Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 6, Issue, 05, pp.6810-6813, May, 2014

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

ON DIOPHANTINE QUADRUPLE WITH PROPERTY D (P^2) WHERE P IS PRIME AND $P^2 \equiv 1(\bmod 6)$

Gopalan, M. A., *Mallika, S. and Vidhyalakshmi, S.

Department of Mathematics, Shrimathi Indira Gandhi College, Trichy, 620002, Tamilnadu, India

ARTICLE INFO

Article History:
Received 15th February, 2014
Received in revised form
19th March, 2014
Accepted 20th April, 2014
Published online 31st May, 2014

Key words:

Diophantine quadruple,
System of equations.

ABSTRACT

We exhibit a method of constructing Diophantine quadruples with property $D(P^2)$,where P is a prime number and P^2 is of the form (6m+1).Some relations between the members in the quadruple and special numbers are given.

INTRODUCTION

The problem of constructing the sets with property that the product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus. A set of m distinct non-zero integers $\{a_1, a_2, ..., a_m\}$ is called a Diophantine m-tuple with property D(n) if $a_i a_j + 1$ is a perfect square for all $1 \le i < j \le m$. Many mathematicians analyzed the construction of different formulations of Diophantine triple and Diophantine quadruples with property D(n) for any arbitrary integer n and also for polynomials in n. There are many formulas with elements represented in Fibonacci numbers. In this context one may refer Dickson (1966), Brown (1985), Morgado (1983-1984) & (1991), Shrividhya (2009) and Gopalan (2012) .It was proved in Dujella (1993) that for any integer n, the Diophantine 2-tuple $(a,b),\{ab \ne square\}$ with property $D(n^2)$ can be extended to Diophantine 4-tuple with the property $D(n^2)$. In particular, the sets {1,33,105,320,18240} and {5,21,64,285,6720} have the property D(256), Dujella (1997). Euler proved that arbitrary rational Diophantine 2-tuples can be extended to a rational Diophantine 5-tuple. For the generalizatio n of Euler's construction to rational Diophantine triples and a review of various articles on quadruples and sextuples one may refer Arkin (1977), Dujella (1998), Mootha (1995), Filipin (2008), Fujita (2008) & (2011), Cerin (2011), Zhang (2011), Bacic (2013) and meena *et al.* (2014).

As we are searching for methods to construct Diophantine quadruples with the property $D(P^2)$ where P is prime, we come across a paper by Dujella [1997] in which he has presented a theorem for constructing Diophantine quintuples with the property $D(q^2)$ where $q \in Q$ .A similar construction holds good when the prime P takes values 2 and 3.In Andrej Dujella (2005), the authors have considered Diophantine m-tuples for primes and in particular, they have obtained an absolute upper bound on the size m of a Diophantine m-tuple with the property $D(\pm P)$ for all primes P. Also, it is noted that the majority of the primes except 2 and 3 have the form $P^2 = 6m+1$.Thus, towards this end, we, in this paper, illustrate a method to construct Diophantine quadruples with property $D(P^2)$ where P is prime and $P^2 \equiv 1(\bmod 6)$.

***Corresponding author: Mallika, S.** *Department of Mathematics, Shrimathi Indira Gandhi College, Trichy, 620002*

*The financial support from the UGC,New Delhi (F.MRP-5122/14(SERO/UGC) dated March 2014) for a part of this work is gratefully acknowledged.

Notations

$$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right) .$$

$$P_n^m = \left(\frac{n(n+1)}{6}\right)[(m-2)n + (5-m)]$$

$$Pt_n = \frac{n(n+1)(n+2)(n+3)}{24}$$

$$SO_n = n(2n^2 - 1)$$

$$S_n = 6n(n-1) + 1$$

$$\mathrm{Pr}_n = n(n+1)$$

Method of Analysis

To start with, it is seen that the pair (a,b) where $a = m, b = 16m + 2$ is a Diophantine 2-tuple with property $D(P^2)$ where $P^2 = 6m + 1$

Let c be any non-zero integer such that

$$mc + P^2 = \sim_1^2 \qquad (1)$$

$$(16m+2)c + P^2 = \ {}_1^2 \qquad (2)$$

Eliminating c between (1) and (2), we get,

$$(16m+2)\sim_1^2 - m\ {}_1^2 = P^2[15m+2] \qquad (3)$$

The substitution of the linear transformations

$$\sim_1 = X_n + mT_n \qquad (4)$$

$${}_1 = X_n + (16m+2)T_n \qquad (5)$$

in (3) leads to the equation

$$X_n^2 = m(16m+2)T_n^2 + P^2 \qquad (6)$$

whose initial solution is

$$T_1 = 1, X_1 = 4m + 1 \qquad (7)$$

Substituting (7) in (4),we get

$$\sim_1 = T_1 + X_1 = 5m + 1 \qquad (8)$$

In view of (1),it is seen that

$$c = 25m + 4$$

Observe that (a,b,c) is a Diophantine triple with property $D(P^2)$,P is a prime and $P^2 = 6m + 1$. Now, choose d in such a way that

$$md + P^2 = \sim_2^2 \qquad (9)$$

$$(16m+2)d + P^2 = \ _2^2 \qquad (10)$$

$$(25m+4)d + P^2 = ¥_2^2 \qquad (11)$$

Eliminating d between (10) and (11) and using the linear transformations

$$_2 = X_n + (16m+2)T_n \qquad (12)$$

$$¥_2 = X_n + (25m+4)T_n \qquad (13)$$

we get

$$X_n^2 = (25m+4)(16m+2)T_n^2 + P^2$$

with initial solution

$T_1 = 1$ and $X_1 = (20m+3)$

Substituting these values in (12) and using (10),we have

$$d = (81m+12)$$

Thus, $(m, 16m+2, 25m+4, 81m+12)$ is a Diophantine quadruple with property $D(P^2)$, P is a prime and $P^2 = 6m+1$.

