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RESEARCH ARTICLE

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A FORMULA FOR THE PRIME COUNTING FUNCTION π(n).

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ABSTRACT

We have created a formula to calculate the number of primes less than or equal to any given positive integer ' n ' . It is denoted by π (n) . This is a fundamental concept in number theory and it is difficult to calculate. A prime number can be divided by 1 and the number itself. The set of all primes can be written as { 2,3,5,7,11,13,17,...............}. The Prime Counting Function was conjectured in the end of 18th century by the famous Mathematician Gauss and Legendre Sir, to be approximately x/(ln x) .But in this paper we are presenting the real formula, by applying the modern approach, that is by applying the basic concept of set theory.

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INTRODUCTION

The main problem in number theory in Mathematics is to understand the distribution of prime numbers. Let π(n) , denote the Prime Counting Function, defined as the number of primes less than or equal to positive integer ' n '. Many Mathematician worked hard including famous Indian Mathematician, RamanujanSir, and G.H. Hardy Sir tried to create the formula for the Prime Counting Function pi(n). A good numbers of deep problem in analytical number theory can be expressed in terms of the Prime Counting Function π(n). For example, the Riemann hypothesis, so Gauss and Legendre Sir's approximation solution x/ln(x) , in the sense that the statement is the prime number theorem. So till now, there is no formula for the Prime Counting Function π(n) ,as we have seen from the end of 18th century to till now. In this paper, we are presenting the real formula and it's proof (examine) by taking examples, we have to find that the formula which I have invented is absolutely correct.

Our perspective:

If we observe, Figures in number theory in Mathematics then we observe those figures often times.

The set consists of all prime numbers { 2,3,5,,7,11,13,17,19,23,29,31,.............} , we observed that there is no distinct common gaps between two serial prime numbers, that is we can not find out any common interval to the primes. How can we formulate the Prime Counting Function π(n), we were so worked hard and hard to formulate it, as it is originally a basic concept of number theory (Arithmetic).

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We have done the formula to the Prime Counting Function π(n) , so that we can give lecture and demonstration to our students in a very understanding and simple way to "the Prime Counting Function π(n).

Creations

In number theory, here we introduce one new formula to calculate the number of primes less than or equal to any given positive integer 'n' , by applying a basic concept of set theory to that number theory. We know that there is no such prime, less than or equal to the positive integer 1, as the smallest prime is 2. So by keeping it in our mind, let's start ,

Let π(n) = number of primes less than or equal to the positive integer 'n'.

Therefore, π(1)=0

Now, we can introduce the formula for π(n) ,as below.

$\pi(n) = 1 + n\ [\ Z_{odd} \setminus (AUBUCUUDU.................)]$

Where, Z_{odd} = the set consists of all the positive odd integers less than or equal to n ,which are greater than 2.

A = the set consists of all positive multiples of the prime 3, which are greater than 3 ,and less than or equal to the positive integer n.
B = the set consists of all the positive multiples of the prime 5, which are greater than 5,and less than or equal to the positive integer n.
C = the set consists of all the positive multiples of the prime 7 ,which are greater than 7,and less than or equal to the positive integer n.
D = the set consists of all the positive multiples of the prime 11,which are greater than 11, and less than or equal to the positive integer n.
.........And so on.

Now , for n =2 ; π(2) = 1 + n[{ } \ { }] ,as there is no odd positive integer less than or equal to 2.

That is , π(2) = 1+ 0 = 1

And , for n=3 ; π(3) = 1 + n[Z \ A] , here Z =:{3} and A = { }.

= 1 + n[{3} \ { }]
= 1+1 = 2.

Which is correct, as the number of primes less than or equal to 3 , are 2 & 3. That is the number of primes 2.

Now, for n=15 , π (15)=?

Here, Z_{odd} = the set consists of all positive odd integers , less than or equal to 15,and which are greater than 2.

= { 3,5,7,9,11,13,15}

A= the set consists of all the positive multiples of the prime 3,which are greater than 3,and less than or equal to 15.

={6,9,12,15}

B= the set consists of all the positive multiples of the prime 5, which are greater than 5, and less than or equal to 15.

={19,15}

C = the set consists of all the positive multiples of the prime 7,which are greater than 7, and less than or equal to 15.

