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RESEARCH ARTICLE

AN ORTHOGONAL GENERALIZED HIGHER REVERSE LEFT (RESP. RIGHT) CENTRALIZER ON SEMIPRIME RINGS

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ABSTRACT

The main object of this paper is prove that: Let R be a 2-torsion free semiprime ring ,$T=(T_i)_{i\in N}$ and $H=(H_i)_{i\in N}$ be two generalized higher reverse left (resp. right) centralizers associated with the higher reverse left (resp. right) centralizers $t=(t_i)_{i\in N}$ and $h=(h_i)_{i\in N}$ resp. of R , where T_n and H_n are commuting. Then T_n and H_n are orthogonal if and only if $T_n(x)\, H_n(y) = t_n(x)\, H_n(y) = 0$, for all x , y $\in$ R and n $\in$ N.

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INTRODUCTION

A ring R is called semiprime if xRx = (0) implies = 0, such that x $\in$ R (3) . Let R be a ring then R is called 2-torsion free if 2x = 0 implies x = 0, for all x $\in$ R (3) . Zalar (5) present the concepts of centralizer and Jordan centralizer of a ring R as follows: A left (resp. right) centralizer of a ring R is an additive mapping $t: R \longrightarrow R$ which satisfies the following equation t (xy) = t(x) y (resp.t (xy) = x t (y)) , for all x , y $\in$ R . t is called a centralizer of R if it is both a left and a right centralizer. A left (resp. right) Jordan centralizer of a ring R is an additive mapping $t: R \longrightarrow R$ which satisfies the following equation $t(x^2) = t(x)\,x$ (resp. $t(x^2) = x\,t(x)$) , for all x $\in$ R . t is called a Jordan centralizer of R if it is both a left and a right Jordan centralizer . Jarullah and Salih (4) introduced the concepts of a generalized higher reverse left (resp. right) centralizer and a Jordan generalized higher reverse left (resp. right) centralizer on rings as follows:

Let $T = (T_i)_{i\in N}$ be a family of additive mappings of a ring R into itself .Then T is called a generalized higher reverse left (resp. right) centralizer associated with the higher reverse left (resp. right) centralizer $t = (t_i)_{i\in N}$ of R if for all x , y $\in$ R and n $\in$ N

$$T_n(xy) = \sum_{i=1}^{n} T_i(y)\, t_{i-1}(x)$$

(resp. $T_n(xy) = \sum_{i=1}^{n} t_{i-1}(y)\, T_i(x)$).

Let $T = (T_i)_{i\in N}$ be a family of additive mappings of a ring R into itself. Then T is called a Jordan generalized higher reverse left (resp. right) centralizer associated with the Jordan higher reverse left (resp. right) centralizer $t = (t_i)_{i\in N}$ of R , if the following equation holds , for all x $\in$ R and n $\in$ N:

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$T_n(x^2) = \sum_{i=1}^{n} T_i(x)\, t_{i-1}(x)$ (resp. $T_n(x^2) = \sum_{i=1}^{n} t_{i-1}(x)\, T_i(x)$).

In this paper , we define and study the concept of orthogonal generalized higher reverse left (resp.right) centralizers of semiprime rings and we prove some of lemmas and theorems about orthogonally one of these Theorems is : Let R be a 2-torsion free semiprime ring ,$T=(T_i)_{i\in N}$ and $H=(H_i)_{i\in N}$ be two generalized higher reverse left (resp.right) centralizers of R , Suppose that $T^2_n = H^2_n$, for all $n \in N$. Then $T_n + H_n$ and $T_n - H_n$ are orthogonal . In our work we need the following Lemmas :

Lemma (1.1): (2)

Let R be a 2-torsion free semiprime ring and x, y be elements of R, then the following conditions are equivalent : **(i)** $xry = 0$, for all $r \in R$ **(ii)** $yrx = 0$, for all $r \in R$ **(iii)** $xry + yrx = 0$, for all $r \in R$ If one of these conditions is fulfilled ,then $xy = yx = 0$.

Lemma (1.2): (1)

Let R be a 2-torsion free semiprime ring and x , y be elements of R if $xry + yrx = 0$, for all $r \in R$,then $xry = yrx = 0$.

Orthogonal Generalized Higher Reverse Left (resp. Right) Centralizers on

Semiprime Rings: In this section we will introduce and study the concept of orthogonal generalized higher reverse left (resp.right) centralizers on semiprime rings.

