin^2\theta)^7}\,cos^4\theta sin\theta d\theta=-\frac{A^9}{12(B^2-A^2)}-\frac{A^9A^2}{40(B^2-A^2)^2}+\frac{A^7B^4}{320(B^2-A^2)^2}+\frac{7A^5B^6}{1920(B^2-A^2)^2}+\frac{7AB^8(2A^2+3B^2)}{3072(B^2-A^2)}+\frac{7B^{12}}{1024\sqrt{(B^2-A^2)}}\times\times\arcsin\frac{\sqrt{B^2-A^2}}{B}$$

$J_{15}=\int_0^{\frac{\pi}{2}}cos^6\theta d\theta=\frac{5\pi}{2^5}$

$J_{16}=\int_0^{\frac{\pi}{2}}cos^4\theta sin^2\theta d\theta=\frac{\pi}{32}$

$J_{17}=\int_0^{\frac{\pi}{2}}sin^4\theta cos^2\theta d\theta=\frac{\pi}{32}$

$J_{18}=\int_0^{\frac{\pi}{2}}sin^6\theta d\theta=\frac{5\pi}{2^5}$

$J_{19}=\int_0^{\frac{\pi}{2}}cos^2\theta d\theta=\frac{\pi}{4}$

$J_{20}=\int_0^{\frac{\pi}{2}}sin^2\theta d\theta=\frac{\pi}{4}$

$$J_{21}=\int_0^{\frac{\pi}{2}}\sqrt{(A^2cos^2\theta d\theta+B^2sin^2\theta)^3}\,cos\theta d\theta=\frac{(3A^2+2B^2)B}{8}+\frac{3A^4}{8\sqrt{(B^2-A^2)}}\cdot\ln\left|\frac{B+\sqrt{B^2-A^2}}{A}\right|$$

$$J_{22}=\int_0^{\frac{\pi}{2}}\sqrt{(A^2cos^2\theta+B^2sin^2\theta)^3}\,sin\theta d\theta==\frac{A(2A^2+3A^2)}{8}+\frac{3B^4}{8\sqrt{(B^2-A^2)}}\cdot\arcsin\frac{\sqrt{B^2-A^2}}{B}+\frac{3B^4}{8\sqrt{(B^2-A^2)}}\cdot\arcsin\frac{\sqrt{B^2-A^2}}{B}$$

Difference $T_{max}-U_{max}$ shall be the function of two independent variables $\alpha_1 u\alpha_2$. In view of this, we shall have the following instead of (4)

$$\frac{\partial(T_{max}-U_{max})}{\partial\alpha_1}\delta\alpha_1+\frac{\partial(T_{max}-U_{max})}{\partial\alpha_2}\delta\alpha_2=0$$

As $\partial\alpha_1\ u\partial\alpha_2$ - are the variations of the independent variables $\alpha_1\ u\alpha_2$, so the last equation will give us the following

$$\frac{\partial(T_{max}-U_{max})}{\partial\alpha}=0,\qquad\frac{\partial(T_{max}-U_{max})}{\partial\alpha_2}\partial\alpha_2=0$$

When we insert the expression $T^{|}_{max}, U^{|}_{max}, T^{||}_{max}, U^{||}_{max}$ into these equations and indicate p^2 in them using ω, we will respectively obtain the following system of two equations:

$(C^{|}_{11}\omega-N^{|}_{11})\alpha_1+(C^{|}_{12}\omega-N^{|}_{12})\alpha_2=0$ (17)
$(C^{|}_{12}\omega-N^{|}_{11})\alpha_1+(C^{|}_{22}\omega-N^{|}_{22})\alpha_2=0$

$(C^{||}_{11}\omega-N^{||}_{11})\alpha_1+(C^{||}_{22}\omega-N^{||}_{22})\alpha_2=0$ (18)
$(C^{||}_{12}\omega-N^{||}_{11})\alpha_1+(C^{||}_{12}\omega-N^{||}_{12})\alpha_2=0$

