



RESEARCH ARTICLE

SELECTION OF SKIP LOT SAMPLING PLAN V WITH MDS (0,1) PLAN AS REFERENCE PLAN THROUGH MINIMUM ANGLE CRITERIA

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ABSTRACT

Under acceptance sampling producer's risk and consumer's risk has become increasingly common in maintaining quality products especially in industries. In this paper a skip lot sampling plan of type (SkSP-V) with Multiple Deferred State Sampling plan MDS (0,1) as reference plan has been studied. Producer's risk and Consumer's risk has been minimized by minimizing the tangent angle passing through $(AQL, 1-\alpha)$ and (LQL, β) . Designing methodologies are provided to illustrate the solution procedures. This paper provides computational analysis that leads to variety of managerial insights. This method seems to be versatile and can be even adopted to the elementary production process, where the stipulated quality level is advisable to fix at a later stage and provides less producer risk and consumer risk.

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INTRODUCTION

Acceptance sampling plan is a procedure for application of statistical techniques to determine whether a quality of material should be accepted or rejected. Most logistic and retailing companies suffer from returned items considered as defective by customers. Generally, the most fundamental reason for defects comes from the manufacturing site is due to imperfect production. In this paper a skip lot sampling plan of type SkSP-V with Multiple Deferring State Sampling Plan (0,1) as reference plan has been proposed. Producer risk and Consumer Risk has been minimized through minimizing the tangent angle passing through $(AQL, 1-\alpha)$ and (LQL, β) . It is discussed how the declination angle of the tangent at the inflection point of the OC curve which discriminates the Multiple Deferred State Sampling plan. Tables are presented for the selection of plans based on Acceptable Quality Level (AQL) and Limiting Quality Level (LQL) with discriminant or declination angle of the tangent. Dodge (1955) has introduced the concept of skip-lot sampling, by applying the principles of a continuous sampling plan of type CSP-1 to a series of lots or batches of material. This plan is designated as the SkSP-1 plan and specifically applicable for bulk materials or products produced in successive lots. Perry (1970) has developed a system of sampling inspection plan known as SkSP-2. This Plan involves inspection of only a fraction 'f' of the submitted lots when quality of the submitted product is good as demonstrated by the quality of the product. Peach and Littaur (1946) have considered two points on the OC curve as $(p_1, 1-\alpha)$, and (p_2, β) and propose another method which minimizes the angle between them. Normal Bush *et al.* (1953) have suggested two points on the OC curve namely $(AQL, 1-\alpha)$, and $(IQL, 0.50)$, and the cosine angle of chord length to describe the direction of OC curve. Suresh (1993) has given for the selection of Skip-lot Sampling Plan of

type SkSP-2 with reference plans $SSP(c=0)$, $SSP(c\neq 0)$ and $DSP(0,1)$ using consumer and producer quality levels. Jayalakshmi (2009) has presented a procedure for designing skip lot sampling plan of type SkSP-2 with STDS as reference plan involving minimum angle method between the lines formed by the points $(AQL, 1-\alpha)$, (AQL, β) and $(AQL, 1-\alpha)$, (LQL, β) are given. Recently Muhammad Aslam *et. al* (2012) has studied optimal designing of an SkSP-V skip lot sampling plan with Double Sampling Plan as the reference plan. The design parameters are determined so as to minimize the average sample number while the specified producer risk and the consumer risks are satisfied. The concept of multiple dependent (or deferred) state sampling (MDS) was introduced by Wortham and Baker (1976). The MDS sampling plan belongs to the group of conditional sampling procedures. In these procedures, acceptance or rejection of a lot is based not only on the sample from that lot, but also on sample results from past lots or from future lots (in the case of deferred state sampling). The operating procedure and characteristics of the attributes MDS sampling plan can be found in Wortham and Baker (1976) and this plan was studied further by Vaerst (1982). Subramani.K and Govindaraju.K (1990) have presented tables for the selection of multiple deferred state MDS – 1 sampling plan for given acceptable and limiting quality levels using Poisson distribution.