Definition (2.1):

Two generalized higher reverse left (resp.right) centralizers $T=(T_i)_{i\in N}$ and $H=(H_i)_{i\in N}$ of a ring R are called orthogonal if $T_n(x)\, R\, H_n(y) = (0) = H_n(y)\, R\, T_n(x)$, for all x , y $\in$ R and $n \in N$. Where $T_n(x)\, R\, H_n(y) = \sum_{i=1}^{n} T_i(x)\, z\, H_i(y)$, for all $z \in R$

Lemma (2.2):

Let R be a semiprime ring , suppose that $T=(T_i)_{i\in N}$ and $H=(H_i)_{i\in N}$ be two generalized higher reverse left (resp.right) centralizers of R , satisfy $T_n(x)\, R\, H_n(x) = (0)$, for all $x \in R$ and $n \in N$. Then $T_n(x)\, R\, H_n(y) = (0)$, for all x , y $\in$ R and $n \in N$.

Proof:

Suppose that $T_n(x)\, R\, H_n(x) = (0)$, for all $x \in R$ and $n \in N$ That is $T_n(x)\, R\, H_n(x) = \sum_{i=1}^{n} T_i(x)\, z\, H_i(x) = 0$, for all x , z $\in$ R ...(1)

Replace x by x + y in (1), we have that $\sum_{i=1}^{n} T_i(x+y)\, z\, H_i(x+y) = 0$ $\sum_{i=1}^{n} T_i(x)\, z\, H_i(x) + T_i(x)\, z\, H_i(y) + T_i(y)\, z\, H_i(x) + T_i(y)\, z\, H_i(y) = 0$ Therefore , by our assumption and Lemma (1.1), we get

$$\sum_{i=1}^{n} T_i(x)\, z\, H_i(x) = 0 \text{ , for all } x , y, z \in R$$

Thus , $T_n(x)\, R\, H_n(y) = (0)$, for all x , y $\in$ R and $n \in N$.

Lemma (2.3):

Let R be a 2-torsion free semiprime ring, $T=(T_i)_{i\in N}$ and $H=(H_i)_{i\in N}$ be two generalized higher reverse left (resp.right) centralizers of R . Then T_n and H_n are orthogonal if and only if $T_n(x)\, H_n(y) + H_n(x)\, T_n(y) = 0$, for all x , y $\in$ R and $n \in N$.

Proof: Suppose that T_n and H_n are orthogonal T.P. $T_n(x)\, H_n(y) + H_n(x)\, T_n(y) = 0$, for all x , y $\in$ R and $n \in N$ Since T_n and H_n are orthogonal , we have that $\sum_{i=1}^{n} T_i(x)\, z\, H_i(y) = 0 = \sum_{i=1}^{n} H_i(y)\, z\, T_i(x)$, for all x , y , z $\in$ R Therefore , by Lemma (1.1) , we get the require result . .

Conversely, , it's clear by using Lemma (1.2)

Theorem (2.4): Let R be a 2-torsion free semiprime ring, $T=(T_i)_{i\in N}$ and $H=(H_i)_{i\in N}$ are orthogonal generalized higher reverse left (resp.right) centralizers associated with the higher reverse left (resp.right) centralizers $t=(t_i)_{i\in N}$ and $h=(h_i)_{i\in N}$ resp. of R, where T_n and H_n are commuting. Then the following relations are holds, for all $x, y \in R$ and $n\in N$: (i) $T_n(x) H_n(y) = H_n(x) T_n(y) = 0$ Hence $T_n(x) H_n(y) + H_n(x) T_n(y) = 0$ (ii) t_n, H_n are orthogonal and $t_n(x) H_n(y) = H_n(x) t_n(y) = 0$ (iii) h_n, T_n are orthogonal and $h_n(x) T_n(y) = T_n(x) h_n(y) = 0$ (iv) t_n, h_n are orthogonal higher reverse left (resp.right) centralizers

Proof:

(i) Suppose that T_n and H_n are orthogonal $\sum_{i=1}^{n} T_i(x) z H_i(y) = 0 = \sum_{i=1}^{n} H_i(y) z T_i(x)$, for all $x, y, z \in R$ By Lemma (1.1), we have that $\sum_{i=1}^{n} T_i(x) H_i(y) = \sum_{i=1}^{n} H_i(x) T_i(y) = 0$, for all $x, y \in R$ Then, we get $\sum_{i=1}^{n} T_i(x) H_i(y) + H_i(x) T_i(y) = 0$, for all $x, y \in R$ Hence $T_n(x) H_n(y) + H_n(x) T_n(y) = 0$, for all $x, y \in R$ and $n \in N$ (ii) Suppose that T_n and H_n are orthogonal