Where

$$C^{|}_{11}=\frac{p\rho}{2}(\frac{9\pi}{2^{12}}\cdot\frac{A^8-a^8}{8}+\frac{5\pi}{2^{12}}\cdot\frac{A^6B^2-a^6b^2}{}+\frac{9\pi}{2^{12}}\cdot\frac{A^2B^6-a^2b^6}{2}+\frac{33\pi}{2^{12}}\cdot\frac{B^8-b^8}{8})$$

$$C^{|}_{12}=\frac{p\rho}{7}\left(\frac{B^9}{12(B^2-A^2)}-\frac{b^9}{12(B^2-A^2)}\right)-\left(\frac{A^2B^9}{40(B^2-A^2)}-\frac{a^2b^9}{40(b^2-a^2)^2}\right)+\left(\frac{A^4B^7}{320(B^2-A^2)^2}--\frac{a^4b^7}{320(b^2-a^2)^2}\right)+\left(\frac{7A^6B^5}{1920(B^2-A^2)^2}-\frac{7a^6b^5}{1920(b^2-a^2)^2}\right)+\left(\frac{7A^8B(3A^2+2B^2)}{3072(B^2-A^2)^2}-\frac{7a^8b(3a^2+2b^2)}{3072(b^2-a^2)^2}\right)++\frac{7A^{12}}{1024\sqrt{(B^2-A^2)^5}}\cdot ln\left|\frac{B+\sqrt{B^2-A^2}}{A}\right|-\frac{7A^{12}}{1024\sqrt{(b^2-a^2)^5}}\cdot ln\left|\frac{b+\sqrt{b^2-a^2}}{a}\right|$$

$C_{22}^{|}=\frac{h\rho}{2}(\frac{3\pi}{2^9}\frac{A^6-a^6}{6}+\frac{3\pi}{2^9}\frac{A^4B^2-a^4b^2}{2}+\frac{7\pi}{2^9}\frac{A^2B^4-a^2b^4}{2}+\frac{63\pi}{2^9}\frac{B^6-a^6}{6})$

$N_{11}^{|}=\frac{D}{2}\{\frac{5\pi(A^4-a^4)}{32}+\frac{\pi((A^2B^2-a^2b^2)+(1-\mu)\cdot(a^4-b^4))}{16}+\frac{\pi[(B^4-b^4)+4(1-\mu)(A^2B^2-a^2b^2)]}{32}++\frac{5\pi(1-\mu)(B^4-b^4)}{16}\}$

$N_{12}^{|}=\frac{4D}{3}((\frac{(3A^2+2B^2)B}{8}-\frac{(3a^2+2b^2)b}{8}+\frac{3A^4}{8\sqrt{B^2-A^2}}\cdot ln\left|\frac{B+\sqrt{B^2-A^2}}{A}\right|-\frac{3A^4}{8\sqrt{b^2-a^2}}\cdot ln\left|\frac{b+\sqrt{b^2-a^2}}{a}\right|)$

$$N_{22}=\frac{D}{2}(\frac{\pi(A^2-a^2)}{2}+\frac{\pi(B^2-b^2)}{2}) \quad (19)$$

$C_{11}^{||}=\frac{p\rho}{2}(\frac{33\pi}{2^{12}}\cdot\frac{A^8-a^8}{8}+\frac{15\pi}{2^{12}}\cdot\frac{A^4B^4-a^4b^4}{4}+\frac{9\pi}{2^{12}}\cdot\frac{B^8-b^8}{8}+\frac{9\pi}{2^{12}}\cdot\frac{A^6B^2-a^6b^2}{2}+\frac{5\pi}{2^{12}}\cdot\frac{A^2B^6-a^2b^6}{2})$

$C_{12}^{|\,|}=\frac{p\rho}{7}(-\frac{1}{12}\left(\frac{A^9}{B^2-A^2}-\frac{a^9}{b^2-a^2}\right)-\frac{1}{40}\cdot\frac{B^2A^9}{(B^2-A^2)^2}-\frac{b^2a^9}{(b^2-a^2)^2})+\frac{1}{320}(\frac{B^4A^7}{(B^2-A^2)^2}-\frac{b^4a^7}{(b^2-a^2)^2})+\frac{7}{1920}(\frac{B^6A^5}{(B^2-A^2)^2}-\frac{b^6a^5}{(b^2-a^2)^2})+\frac{7(B^8-b^8)}{3072}\cdot(\frac{A(2A^2+3B^2)}{(B^2-A^2)^2}-\frac{a(2a^2+3b^2)}{(b^2-a^2)^2})+\frac{21}{3072}\cdot(\frac{B^{12}\sqrt{B^2-A^2}}{(B^2-A^2)^3}\cdot \arcsin\frac{\sqrt{B^2-A^2}}{B}-\frac{b^{12}\sqrt{b^2-a^2}}{(b^2-a^2)^2}\cdot arcsin\frac{\sqrt{b^2-a^2}}{b}))$