={14}

Thus , AUBUC ={6,9,10,12,14,15}

Z_{odd} \(AUBUC)={3,5,7,9,11,13,15} \{6,9,10,12,14,15}
={3,5,7,11,13} ; so that n [Z_{odd} \ (AUBUC)] =n {3,5,7,11,13} = 5 .
Thus , π(15)= 1+ n [Z_{odd}\》(AUBUC)]
= 1 + 5 = 6 ,

Which is correct, as the prime numbers less than or equal to 15 are 2,3,5,7,11 &13. That is 6.

Now, for n = 100 ; π(100) = ?

Let,

Z_{odd} = {3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39,41,43,45,47,49,51,53,55,57,59,61,63,65, 67,69,71,73,75,77,79,81,83,85,87,89,91,93,95,97,99}

A_1= the set consists of all the positive multiples of the prime 3, say xi, 3<xi <100.

={6,9,12,15,18,21,24,27,30,33,36,39,42,45,48,51,54,57,60,63,66,69,72,75,78,81,84,87,90,93,96,99}

A_2 = the set consists of all the positive multiples of the prime 5,say xi; xi's are greater than 5 and less than or equal to 100.
={10,15,20,25,30,35,40,45,50,55,60,65,70,75,80,85,90,95,100}

A_3 = the set consists of all the positive multiples of the prime 7,say xi; 7<xi <100.

= {14,21,28,35,42,49,56,63,70,77,84,91,98}

A_4 = the set consists of all the positive multiples of the prime 11,say xi; 11<xi <100.

={22,33,44,55,66,77,88,99}

A_5 = the set consists of all the positive multiples of the prime 13,say xi; 13<xi <100.

={26,39,52,65,78,91}

A_6 = the set consists of all the positive multiples of the prime 17,say xi; 17<xi <100.

={34,51,68,85}

A_7 = the set consists of all the positive multiples of the prime 19,say xi; 19<xi <100.

={38,57,76,95}

A_8 = the set consists of all the positive multiples of the prime 23,say xi;23<xi <100.

={46,69,92}

A_9 = the set consists of all the positive multiples of the prime 29 ,say xi; 29<xi <100.

={58,87}

A_{10} = the set consists of all the positive multiples of the prime 31,say xi;31<xi <100.

={62,93}

A_{11} = the set consists of all the positive multiples of the prime 37,say xi; 37<xi <100.

={74}

A_{12} = the set consists of all the positive multiples of the prime 41,say xi; 41<xi <100.

={82}

A_{13} = the set consists of all the positive multiples of the prime 43,say xi; 43<xi <100.

={86}

A_{14} = the set consists of all the positive multiples of the prime 47,say xi; 47<xi <100.

={94}

$Z_{odd} \cap ((A_1 UA_2 UA_3 UA_4 U.............................UA_{14})$.

={9,15,21,25,27,33,35,39,45,49,51,55,57,63,65,69,75,77,81,85,87,91,93,95,99}

Thus , $n[Z_{odd} \backslash (A_1 UA_2 UA_3 UA_4 U.............................UA_{14})$.

$= n\{ Z_{odd} \} \backslash n \{ Z_{odd} \cap (A_1 UA_2 UA_3 UA_4 U.............................UA_{14})$.

= 49 – 25 =24

Hence π (100) = 1 + $n[Z_{odd} \backslash (A_1 UA_2 UA_3 UA_4 U.............................UA_{14})$.

= 1 + 24 =25 .

Which is correct ,actual Counting we have the number of primes less than or equal to the positive integer 100 is 25.

Let us assume that n= 1000 .

We have to find out π (1000) .
Let the set Z_{odd} = the set of all positive odd integers greater than 2 , which are less than or equal to 1000.