By (i), we have that $\sum_{i=1}^{n} T_i(x) H_i(y) = 0$, for all $x, y \in R$ Replace x by zx and since H_n is a commuting, we have that $\sum_{i=1}^{n} H_i(y) T_i(zx) = 0$ $\sum_{i=1}^{n} H_i(y) T_i(x) t_{i-1}(z) = 0$ By Lemma (1.1), we have that

$$\sum_{i=1}^{n} H_i(y) t_{i-1}(z) = 0$$

Right multiply by $t_i(x)$, we have that

$\sum_{i=1}^{n} H_i(y) t_{i-1}(z) t_i(x) = 0$, for all $x, y, z \in R$ ... (1) Since H_n is a commuting, we have that $\sum_{i=1}^{n} t_i(x) t_{i-1}(z) H_i(y) = 0$, for all $x, y, z \in R$ ... (2) By (1) and (2), we get t_n and H_n are orthogonal. From (2), we have that $\sum_{i=1}^{n} t_i(x) t_{i-1}(z) H_i(y) = 0$, for all $x, y, z \in R$ By Lemma (1.1), we have that

$\sum_{i=1}^{n} t_i(x) H_i(y) = \sum_{i=1}^{n} H_i(x) t_i(y) = 0$, for all $x, y \in R$ Thus, $t_n(x) H_n(y) = H_n(x) t_n(y) = 0$, for all $x, y \in R$ and $n \in N$.

(iii) By the same method as (ii). **(iv)** Since that T_n and H_n are orthogonal By (ii), we have that $t_n(x) H_n(y) = 0$, for all $x, y \in R$, and $n \in N$ $\sum_{i=1}^{n} t_i(x) H_i(y) = 0$ Replace y by yz, we have that $\sum_{i=1}^{n} t_i(x)\, \alpha\, H_i(yz) = 0$ $\sum_{i=1}^{n} t_i(x) H_i(z) h_{i-1}(y) = 0$ Replace $h_{i-1}(y)$ by $h_i(y)$, we have that $\sum_{i=1}^{n} t_i(x) H_i(z) h_i(y) = 0$ By Lemma (1.1), we get the require result.

Theorem (2.5)

Let R be a 2-torsion free semiprime ring, $T=(T_i)_{i\in N}$ and $H=(H_i)_{i\in N}$ are orthogonal generalized higher reverse left (resp.right) centralizers associated with the higher reverse left (resp.right) centralizers $t=(t_i)_{i\in N}$ and $h=(h_i)_{i\in N}$ resp. of R. Then the following relations are hold, for all $n\in N$: **(i)** $t_nH_n = H_nt_n = 0$ **(ii)** $h_nT_n = T_nh_n = 0$ **(iii)** $T_nH_n = H_nT_n = 0$

Proof : (i) Since that T_n and H_n are orthogonal By Theorem (2.4)(ii), we have that $t_n(x) H_n(y) = 0$, for all $x, y \in R$ and $n \in N$

$\sum_{i=1}^{n} t_i(x) H_i(y) = 0$, for all $x, y \in R$ $\sum_{i=1}^{n} t_i\,(t_i(x) H_i(y)) = 0$ $\sum_{i=1}^{n} t_i(H_i(y))\, t_{i-1}(t_i(x)) = 0$ Right multiply by $t_i(H_i(y))$, we have that $\sum_{i=1}^{n} t_i(H_i(y))\, t_{i-1}(t_i(x))\, t_i(H_i(y)) = 0$, for all $x, y \in R$ Since R is a semiprime ring, we have that $\sum_{i=1}^{n} t_i(H_i(y)) = 0$, for all $y\in R \Rightarrow t_n H_n = 0$, for all $n \in N$ ... (1) Also, by Theorem (2.4)(ii), we have that
$H_n(x) t_n(y) = 0$, for all $x, y \in R$ and $n \in N$