$C_{22}^{||}=\frac{h\rho}{2}(\frac{63\pi}{2^9}\cdot\frac{A^6-a^6}{6}+\frac{7\pi}{2^9}\cdot\frac{A^4B^2-a^4b^2}{2}+\frac{3\pi}{2^9}\cdot\frac{A^2B^4-a^2b^4}{2}+\frac{3\pi}{2^9}\cdot\frac{B^6-b^6}{6})$

$N_{11}^{||}=\frac{D}{4}(\frac{\pi}{16}(A^4-a^4)+\frac{\pi}{8}(A^2B^2-a^2b^2)+\frac{5\pi}{16}(B^4-b^4)+\frac{5\pi}{8}(1-\mu)(A^4-a^4)+\frac{\pi}{4}(A^2B^2-a^2b^2)+\frac{\pi}{8}(B^4-b^4))$

$$N_{12}^{||}=\frac{D}{6}(\frac{A^3-a^3}{4}+\frac{3(AB^2-ab^2)}{8}+\frac{3B^4}{8\sqrt{B^2-A^2}}\cdot arcsin\frac{\sqrt{B^2-A^2}}{B}-\frac{3B^4}{\sqrt{b^2-a^2}}\times \times arcsin\frac{\sqrt{b^2-a^2}}{b})$$

$$N_{22}^{||}=\frac{D}{4}(\pi(A^2-a^2)+\pi(B^2-b^2))$$

$$\begin{vmatrix} C_{11}^{||}\omega-N_{11}^{|} & C_{12}^{|}\omega-N_{12}^{|} \\ C_{12}^{|}\omega-N_{12}^{|} & C_{22}^{|}\omega-N_{22}^{|} \end{vmatrix}=0$$

$$\begin{vmatrix} C_{11}^{|}\omega-N_{12}^{|} & C_{12}^{|}\omega-N_{12}^{||} \\ C_{12}^{|}\omega-N_{12}^{||} & C_{22}^{||}\omega-N_{22}^{||} \end{vmatrix}=0$$

Or

$$(C_{11}^{|}\omega-N_{11}^{|})(C_{22}^{|}\omega-N_{22}^{|})-(C_{12}^{|}\omega-N_{12}^{|})^2=0$$
$$(C_{11}^{||}\omega-N_{11}^{||})(C_{22}^{||}\omega-N_{22}^{||})-(C_{12}^{||}\omega-N_{12}^{||})^2=0 \quad (20),(21)$$

As the equation systems (17) and (18) are homogeneous in independent values $\alpha_1 u \alpha_2$, which cannot become zero simultaneously, so based on the known theory of algebra, their determinants shall be equal to zero.

$$W^{|}=(\alpha_1x+\alpha_1)y^2=0$$

$$W^{||}=(\alpha_1y+\alpha_1)x^2=0 \quad (21)$$

The last expressions are the equations for the frequencies of the analyzed packer system packing element. Now let's pass over to generation of nodal line equations.

The deflection function shall be equal to zero:

$\alpha_1x+\alpha_2=0$ (22)

$\alpha_1y+\alpha_2=0$(23)

Among these equations, *y=0, x =0* express the nodal line equations only for the first equality mode. We shall also use the nodal line equation expressions for generating the nodal line equations of the second mode. We will use (17), (18), (22), (23), and will obtain the following;

$$X=\frac{C_{11}^{|}\omega_2-N_{11}^{|}}{C_{12}^{|}\omega_2-N_{12}^{|}} \quad (24)$$

$$Y=\frac{C_{11}^{||}\omega_2-N_{11}^{||}}{C_{12}^{||}\omega_2-N_{12}^{||}} \quad (25)$$

Thus, for the considered problems, the second mode nodal lines are expressed as straight lines, which are parallel to *Y*-axis in case one, and to *X*-axis in case two.

Analysis of the results

Let's illustrate this solution using a numerical example.