={3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39,41,43,45,,47,49,51,53,55,57,59,61,63,65,67,69, 71,73,75,77,79,81,83,85,87,89,91,93,95,97,99,101,103,105,107,109,111,113,115,117,119,121,123,125, 127,129,131,133,135,137,139,141,143,145,147,149,151,153,155,157,159,161,163,165,167,169,171,173,175,177,179,181,183,185,187,189,191,193,195,197,199,201,203,205,207,209,211,213,215,217,219,221,223,225,227,229,231,233,235,237,239,241,243,245,247,249,O251,253,255,257,259,261,263,265,267, 269,271,273,275,277,279,281,283,285,287,289,291,293,295,297,299,301,303,305,307,309,311,313,315,317,319,321,323,325,327,329,331,333,335,337,339,341,343,345,347,349,351,353,355,357,359,361,363,365,367,379,371,373,375,377,379,381,383,385,387,389,391,393,395,397,399,401,403,405,407,409,411,413,415,417,419,421,423,425,427,429,431,433,435,437,439,441,443,445,447,449,451,453,455, 457,459,461,463,465,467,469,471,473,475,477,479,481,483,485,487,489,491,493,495,497,499,501,503,505,507,509,511,513,515,517,519,521,523,525,527,529,531,533,535,537,539,541,543,545,547,549,551,553,555,557,559,561,563,565,567,569,571,573,575,577,579,581,583,585,587,589,591,593,595,597,599,601,603,605,607,609,611,613,615,617,619,621,623,625,627,629,631,633,635,637,639,641,643, 645,647,649,651,653,655,657,659,661,663,665,667,669,671,673,675,677,679,681,683,685,687,689,691,693,695,697,699,701,703,705,707,709,711,713,715,717,719,721,723,725,727,729,731,733,735,737,739,741,743,745,747,749,751,753,755,757,759,761,763,765,767,769,771,773,775,777,779,781,783,785,787,789,791,793,795,797,799,801,803,805,807,809,811,813,815,817,819,821,823,825,827,829,831, 833,835,837,839,841,843,845,847,849,851,853,855,857,859,861,863,865,867,869,871,873,875,877,879,881,883,885,887,889,891,893,895,897,899,901,903,905,907,909,911,913,915,917,919,921,923,925,927,929,931,933,935,937,939,941,943,945,947,949,951,953,955,957,959,961,963,965,967,969,971,973,975,977,979,981,983,985,987,989,991,993,995,997,999}

Also , A_1 = the set consists of all the positive multiples of the prime 3 ,which are greater than 3 and less than or equal to 1000.

={ 6,9,12,15,18,21,24,27,30,33,36,39,42,45,48,51,54,57,60,63,66,69,72,75,78,81,84,87,90,93,96,99,102 ,105,108,111,114,117,120,123,126,123,129,132,135,138,141,144,147,150,153,156,159,162,165,168,171,174,177,180,183,186,189,192,195,198,201,204,207,210,213,216,219,222,225,228,231,234,237,240,243,246,249,252,255,258,261,264,267,270,273,276,279,282,285,288,291,294,297,300,303,306,309,312,315,318,321,324,327,330,333,336,339,342,345,348,351,354,357,360,363,366,369,372,375,378,381, 384,387,390,393,396,399,402,405,408,411,41,417,420,423,426,429,432,435,438,441,444,447,450,453,456,459,462,465,468,471,474,477,480,483,486,489,492,495,498,501,504,507,510,513,516,519,522,525,528,531,534,537,540,543,546,549,552,555,558,561,564,567,570, 573,576,579,582,585,588,591,594,597,600,603,606,609,612,615,618,621,624,627,630,633,636,639,642,645,648,651,654,657,660,663,666,669,672,675,678,681,684,687,690,693,696,699,702,705,708,711,714,717,720,723,726,729,732,735,738,741,744,747,750,753,756,759,762,765,768,771,774,777,780,783,786,789,792,795,798,801,804,807,810,813,816,819,822,825,828,831,834,837,840,843,846,849,852, 855,858,861,864,867,870,873,876,879,882,885,888,891,894,897,900,903,906,909,912,915,918,921,924,927,930,933,936,939,942,945,948,951,954,957,960,963,966,969,972,975,978,981,984,987,990,993,996,999}

A_2 = the set consists of all the positive multiples of the prime 5 , which are greater than 5 ,and less than or equal to 1000.

={10,15,20,25,30,35,40,45,50,55,60,65,70,75,80,85,90,95,100,105,110,115,120,125,130,135,140,145, 150,155,160,165,170,175,180,185,190,195,200,205,210,215,220,225,230,235,240,245,250,255,260,265,270,275,280,285,290,295,300,305,310,315,320,325,330,335,340,345,350,355,360,365,370,375,380,385,390,395,400,405,410,415,420,425,430,435,440,445,450,455,460,465,470,475,480,485,490,495,500,505,,510,515,520,525,530,535,540,545,550,555,560,565,570,575,580,585,590,595,600,605,610,615 ,620,625,630,635,640,645,650,655,660,665,670,675,680,685,690,695,700,705,710,715,720,725,730,735,740,745,750,755,760,765,770,775,780,785,790,795,800,805,810,815,820,825,830,835,840,845,850,855,860,865,870,875,880,885,890,895,900,905,910,915,920,925,930,935,940,945,950,955,960,965,970,975,980,985,990,995,1000}.