$\sum_{i=1}^{n} H_i(x)\, t_i(y) = 0$ $\sum_{i=1}^{n} H_i(H_i(x)\, t_i(y)) = 0$ $\sum_{i=1}^{n} H_i(t_i(y))\, h_{i-1}(H_i(x)) = 0$ Right multiply by $H_i(t_i(y))$, we have that $\sum_{i=1}^{n} H_i(t_i(y))\, h_{i-1}(H_i(x))\, H_i(t_i(y)) = 0$ Since R is a semiprime ring , we have that $\sum_{i=1}^{n} H_i(t_i(y)) = 0$, for all $y \in R \Rightarrow H_n\, t_n = 0$, for all $n \in N$ ... (2) From (1) and (2), we get $t_n H_n = H_n t_n = 0$, for all $n \in N$. **(ii)** By the same method as (i) **(iii)** Since that T_n and H_n are orthogonal By Theorem (2.4)(i) , we have that $T_n(x)\, H_n(y) = 0$, for all $x , y \in R$ and $n \in N$ $\sum_{i=1}^{n} T_i(x)\, H_i(y) = 0$ $\sum_{i=1}^{n} T_i(T_i(x)\, H_i(y)) = 0$ $\sum_{i=1}^{n} T_i(H_i(y))\, t_{i-1}(T_i(x)) = 0$ Right multiply by $T_i(H_i(y))$, we have that $\sum_{i=1}^{n} T_i(H_i(y))\, t_{i-1}(T_i(x))\, T_i(H_i(y)) = 0$ Since R is a semiprime ring , we have that $\sum_{i=1}^{n} T_i(H_i(y)) = 0$, for all $y \in R \Rightarrow T_n\, H_n = 0$, for all $n \in N$ ... (1) Also, By Theorem (2.4)(i) , we have that

$H_n(x)\, T_n(y) = 0$, for all $x , y \in R$ and $n \in N$

$\sum_{i=1}^{n} H_i(x)\, T_i(y) = 0$ $\sum_{i=1}^{n} H_i(H_i(x)\, T_i(y)) = 0$

$\sum_{i=1}^{n} H_i(T_i(y))\, h_{i-1}(H_i(x)) = 0$ Right multiply by $H_i(T_i(y))$, we have that

$\sum_{i=1}^{n} H_i(T_i(y))\, h_{i-1}(H_i(x))\, H_i(T_i(y)) = 0$ Since R is a semiprime ring , we have that $\sum_{i=1}^{n} H_i(T_i(y)) = 0$, for all $y \in R \Rightarrow H_n\, T_n = 0$, for all $n \in N$ ... (2) From (1) and (2), we get $T_n H_n = H_n T_n = 0$, for all $n \in N$.

Theorem (2.6):

Let R be a 2-torsion free semiprime ring , $T=(T_i)_{i\in N}$ and $H=(H_i)_{i\in N}$ be two generalized higher reverse left (resp.right) centralizers associated with the higher reverse left (resp.right) centralizers $t=(t_i)_{i\in N}$ and $h=(h_i)_{i\in N}$ resp. of R , where T_n and H_n are commuting . Then T_n and H_n are orthogonal if and only if the following relations are holds , for all $x , y \in R$ and $n\in N$: **(i)** $T_n(x)\, H_n(y) + H_n(x)\, T_n(y) = 0$ **(ii)** $t_n(x)\, H_n(y) = h_n(x)\, T_n(y) = 0$

Proof:

Suppose that T_n and H_n are orthogonal T.P. **(i)** $T_n(x)\, H_n(y) + H_n(x)\, T_n(y) = 0$

(ii) $t_n(x)\, H_n(y) = h_n(x)\, T_n(y) = 0$, for all $x , y \in R$ and $n\in N$

Since T_n and H_n are orthogonal

By Lemma (2.3) , we get (i) .
Now, Since T_n and H_n are orthogonal
By Theorem (2.4)(i) , we have that
$T_n(x)\, H_n(y) = 0$

Replace $H_i(y)$ by x , we have that

$$\sum_{i=1}^{n} T_i(x)\, x = 0$$

$$\sum_{i=1}^{n} t_i\, (T_i(x)\, x) = 0$$

$$\sum_{i=1}^{n} t_i(x)\, t_{i-1}(T_i(x)) = 0$$

Left multiply by $H_i(y)$ and since H_n is a commuting, we have that

$$\sum_{i=1}^{n} t_i(x)\, H_i(y)\, t_{i-1}(T_i(x)) = 0$$

Right multiply by $t_i(x)\, H_i(y)$, we have that

$$\sum_{i=1}^{n} t_i(x)\, H_i(y)\, t_{i-1}(T_i(x))\, t_i(x)\, H_i(y) = 0$$

Since R is a semiprime ring, we have that $\sum_{i=1}^{n} t_i(x) H_i(y) = 0 \Rightarrow t_n(x) H_n(y) = 0$, for all $x, y \in R$ and $n \in N$ …(1)