SKIP LOT SAMPLING PLAN V

Based on the principles of CSP-V plan, a new system of skip-lot sampling plan designated as SkSP-V skip-lot sampling procedure is developed for the quality inspection of continuous flow of bulk products. The SkSP-V plan, like other skip-lot plans, has both a continuous sampling part for choosing which lots to inspect, and a lot sampling part called 'reference plan' for inspecting the chosen lots.

THE OPERATING PROCEDURE FOR SKSP-V PLAN

- At the outset, start with normal inspection using the reference plan. During the normal inspection, lots are inspected one by one in the

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order of production or in the order of being submitted to inspection.

- When i consecutive lots are accepted on normal inspection, discontinue the normal inspection and switch to skipping inspection.
- During skipping inspection, inspect only a fraction ' f ' of the lots selected at random. Skipping inspection is continued until sampled lot is rejected.
- When a lot is rejected on skipping inspection before k consecutively sampled lots are accepted, revert to normal inspection as per (1) above.
- When a lot is rejected after k consecutive lots have been accepted revert to normal inspection with reduced clearance number x as per (6) given below.
- During normal inspection with clearance number x , lots are inspected one by one in the order of being submitted to inspection. This continues until either a lot is rejected or x lots are accepted, whichever occurs earlier.
- When a lot is rejected, immediately revert to normal inspection with clearance number i as per (1) given above.
- When x lots are accepted, discontinue normal inspection and switch to skipping inspection as per (3) above.
- When a lot is rejected, perform 100% inspection (screening) and replace all the non-conforming units found with conforming units in the rejected lots in the case of non-destructive testing.

Operating Procedure for MDS-1 (c_1, c_2)

- For each lot, select a sample of n units and test each unit for conformance to the specified requirements.
- Accept the lot if d (the observed number of observation of defectives) is less than or equal to c_1 ; reject the lot if d is greater than c_2 .
- If $c_1 < d < c_2$, accept the lot preceding or succeeding i lots are accepted with $d \leq c_1$, otherwise reject the lot.

Operating procedure for MDS (0,1) plan

A multiple deferred state sampling plan of Wortham and Baker (1976) with $r = 0$ and $b = 1$ is operated as follows:

- From each lot, take a random sample of n units and observe the non-conforming units, d .
- If $d = 0$, accept the lot; if $d > 1$, reject the lot. If $d = 1$, accept the lot, provided the forthcoming m lots in succession are all accepted (previous m lots in case of multiple dependent state sampling).

The probability of acceptance based on poisson model is

$$P_a(p) = e^{-np} + np_1 e^{-np} e^{-npm}$$

The Operating characteristic function for MDS (r, b) is as follows

$$P = P_a(p) = P_{a,r}(p) + [P_{a,r+b}(p) - P_{a,r}(p)] [P_a(p)]^m$$

When the parameters $r=0$ and $b=1$. The Probability of Acceptance will be

$$P_a(p) = e^{-np} + np_1 e^{-np} e^{-npm}$$

DESIGNING METHOD USING MINIMUM ANGLE CRITERIA

Norman Bush et. al. have considered two points on the OC curve as $(AQL, 1-\alpha)$ and $(IQL, 0.05)$ for minimizing the consumer's risk. Here another approach of minimization of angle between the lines joining the points (AQL, β) , $(AQL, 1-\alpha)$ and $(AQL, 1-\alpha)$, (LQL, β) was given

by Singaravelu (1993). Applying this method one can get a better plan which has an OC curve approaching to the ideal OC curve.

The formula for $\tan\theta$ is given as

$$\tan \theta = \frac{\text{Opposite side}}{\text{Adjacents side}}$$

$$n \tan \theta = \frac{np_2 - np_1}{P_a(p_1) - P_a(p_2)}$$

Using this formula, the angle θ is minimized for the given np_1 and np_2 values.