A_3 = the set consists of all the positive multiples of the prime 7 , say xi,where $7 < xi < 1000$.

={14,21,28,35,42,49,56,63,70,77,84,91,98,105,112,119,126,133,140,147,154,161,168,175,182,189,196, ,203,210,217,224,231,238,245,252,259,266,273,280,287,294,301,308,315,322,329,336,343,350,357,364,371,378,385,392,399,406,413,420,427,434,441,448,455,462,469,476,483,490,497,504,511,518,525,532,539,546,553,560,567,574,581,588,595,602,609,616,623,630,637,644,651,658,665,672,679,686,693,700,707,714,721,728,735,742,749,756,763,770,777,784,791,798,805,812,819,826,833,840,847,854, 861,868,875,882,889,896,903,910,917,924,931,938,945,952,959,966,973,980,987,994}

A_4 = the set consists of all the positive multiples of the prime 11 ,say xi ,where $11 < xi < 1000$.

={22,33,44,55,66,77,88,99,110,121,132,143,154,165,176,187,198,209,220,231,242,253,264,275,286,297,308,319,330,341,352,363,374, 385396,407,418,429,440,451,462,473,484,495,506,517,528,539,550,561,572,583,594,605,616,627,638,649,660,671,682,693,704,715,726,737,748,759,770,781,792,803,814,825,836,847,858,869,880,891,902,913,924,935,946,957,968,979,990}

A_5 = the set consists of all the positive multiples of the prime 13 , say xi, where $13 <xi <1000$.
={26,39,52,65,78,91,104,117,130,143,156,169,182,195,208,221,234,247,260,273,286,299,312,325,338, 351,364,377,390,403,416,429,442,455,468,481,494,507,520,533,546,559,572,585,598,611,624,637,650,663,676,689,702,715,728,741,754,767,780,793,806,819,832,845,858,871,884,897,910,923,936,949,962,975,988}
A_6 = the set consists of all the positive multiples of the prime 17, say xi, where $17 < xi < 1000$.
={34,51,68,85,102,119,136,153,170,187,204,221,238,255,272,289,306,323,340,357,374,391,408,425, 442,459,476,493,510,527,544,571,588,605,622,639,656,673,690,707,724,741,758,775,792,809,826,843,860,877,894,911,928,945,962,979,996}

A_7 = the set consists of all the positive multiples of the prime 19, say xi, where 19< xi <1000.
={38,57,76,95,114,133,152,171,190,209,228,247,266,285,304,323,342,361,380,399,418,437,456,475,
494,513,532,551,570,589,608,627,646,665,684,703,722,741,760,779,798,817,836,855,874,893,912,931,950,969,988}
A_8 = the set consists of positive multiples of the prime 23, say xi, where 23 < xi < 1000.
={46,69,92,115,138,161,184,207,230,253,276,299,322,345,368,391,414,437,460,483,506,529,552,575,
598,621,644,667,690,713,736,759,782,805,828,851,874,897,920,943,966,989}
A_9 =the set consists of all the positive multiples of the prime 29 ,say xi, where 29 < xi < 1000.
={58,87,116,145,174,203,232,261,290,319,348,377,406,435,464,493,522,551,580,609,638,667,696,725,754,783,812,841,870,899,928,9
57,986}

A_{10} =the set consists of all the positive multiples of the prime 31, say xi ;31<xi <1000.
={62,93,124,155,186,217,248,279,310,341,372,403,434,465,496,527,558,589,620,651,682,713,744,7775,806,837,868,899,930,961,992}
A_{11} = the set consists of all the positive multiples of the prime 37 ,say xi; 37<xi <1000.
={74,111,148,185,222,258,296,333,370,407,444,481,518,555,592,629,666,708,740,777,814,851,888,925,963,889}
A_{12}= the set consists of all the positive multiples of the prime 41, say xi ; 41<xi <1000.
={82,123,164,205,246,287,328,369,410,451,498,533,574,615,656,697,738,779,820,861,902,943,984}