Also, by Theorem (2.4)(i), we have that $H_n(x) T_n(y) = 0$, for all $x, y \in R$ and $n \in N$

$\sum_{i=1}^{n} H_i(x) T_i(y) = 0$ Replace $T_i(y)$ by x, we have that

$$\sum_{i=1}^{n} H_i(x)\, x = 0$$

$$\sum_{i=1}^{n} h_i (H_i(x)\, x) = 0$$

$$\sum_{i=1}^{n} h_i (x)\, h_{i-1}(H_i(x)) = 0$$

Left multiply by $T_i(y)$ and since T_n is a commuting, we have that

$$\sum_{i=1}^{n} h_i (x) T_i(y)\, h_{i-1}(H_i(x)) = 0$$

Right multiply by $h_i (x) T_i(y)$, we have that

$\sum_{i=1}^{n} h_i (x) T_i(y)\, h_{i-1}(H_i(x))\, h_i (x) T_i(y) = 0$ Since R is a semiprime ring, we have that

$\sum_{i=1}^{n} h_i (x) T_i(y) = 0 \Rightarrow h_n (x) T_n(y) = 0$, for all $x, y \in R$ and $n \in N$ …(2)

From (1) and (2), we get (ii).

Conversely, Suppose that the relations are hold, for all $x, y \in R$ and $n \in N$:

(i) $T_n(x) H_n(y) + H_n(x) T_n(y) = 0$
(ii) $t_n(x) H_n(y) = h_n(x) T_n(y) = 0$ T.P. T_n and H_n are orthogonal From (i), we have that $T_n(x) H_n(y) + H_n(x) T_n(y) = 0$, for all $x, y \in R$ and $n \in N$ By Lemma (2.3), we get the require result.

Theorem (2.7): Let R be a 2-torsion free semiprime ring, $T=(T_i)_{i\in N}$ and $H=(H_i)_{i\in N}$ be two generalized higher reverse left (resp.right) centralizers associated with the higher reverse left (resp.right) centralizers $t=(t_i)_{i\in N}$ and $h=(h_i)_{i\in N}$ resp. of R, where T_n and H_n are commuting. Then T_n and H_n are orthogonal if and only if $T_n(x) H_n(y) = t_n(x) H_n(y) = 0$, for all $x, y \in R$ and $n \in N$

Proof: Suppose that T_n and H_n are orthogonal T.P. $T_n(x) H_n(y) = t_n(x) H_n(y) = 0$, for all $x, y \in R$ and $n \in N$ Since T_n and H_n are orthogonal By Theorem (2.4)(i), we have that $T_n(x) H_n(y) = 0$, for all $x, y \in R$ and $n \in N$ ...(1) $\sum_{i=1}^{n} T_i(x) H_i(y) = 0$, for all $x, y \in R$ Replace $H_i(y)$ by x, we have that $\sum_{i=1}^{n} t_i (T_i(x)\, x) = 0$ $\sum_{i=1}^{n} t_i (x)\, t_{i-1}(T_i(x)) = 0$ Left multiply by $H_i(y)$ and since H_n is a commuting, we have that $\sum_{i=1}^{n} t_i(x) H_i(y)\, t_{i-1}(T_i(x)) = 0$ Right multiply by $t_i(x) H_i(y)$, we have that

$$\sum_{i=1}^{n} t_i(x) H_i(y)\, t_{i-1}(T_i(x))\, t_i(x) H_i(y) = 0$$

Since R is a semiprime ring, we have that

$\sum_{i=1}^{n} t_i(x) H_i(y) = 0 \Rightarrow t_n(x) H_n(y) = 0$, for all $x, y \in R$ and $n \in N$ …(2)

From (1) and (2), we get the require result.