A few numerical examples are given in the Table 1 below.

Table 1. Diophantine quadruple with property $D(P^2)$

a	b	c	d	$D(P^2)$
4	66	104	336	$D(5^2)$
8	130	204	660	$D(7^2)$
280	4482	7004	22692	$D(41^2)$
580	9282	14504	46992	$D(59^2)$
1908	30530	47704	154560	$D(107^2)$
16328	261250	408204	1322580	$D(313^2)$
245228	3923650	6130704	19863480	$D(1213^2)$
441188	7059010	11029704	35736240	$D(1627^2)$
678048	10848770	16951204	54921900	$D(2017^2)$
2365048	37840770	59126204	191568900	$D(3767^2)$
2433340	38933442	60833504 1	197100552	$D(3821^2)$
5080240	81283842	127006004	411499452	$D(5521^2)$
7618520	121896322	190463004	617100132	$D(6761^2)$
11841340	189461442	296033504	959148552	$D(8429^2)$
374760260	5996164162	9369006504	30355581072	$D(47419^2)$

Representing a,b,c,d by a(m),b(m),c(m),d(m) respectively the following results are observed.

1. $6[a(m)b(m) - 10a(m) + 1]$ is a nasty number.
2. $\{a(m)[d(m)]\}^2 - 972So_a - 288t_{3,a} - 828a$ is a biquadratic integer.
3. $a(m)[(b(m) - 2)]$ is a perfect square.
4. $16c(m) - 25b(m) = 14$
5. $25b(m) + 65c(m) - 25d(m) = 10$
6. $81c(m) - 25d(m) = 24$
7. $7a(m) - 2b(m) + c(m) = 0$

8. $a(m)[b(m)]^2 - 128So_a - 64\,\mathrm{Pr}_a - 68a(m) = 0$
9. $[a(m)]^2[b(m)] - 24[OH_m] + 16t_{3,m} + 10t_{4,m} = 0$
10. $a(m)b(m)c(m) + t_{4,m} + 2t_{12,m} = 800P_m^5$
11. $[a(m)]^2 d(m) - 6P_m^4 - 117[OH_m] - 18t_{3,m} + 49a(m)$ is a cubical integer.

Conclusion

To conclude one may search for Diophantine quadruple consisting of special numbers with property $D(P^2)$ for all primes P.

REFERENCES

Andrej Dujella and Florian Luca, 2005, *Diophantine m-tuples for* primes Intern. Math.Research, Notes 47,2913-2940.

Arkin.J, Hoggatt.V.E, and Strauss .E.G.1979 *On Euler's solution of a problem of Diophantine, Fibonacci* Quart 17, 333-339.

Bacic, Lj, Filipin.A, 2013. *The extendibility of D(4)-pairs* Math,Commun,18,447-456.

Brown E, 1985, *Sets in which xy+k is always a square,*Math.Comp.,45,613-620.

Cerin. Z 2011. *On extended Euler quadruples,* Atti Semin Mat Fis.Univ Modena Reggio Emilia 58 ,101-120

Dickson. I.E. 1966. History of the numbers, Vol.2 Chelsea,Newyork,513-520.

Dujella, A 1998. *A problem of Diophantus and pell numbers, Application of Fibanacci Numbers,* Vol 7 (G.E. Bergum, A.N. Philippou, A.F. Horadan eds.) Khowet Dordrecht, pp 61-68.

Dujella, A, 1993. *Generalization of a problem of diophantus,* Acta Arith 65, 15-27.

Dujella, A, 1997. *On Diophantine quintuples,* Acta Arith 81, 870.

Filipin, A. 2008. *There does not exist a D(4)-sextuple* J.Number Theory 128, 1555-1565

Fujita, Y 2008. *The extensibility of Diophantine pairs [k-l,k+l],*J.Number Theory,128, 322-353.

Fujita, Y, Togbe.A, 2011. *Uniqueness of the extension of the* $D(4k^2)$*-triple* $[k^2-4, k^2, 4k^2-4]$, Notes Number Theory, Discrete Math,17, 42-49.

Gopalan, M.A., Shrividhya.G, 2012. *Diophantine quadraples for Fibanacci and Lucas numbers with property D(4)* Diophantus *J.Math* 1(2012),15-18.

Meena. K. Vidhyalakshmi.S,Gopalan.M.A.,Akila.G,Presenna.R, 2014, *Formation of special Diophatine quadraples with property* $D(6kpq)^2$,The *international journal of Science and* Technology 2, 11-14.

Mootha,V.K, 1995. *On the set of numbers {14,22,30,42,90},*Acta Arith,71, 259-263.

Morgado, J.1983-1984. *Generalization of a result of Hoggatt and Bergum on Fibonacci numbers* Portugaliae Math,42, 441-445.

Morgado, J, 1991. *Note on a Shannon's theorem concerning the Fibonacci numbers and Diophantine quadruples,*Portugaliae Math,48,429-439.

Srividhya, G, 2009. *Diophantine quadruples for Fibonacci numbers with property D(1),*Indian *Journal of Mathematics and Mathematical Sciences,5,57-59.*

Zhang,Y, 2011. *Diophantine triples and extendibility of [1,2,5] and [1,5,10],*Master Thesis,Central Michigan University.

. *******