A_{13}=the set consists of all the positive multiples of the prime 43 , say xi; 43<xi <1000.
={86,129,172,215,258,301,344,387,430,473,516,559,602,645,688,731,774,817,860,903,946,989}
A_{14}= the set of all the positive multiples of the prime47 , say xi; 47<xi <1000.
={94,141,188,235,282,329,376,423,470,517,564,611,658,705,752,799,846,893,940,987}
A_{15}=the set consists of all the positive multiples of the prime 53, say xi; 53<xi <1000.
={ 106,159,212,265,318,371,424,477,530,583,636,689,742,795,848,901,954}
A_{16}= the set consists of all the positive multiples of the prime 59 , say xi; 59<xi <1000.
= { 118,177,236,295,354,413,472,531,590,649,708,767,826,885,944}
A_{17} =the set of all the positive multiples of the prime 61 ,say xi; 61<xi <1000.
= {122,183,244,305,366,427,488,549,610,671,732,793,854,815,976}
A_{18} = the set of all the positive multiples of the prime 67 , say xi; 67<xi <1000.
={134,201,268,335,402,469,536,603,670,737,804,871,938}
A_{19} = the set consists of all the positive multiples of71 ,say xi ; 71<xi <1000.
= {142,213,284,355,426,497,568,639,710,781,852,923,994}
A_{20} = the set consists of all the positive multiples of the prime 73 , say xi; 73<xi <1000.
={ 146,219,292,365,438,511,584,657,730,803,876,949}
A_{21} = the set consists of all the positive multiples of the prime 79 ,say xi; 79<xi <1000.
={ 158,237,316,395,474,553,632,711,790,869,948}
A_{22} = the set consists of all the positive multiples of the prime 83 , say xi; 83<xi <1000.
= { 166,249,332,415,498,581,664,747,830,913,996}
A_{23} = the set consists of all the positive multiples of the prime 89 ,say xi; 89<xi <1000.
={ 178,267,356,445,534,623,712,801,890,979}
A_{24} = the set consists of all the positive multiples of the prime 97, say xi; 97<xi <1000.
= { 194,291,388,485,582,679,776,873,970}
A_{25} = the set consists of all the positive multiples of the prime 101, say xi; 101<xi <1000.
={ 202,303,404,505,606,707,808,909}
A_{26} = the set consists of all the positive multiples of the prime 103 ,say xi; 103<xi <1000.
={206,309,412,515,618,721,824,927}
A_{27} = the set consists of all the positive multiples of the prime 107 , say xi; 107<xi <1000.
={ 214,321,428,535,642,749,856,963}
A_{28} = the set consists of all the positive multiples of the prime 109, say xi; 109<xi <1000.
= { 218,327,436,545,654,763,872,981}
A_{29} = the set consists of all the positive multiples of the prime 113 , say xi; 113<xi <1000.
={ 226,339,452,565,678 ,791,904}
A_{30} = the set consists of all the positive multiples of the prime 127, say xi; 127<xi <1000.
= { 254,381,508,635,762,889}
A_{31} = the set consists of all the positive multiples of the prime 131 , say xi ; 131<xi 1000.
= { 262,393,524,655,786 ,917}
A_{32} = the set consists of all the positive multiples of the prime 137 ,say xi; 137<xi <1000.
= { 274,411,548,685,822,959}
A_{33} = the set consists of all the positive multiples of the prime 139 , say xi ; 139<xi <1000.
= { 278,417,556,695,834,973}
A_{34} = the set consists of all the positive multiples of the prime 149 , say xi ; 149<xi <1000.
= { 298,447,596,745,894}
A_{35} = the set consists of all the positive multiples of the prime 151 ,say xi; 151<xi <1000.
={ 302,453,604,755,906}
A_{36} = the set consists of all the positive multiples of the prime 157 ,say xi; 157<xi <1000.
={ 314,471,628,785,942}
A_{37} = the set consists of all the positive multiples of the prime 163 , say xi; 163<xi <1000.
= { 326,489,652,815,978}
A_{38} = the set consists of all the positive multiples of the prime 167 , say xi ; 167<xi <1000.