Conversely , Suppose that $T_n(x) H_n(y) = t_n(x) H_n(y) = 0$, for all x , y $\in$ R and n$\in$N T.P. T_n and H_n are orthogonal By assumption , we have that $T_n(x) H_n(y) = 0$ $\sum_{i=1}^{n} T_i(x) H_i(y) = 0$ Replace x by zx , we have that $\sum_{i=1}^{n} T_i(zx) H_i(y) = 0$ $\sum_{i=1}^{n} T_i(x) t_{i-1}(z) H_i(y) = 0$, for all x , y , z $\in$ R …(3) Since T_n and H_n are commuting , we have that $\sum_{i=1}^{n} H_i(y) t_{i-1}(z) T_i(x) = 0$, for all x , y , z $\in$ R …(4) From (3) and (4), we have that $T_n(x) R H_n(y) = (0) = H_n(y) R T_n(x)$, for all x , y , z $\in$ R and n $\in$ N
Hence T_n and H_n are orthogonal .

Theorem (2.8)

Let R be a 2-torsion free semiprime ring , $T=(T_i)_{i\in N}$ and $H=(H_i)_{i\in N}$ be two generalized higher reverse left (resp.right)centralizers associated with the higher reverse left(resp.right) centralizers $t=(t_i)_{i\in N}$ and $h=(h_i)_{i\in N}$ resp. of R , where T_n and H_n are commuting . Then T_n and H_n are orthogonal if and only if $T_n(x) H_n(y) = 0$, and $t_n H_n = t_n h_n = 0$, for all x , y $\in$ R and n$\in$ N .

Proof:

Suppose that T_n and H_n are orthogonal T.P. $T_n(x) H_n(y) = 0$ and $t_n H_n = t_n h_n = 0$, for all x , y $\in$ R and n$\in$N Since T_n and H_n are orthogonal By Theorem (2.4)(i) , we have that $T_n(x) H_n(y) = 0$, for all x , y $\in$ R and n$\in$N ...(1) By Theorem (2.5)(i) , we have that $t_n H_n = 0$, for all n $\in$N …(2) By Theorem (2.4)(iii) , we have that $T_n(x) h_n(y) = 0$, for all x , y $\in$ R and n $\in$ N $\sum_{i=1}^{n} t_i (T_i(x)h_i(y)) = 0$, for all x , y $\in$ R $\sum_{i=1}^{n} t_i (h_i(y)) t_{i-1}(T_i(x)) = 0$ Right multiply by $t_i(h_i(y))$, we have that $\sum_{i=1}^{n} t_i (h_i(y)) t_{i-1}(T_i(x)) t_i (h_i(y)) = 0$ Since R is a semiprime ring , we have that $\sum_{i=1}^{n} t_i(h_i(y)) = 0$, for all y $\in$ R $\Rightarrow$ $t_n h_n = 0$, for all n $\in$N …(3)

From (1), (2) and (3) , we get the required result . Conversely , Suppose that $T_n(x) H_n(y) = 0$ and $t_n H_n = t_n h_n = 0$, for all x , y $\in$ R and n $\in$ N . T.P. T_n and H_n are orthogonal By assumption , we have that $T_n(x) H_n(y) = 0$ $\sum_{i=1}^{n} T_i(x) H_i(y) = 0$ Replace x by zx , we have that $\sum_{i=1}^{n} T_i(zx) H_i(y) = 0$ $\sum_{i=1}^{n} T_i(x) t_{i-1}(z) H_i(y) = 0$, for all x , y , z $\in$ R …(4) Since T_n and H_n are commuting , we have that $\sum_{i=1}^{n} H_i(y) t_{i-1}(z) T_i(x) = 0$, for all x , y, z $\in$ R …(5) From (4) and (5) , we get T_n and H_n are orthogonal .

Theorem (2.9): Let R be a 2-torsion free semiprime ring ,$T=(T_i)_{i\in N}$ and $H=(H_i)_{i\in N}$ be two generalized higher reverse left (resp.right) centralizers of R , Suppose that $T^2_n = H^2_n$, for all n $\in$ N . Then $T_n + H_n$ and $T_n - H_n$ are orthogonal.

Proof:

$$((T_n + H_n)(T_n - H_n) + (T_n - H_n) (T_n + H_n)) (x)$$
$$= \sum_{i=1}^{n} T^2_i(x) - T_i(x) H_i(x) + H_i(x)T_i(x) - H^2_i(x) + T^2_i(x) + T_i(x)H_i(x) - H_i(x)T_i(x) - H^2_i(x) = 0$$

Therefore, $((T_n + H_n)(T_n - H_n) + (T_n - H_n)(T_n + H_n))(x) = 0$
By Lemma (2.3), we get the require result.

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