A) for the first setting option we insert value $C_{11}^{|}, C_{12}^{|}, C_{22}^{|}, N_{11,}^{|}N_{12,}^{|}N_{22}^{|}$ into (20), and after some calculations we shall obtain the following:

$$(28.640\text{h}\rho\omega-41{,}055D)(1{,}185h\rho\omega-0.1742D)-(4{,}844h\rho\omega-D)^2=0 \quad (26)$$

Inserting values

h=1,5sm, $\boldsymbol{\rho}=\frac{\gamma}{g}=\frac{0,0075}{981}$

D=27,777 into equation (26) we shall obtain the following
$0{,}142\boldsymbol{\omega^2}-\mathbf{45{,}210}\boldsymbol{\omega}+\mathbf{474{,}576{,}600}=\mathbf{0}$

The solution of the equation shall be as follows
$\boldsymbol{\omega_1}=\mathbf{10{,}873},\qquad \boldsymbol{\omega_2}=\mathbf{307{,}507}$

Hence, the frequencies:

$\boldsymbol{P_1}=\mathbf{104};\ \boldsymbol{P_2}=\mathbf{554}$

$$\frac{P_1}{2\pi}=\frac{104}{6{,}2832}\approx 17\,,\frac{P_2}{2\pi}=\frac{554}{6{,}2832}\approx 88$$

Based on the nodal line equation, corresponding

$X=\frac{286440h\rho\omega-41.05D}{4844h\rho\omega-D}=\mathbf{4{,}14}\boldsymbol{sm}$

б) for the second setting configuration, the equation (21) shall be as follows:

$$(71{,}538\boldsymbol{h\rho\omega}-\mathbf{223.16}\boldsymbol{D})\cdot(866.05\text{h}\boldsymbol{\rho\omega}-\mathbf{1.162}\boldsymbol{D})-(\mathbf{1290.5}\boldsymbol{h\rho\omega}-\boldsymbol{D})^2=\mathbf{0} \quad \mathbf{(27)}$$

Now we insert h value, $\boldsymbol{\rho u D}$

And obtain the following root values:

$\boldsymbol{\omega_1 = 971,181}$, $\boldsymbol{\omega_2}$=232,810

The frequencies shall be as follows

$\boldsymbol{P_1 = 312}$; $\boldsymbol{P_2 = 483}$

The first mode and second mode circular frequency, Hz:

$\frac{P_1}{2\pi} = 50$; $\frac{P_2}{2\pi} = 77$

Based on (25), the nodal line equation corresponding to the second mode shall be as follows

$$Y=\frac{1290.5h\rho\omega - D}{866.05h\rho\omega - 1.162D} = 1.92sm$$

Node lines are shown in Fig. 2 and Fig. 3 with dotted lines

Conclusion

1. Application of the Lagrange's variational equation allows for convenient method of study for the natural bending oscillations:
2. Based on the analytical study of the natural oscillations we have obtained formulas for determination of the oscillation frequencies and waveforms.
   - packer element (packer 1) is set in horizontal interval of the string
   - packer element (packer 2) is set in vertical interval of the string

REFERENCES

Babanly M.B., Mamedov G.A., Mammadov V.T., Aslanov J.N. 2016. Features of the calculation while nonstationary dynamic loadings for the downhole packer sealing. / Science Innovators. International Conference on European Science and Technology. Materials of the XII International Research and Practice Conference. Munich, Germany, Yuly 1st-2nd, p-p. 42-54.

Filatov A.N. and Sharova L.V. 1976.Integral inequalities and theory of nonlinear oscillations. *Science*, 152 p.

Kocoshvili S.M. 1978. Methods of dynamic testing rigid polymeric materials. Riga. Knowledge.182 p.

Leybenzon L.J.Collection of works. in 4 volumes. - M.:, 1951-1955.

Malsev L.E. 1978. Approximate solution of some dynamic problems of viscoelasticity. *Mechanics of Polymers*, N2. p. 210-218.

Mammadov V.T. and Gurbanov S.R. 2015. The impact of the relatively low compression rubber to functions sealing elements of wellbore packers ICSCCW-2015, Eighth International Conference on Soft Computing, Computing with Words and Perceptions in System Analysis, Decision and Control, Antalya, Turkey September 3-4, p-p. 397-399.

Ponovka Y.G. 1976. Fundamentals of the applied theory of oscillation and impact. L: Mechanical Engineering. 320 p.

Timoshenko S.P. 1967. Oscillations in engineering. M.: Science, 444.p.

Tonti E. 1967. Variational principles in elastostatics – Meccanics.Vol.2, №4.p.201-202.

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