= { 334,501,668,835}
A_{39} = th set consists of all the positive multiples of the prime 173 , say xi; 173<xi <1000.
= { 346,519,692,865}
A_{40} = the set consists of all the positive multiples of the prime 179 , say xi; 179<xi <1000.
= { 358,537,716,895}
A_{41} = the set consists of all the positive multiples of the prime 181 ,say xi ; 181<xi <1000.
= { 362,543,724,905}
A_{42} = the set consists of all the positive multiples of the prime 191 , say xi; 191<xi <1000.
= { 382,573,764,955}
A_{43} = the set consists of all the positive multiples of the prime 193 ,say xi; 193<xi <1000.
= {386,579,772,965}
A_{44} = the set consists of all the positive multiples of the prime 197 ,say xi ; 197<xi <1000.
= { 394,591,788,985}
A_{45} = the set consists of all the positive multiples of the prime 199 ,say xi ; 199<xi <1000.
= { 398,597,796,995}
A_{46} = the set consists of all the positive multiples of the prime 211 , say xi ; 211<xi <1000.
= { 422,633,844}
A_{47} = the set consists of all the positive multiples of the prime 223 , say xi, 223<xi <1000.
={ 446,669,892}
A_{48} =the set consists of all the positive multiples of the prime 227 ,say xi, 227<xi <1000.
= {454,681,908}
A_{49} = the set consists of all the positive multiples of the prime 229 ,say xi; 229<xi <1000.
={ 458,687,916}
A_{50} = the set consists of all the positive multiples of the prime 233 , say xi; 233<xi <1000.
= { 466 ,699,932}
A_{51} = the set consists of all the positive multiples of the prime 239 , say xi ; 239<xi <1000.
= {478,717,956}
A_{52} = the set consists of all the positive multiples of the prime 241, say xi; 241<xi <1000.
= {482,723,964}
A_{53} = the set consists of all the positive multiples of the prime 251 ,say xi; 251<xi <1000.
= {502,753}
A_{54} = the set consists of all the positive multiples of the prime 257, say xi; 257<xi <1000.
= {514,771}

A_{55} = the set consists of all the positive multiples of the prime 263 , say xi; 263<xi <1000.
= {526,789}
A_{56} = the set consists of all the positive multiples of the prime 269 , say xi; 269<xi <1000.
= {538,807}
A_{57} = the set consists of all the positive multiples of the prime 271 , say xi; 271<xi <1000.
= { 542,813}
A_{58} = the set consists of all the positive multiples of the prime 277 ,say xi; 277<xi <1000.
= {554,831}
A_{59} =the set consists of all the positive multiples of the prime 281 , say xi;281<xi <1000.
= { 562,843}
A_{60} = the set consists of all the positive multiples of the prime 283 ,say xi; 283<xi <1000.
={ 566,849}
A_{61} = the set consists of all the positive multiples of the prime 293 , say xi; 293<xi <1000.
= {586,879}
A_{62} = the set consists of all the positive multiples of the prime 307 , say xi; 307<xi <1000.
= {614,921}
A_{63} = the set consists of all the positive multiples of the prime 311, say xi; 311<xi <1000.
={622,933}
A_{64} = the set consists of all the positive multiples of the prime 313 ,say xi; 313<xi <1000.
={626,939}
A_{65} = the set consists of all the positive multiples of the prime 317, say xi ; 317<xi <1000.
={634,951}
A_{66} = the set consists of all the positive multiples of the prime 331, say xi ; 331<xi <1000.
={662,993}
A_{67} = the set consists of all the positive multiples of the prime 337 ,say xi ; 337<xi <1000.
={674}
A_{68} = the set consists of all the positive multiples of the prime 347 , say xi ; 347<xi <1000.
={694}

A_{69} = the set consists of all the positive multiples of the prime 349 ,say xi ; 349<xi <1000.
={698}
A_{70} = the set consists of all the positive multiples of the prime 353 ,say xi ; 353<xi <1000.
={706}
A_{71} = the set consists of all the positive multiples of the prime 359 , say xi; 359<xi <1000.

={718}.
A_{72} = the set consists of all the positive multiples of the prime 367 , say xi ; 367<xi <1000.
= {734}
A_{73} = the set consists of all the positive multiples of the prime 373 , say xi ; 373<xi <1000.
={746}
A_{74} = the set consists of all the positive multiples of the prime 379, say xi ; 379<xi <1000.
={758}
A_{75} = the set consists of all the positive multiples of the prime 383 ,say xi; 383<xi <1000.
={766}
A_{76} = the set consists of all the positive multiples of the prime 389 , say xi;383<xi <1000.
={778}
A_{77} = the set consists of all the positive multiples of the prime 397 ,say xi ;397<xi <1000.
={794}
A_{78} = the set consists of all the positive multiples of the prime 401,say xi; 401<xi <1000.
={802}
A_{79} = the set consists of all the positive multiples of the prime 409, say xi; 409<xi <1000.
={818}
A_{80} = the set consists of all the positive multiples of the prime 419 ,say xi; 419<xi <1000.
= {838}
A_{81} = the set consists of all the positive multiples of the prime 421, say xi; 421<xi <1000.
={842}
A_{82} = the set consists of all the positive multiples of the prime 431, say xi;431<xi <1000.
={862}