SELECTION PROCEDURE FOR SkSP-V WITH MULTIPLE DEFERRED STATE SAMPLING PLAN MDS(0,1)

Table 1 can be used for obtaining plan parameters with the minimum tangent angle ($n \tan\theta$) between the lines formed by the points $(AQL, 1-\alpha)$, (AQL, β) and $(AQL, 1-\alpha)$, (LQL, β) . One can find the sampling plan from the tables with minimum tangent angle ($n \tan\theta$) by the following procedures:

- Compute the operating ratio p_2/p_1
- With the computed values of p_2/p_1 enter the value from the table headed by p_2/p_1 this is equal to or just greater than the computed ratio.
- The sample size is then obtained as $n = np_1/p_1$, since θ is known, the parameter n can be computed.
- Thus the minimum angle can be found as $\{\theta = \tan^{-1} \theta/n\}$

SELECTION OF PLAN FOR GIVEN, i, k, f, p_1 and p_2

To select a plan for given i, k, f, p_1 and p_2 , first calculate the operating ratio p_2/p_1 . Select and then the table corresponding to the given i, k, f and ' m ' and locate the value or in the row headed with OR which is very close to the desired ratio. The parameter np_1 and $n \tan\theta$ are can obtained from the selected table corresponding to given i, k, f and ' m ' along with producers and consumers risk. The sample size thus obtained as $n = np_1/p_1$ and the minimum angle $\theta = \tan^{-1} \{(\tan\theta)/n\}$. For example for given $p_1=0.01, p_2=0.5$ one can compute $p_2/p_1=0.50/0.01=50$. The OR value exactly equal to 50 with $i = 1, k = 2, f = 2/3$ and $m = 1$ one find the following values for skip-lot plans from the constructed Table 1.

$n \tan\theta = 4.9967$	$m=1$	$\alpha = 0.89\beta = 1.04$
$n \tan\theta = 7.4996$	$m=1$	$\alpha = 1.91\beta = 0.08$
$n \tan\theta = 10.1287$	$m=1$	$\alpha = 3.24\beta = 0.01$
$n \tan\theta = 2.8717$	$m=1$	$\alpha = 4.83\beta = 0.00$

The skip lot plans corresponding to minimum angle from the above set of values are

(1, 2, 2/3, 0.10)	with $\theta = 26.55\alpha = 0.89\beta = 1.04$
(1, 2, 2/3, 0.15)	with $\theta = 26.56\alpha = 1.91\beta = 0.08$
(1, 2, 2/3, 0.20)	with $\theta = 26.86\alpha = 3.24\beta = 0.01$
(1, 2, 2/3, 0.25)	with $\theta = 27.24\alpha = 4.83\beta = 0.00$

Thus for given $i = 1, k = 2, f = 2/3$ and $m = 1$ the minimum angle plan is (1,2, 2/3, 0.10)

CONSTRUCTION OF TABLES

The probability of acceptance for SkSP-V with reference plan is

$$P_a(p) = \frac{fP + (1-f)P^i + fP^{k+1}(P^i - P^k)}{f(1 + P^{i+k} - P^{2k}) + (1-f)P^i} \quad (1)$$

Table 1 - Minimum Angle SkSP – V with MDS(0,1) plans for given OR and np₁ for i= 1, k=2,

OR	f	2/3		1/2		1/3		1/4					
	np ₁	ntanθ	α	β	ntanθ	α	β	ntanθ	α	β	ntanθ	α	β
60	0.07	4.2500	0.45	2.37	4.2788	0.34	3.14	4.3410	0.23	4.63	4.4056	0.17	6.09
	0.08	4.8095	0.58	1.28	4.8230	0.44	1.70	4.8568	0.29	2.52	4.8941	0.22	3.34
	0.09	5.3867	0.73	0.69	5.3893	0.55	0.92	5.4042	0.37	1.38	5.4240	0.28	1.83
	0.1	5.9759	0.89	0.38	5.9701	0.67	0.50	5.9718	0.45	0.75	5.9800	0.34	1.00
	0.15	9.0242	1.91	0.02	8.9822	1.45	0.02	8.9403	0.97	0.04	8.9198	0.73	0.05
	0.2	12.1950	3.24	0.00	12.0983	2.46	0.00	12.0003	1.67	0.00	11.9509	1.26	0.00
	0.25	15.4985	4.83	0.00	15.3165	3.70	0.00	15.1311	2.52	0.00	15.0372	1.91	0.00
	0.08	4.4313	0.58	1.93	4.4534	0.44	2.56	4.5037	0.29	3.78	4.5571	0.22	4.98
55	0.09	4.9505	0.73	1.10	4.9595	0.55	1.46	4.9865	0.37	2.17	5.0179	0.28	2.87
	0.1	5.4833	0.89	0.63	5.4825	0.67	0.83	5.4930	0.45	1.24	5.5096	0.34	1.65
	0.15	8.2612	1.91	0.04	8.2233	1.45	0.05	8.1861	0.97	0.08	8.1684	0.73	0.10
	0.2	11.1617	3.24	0.00	11.0733	2.46	0.00	10.9837	1.67	0.01	10.9386	1.26	0.01
50	0.25	14.1851	4.83	0.00	14.0185	3.70	0.00	13.8488	2.52	0.00	13.7629	1.91	0.00
	0.1	4.9967	0.89	1.04	5.0029	0.67	1.38	5.0262	0.45	2.06	5.0551	0.34	2.73
	0.15	7.4996	1.91	0.08	7.4663	1.45	0.11	7.4347	0.97	0.17	7.4209	0.73	0.22
	0.2	10.1287	3.24	0.01	10.0486	2.46	0.01	9.9676	1.67	0.01	9.9269	1.26	0.02
45	0.25	12.8717	4.83	0.00	12.7205	3.70	0.00	12.5666	2.52	0.00	12.4887	1.91	0.00
	0.15	6.7408	1.91	0.18	6.7129	1.45	0.24	6.6887	0.97	0.35	6.6804	0.73	0.47
	0.2	9.0963	3.24	0.02	9.0246	2.46	0.02	8.9526	1.67	0.04	8.9168	1.26	0.05
	0.25	11.5584	4.83	0.00	11.4227	3.70	0.00	11.2847	2.52	0.00	11.2147	1.91	0.01
40	0.15	5.9870	1.91	0.38	5.9662	1.45	0.50	5.9526	0.97	0.75	5.9531	0.73	1.00
	0.2	8.0652	3.24	0.05	8.0026	2.46	0.07	7.9404	1.67	0.10	7.9103	1.26	0.13
	0.25	10.2455	4.83	0.01	10.1254	3.70	0.01	10.0033	2.52	0.01	9.9416	1.91	0.02
35	0.2	7.0376	3.24	0.14	6.9850	2.46	0.18	6.9347	1.67	0.27	6.9124	1.26	0.37
	0.25	8.9335	4.83	0.02	8.8293	3.70	0.03	8.7239	2.52	0.05	8.6711	1.91	0.06

When P is MDS (0,1) reference plan and its OC function as:

$$P_a(p) = P_{a,r}(p) + [P_{a,r+b}(p) - P_{a,r}(p)] [P_a(p)]^m \quad (2)$$

When np₁ and p₂/p₁ are known np₂ can be calculated from np₂ = np₁ (p₂/p₁).

The following search procedure is used to obtained the parametric value, fixing α = 0.05 and β = 0.10

- Set m=1
- Compute α and β using equation 1 and 2 for given i, k, f, np₁ and OR
- If P_a(p₁) ≥ 1- α and P_a(p₂) ≤ β go to step (6)
- Find n tan θ using np₁, α and β and computed np₂ = OR x np₁
- Record minimum of n tan θ
- Increase m by 1 go to step (2)
- If the current value of m > 1 step the process otherwise repeat steps 2 to 7
- Record the m value for which n tan θ is minimum.

The above search procedure is used to obtain the optimum value for 'm' which minimize the tangent angle for certain specific values of np₁ and np₂ by keeping the producer's risk below 5% and consumer's risk below 10%.

Conclusion

Acceptance sampling is the techniques which deals with the procedure in which a decision either to accept or reject lots or processes which are based on the examination of samples. In acceptance sampling the producer and consumer plays a dominant role and hence one allows certain level of risk for both producer and consumer, namely α=0.05, β=0.10. In practice it is desirable to design any such sampling plan with the associated quality levels, which has concern to both producer and consumer. The result presented in this paper are mainly related with new procedure for designing sampling plan and necessary tables for selection of sampling system through minimum angle method involving producer and consumer quality levels.

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