A_{83} = the set consists of all the positive multiples of the prime 433, say xi;433<xi <1000.
={866}
A_{84} = the set consists of all the positive multiples of the prime 439, say xi;439<xi <1000.
={878}
A_{85} = the set consists of all the positive multiples of the prime 443, say xi; 443<xi <1000.
={886}
A_{86} = the set consists of all the positive multiples of the prime 449,say xi;449<xi <1000.
={898}
A_{87} = the set consists of all the positive multiples of the prime 457,say xi;457<xi <1000.
={914}
A_{88} = the set consists of all the positive multiples of the prime 461, say xi; 461<xi <1000.
={922}
A_{89} = the set consists of all the positive multiples of the prime 463, say xi;463<xi <1000.
={926}
A_{90} = the set consists of all the positive multiples of the prime 467, say xi; 467<xi <1000.
={934}
A_{91} = the set consists of all the positive multiples of the prime 479, say xi; 479<xi <1000.
={958}
A_{92} = the set consists of all the positive multiples of the prime 487, say xi; 487<xi <1000.
={974}
A_{93} = the set consists of all the positive multiples of the prime 491, say xi; 491<xi <1000.
={982}
A_{94} = the set consists of all the positive multiples of the prime 499, say xi; 499<xi <1000.
={998}.

Therefore,

$\{Z_{odd} \cap (A_1 UA_2 UA_3 U..........................UA_{94})\}$

9	15	21	25	27	33	35	39	45	49	51	55	57	63	65	69	75	77	81	85
87	91	93	95	99	105	111	115	117	119	121	123	125	129	133	135	141	143	145	147
153	155	159	161	165	169	171	175	177	183	185	187	189	195	201	203	205	207	209	213
215	217	219	221	225	231	235	237	243	245	247	249	253	255	259	261	265	267	273	275
279	285	287	289	291	295	297	299	301	303	305	309	315	319	321	323	325	327	329	333
335	339	341	343	345	351	355	357	361	363	365	369	371	375	377	381	385	387	391	393
395	399	403	405	407	411	413	415	417	423	425	427	429	435	437	441	445	447	451	453
455	459	465	469	471	473	475	477	481	483	485	489	493	495	497	501	505	507	511	513
515	517	519	525	527	529	531	533	535	537	539	543	545	549	551	553	555	559	561	565
567	573	575	579	581	583	585	589	591	595	597	603	605	609	611	615	621	623	625	627
629	633	635	637	639	645	649	651	655	657	663	665	667	669	671	675	679	681	685	687
689	693	695	697	699	703	705	707	711	713	715	717	721	723	725	729	731	735	737	741
745	747	749	753	755	759	763	765	767	771	775	777	779	781	783	785	789	791	793	795
799	801	803	805	807	813	815	817	819	825	831	833	835	837	841	843	845	847	849	851
855	861	865	867	869	871	873	875	879	885	889	891	893	895	897	899	901	903	905	909
913	915	917	921	923	925	927	931	933	935	939	943	945	949	951	955	957	959	961	963
965	969	973	975	979	981	985	987	989	993	995	999								

= Total Number 332.

Thus, $n[\, Z_{odd} \setminus (A_1 UA_2 UA_3 U\ldots\ldots UA_{94})\}$

$=n\{Z_{odd} \setminus n\{ Z_{odd} \cap (A_1 UA_2 UA_3 U\ldots\ldots UA_{94})\}$

$= 449 - 332$

$=167$

Therefore, $\pi\,(1000)=1+n\{Z_{odd} - (A_1 UA_2 UA_3 U\ldots\ldots UA_{94})\}$

$= 1+ 167$

$= 168$

CONCLUSION

The Prime Counting Function pi (n) has many applications in number theory and it's related to one of the famous problem in Mathematics, for example the Riemann Hypothesis because the Prime Counting Function is related to Riemann Function and it has many thousands of applications accross Science and Mathematics.

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