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RESEARCH ARTICLE

A NEW APPROACH FOR FINDING MINIMUM PATH IN A NETWORK USING TRIANGULAR INTUITIONISTIC FUZZY NUMBER

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ABSTRACT

In this paper, we proposed a new approach that can obtain a minimum path in a network using Triangular Intuitionistic Fuzzy number (TIFN). The path labelling algorithm is illustrated with the help of numerical example. The minimum paths obtained from one node to each node of a network can be helpful to decision makers as they make decision to use minimum number of nodes.

INTRODUCTION

Many researchers have paid much attention to the fuzzy Shortest Path Problem (SPP) since it is central to lots of applications. In the fuzzy SPP, the fuzzy shortest length and the corresponding shortest paths are the useful information for the decision makers. Determination of shortest distance and shortest path between two vertices is one of the most fundamental problems in graph theory. Let G = (V, E) be a graph with V as the set of vertices and E as the set of edges. A path between two vertices is an alternating sequence of vertices and edges starting and ending with vertices, and no vertices or no edges appear more than once in the sequence. The length of a path is the sum of the weights of the edges on the path. There may exist more than one path between a pair of vertices. The SPP is to find the path with minimum length between a specified pair of vertices. The fuzzy SPP was first analyzed by Dubois and Prade (1980) using fuzzy number instead of a real number is assigned to each edges. Okada (2000) concentrated on a SPP and introduced the concept of degree of possibility in which an arc is on the shortest path. Takahasi and Yamakami (2005) discussed the SPP from a specified node to every other node on a network. Chaung and Kung (2005) pointed out there are several methods to solve this kind of problem in the open literature. Amit Kumar and Manjot Kaur (2011) introduced a existing algorithm about SPP with fuzzy arc length. Pandian and Rajendiran (2010) have proposed a new algorithm to find the minimum path in a network. Sophia Porchelvi and Sudha (2013) also did the same with a SPP using TIFN. This paper is organized as follows. In section 2, preliminary concepts and definitions are given. The procedure for finding minimum path using TIFN is developed in section 3. An illustrative example is provided in section 4 to find the minimum path from a specified node to every node in a network having imprecise weights. The last section draws some concluding remarks.

2. Prerequisites

2.1 Acyclic Digraph

A digraph is a graph each of whose edges are directed. Hence an acyclic digraph is a directed graph which does not contain a cyclic tour.

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2.2 Intuitionistic Fuzzy Set (IFS)

Let X be a Universe of discourse, then an Intuitionistic fuzzy set A in X is given by
A = $\{(x, \mu_A(x), \gamma_A(x)) / x \in X\}$ Where the function $\mu_A(x): X \to [0,1]$ and $\gamma_A(x): X \to [0,1]$ determines the degree of membership and non membership of the element $x \in X, 0 \le \mu_A(x) + \gamma_A(x) \le 1$.

2.3 Intuitionistic Fuzzy Number (IFN)

Let $A = \{x, \mu_A(x), \gamma_A(x) / x \in X\}$ be an IFS, then we call $(\mu_A(x), \gamma_A(x))$ an IFN .We denote it by $(\langle a,b,c\rangle, \langle e,f,g\rangle)$ where $\langle a,b,c\rangle$ and $\langle e,f,g\rangle \in F(I)$

2.4 Triangular Intuitionistic Fuzzy Number(TIFN)

A triangular Intuitionistic fuzzy number A is denoted by $\{\mu_A, \gamma_A / x \in R\}$ Where μ_A and γ_A are triangular fuzzy number with $\gamma_A \le \mu_A{}^C$. So a TIFN 'A' is given by $\widetilde{A} = (\langle a_1, a_2, a_3\rangle \langle a_1', a_2, a_3'\rangle)$ that is either $a_1' \ge a_2$ and $a_3' \ge a_3$ (or) $a_2 \le a_1$ and $a_3' \le a_2$ are membership and non-membership fuzzy numbers of A.

2.5 Minimum path

A minimum path from a node **p** to another node **q** in a network is a shortest path from p to q with the property that no other such path has a lower length.

Remark 1: The weight of the shortest path is the same as the weight of the minimum path.
Remark 2: If there is only one shortest path from a node p to a node q, then the minimum path from p to q is the shortest path.
Remark 3: If (k; $l(p_1, p_s)$) is a minimum path at the node p_s from p_1 and (m; $l(p_s, p_k)$) is a minimum path at the node p_k from p_s, then (k + m; $l(p_1, p_s) + l(p_s, p_k)$) is a minimum path at the node p_k from p_1, where $l(p_1, p_s) + l(p_s, p_k)$ is the path joining the two paths $l(p_1, p_s)$ and $l(p_s, p_k)$ at the node p_s.

3. Minimum Path labelling algorithm

In a network, a label at a node p_s from p_1 can be represented as follows: (k ; $l(p_1, p_s)$)
Where $l(p_1, p_s)$ is a path from p_1 to p_s and k is the weight of the path $l(p_1, p_s)$ which may be a intuitionistic fuzzy number or a fuzzy number or a number.

Let (k_i ; $l_i(p_1, p_s)$), i = 1,2,3,.....,m be m labels. The label (k_n ; $l_n(p_1, p_s)$),), $n \in \{1,2,......., m\}$ is said to be a minimum label at the node p_s from p_1 if $k_n \le k_i$, for all i . The path $l_n(p_1, p_s)$ is the shortest path to p_s from p_1.

Algorithm

Step 1: Find the collection of nodes N_1 in the network which is adjacent to p_1. If N_1 is empty, then go to step 7.
Step 2: Find out the minimum path at every node of N_1 from p_1.
Step 3: Find the collection of nodes N_2 in the network which is adjacent to N_1. If N_2 is empty, then go to step 7. Otherwise go to step 4.
Step 4: Compute the minimum path at every node of N_2 from p_1 using the Remark 3. If the shortest path between p_1 and any node of N_2 with negative weight is a cycle, go to step 7. Otherwise go to step 5.
Step 5: Repeat the step 1 to step 4 until to compute the set of collection of nodes in the network which are adjacent to each of the minimum label node is empty.
Step 6: Find the minimum path from p_1 to each of the nodes in the network in step 5. Stop.
Step 7: There is no path from the node p_1 to the label node. Stop.

Example 1: Consider a mobile service company which handles 23 geographical centres. A configuration of a telecommunication network is presented in Figure 1. Assume that the distance between any two centres is a Intuitionistic Triangular Fuzzy number (the arc lengths are given in Table 1). The company wants to find a shortest path for an effective message flow amongst the centres. The shortest path and the corresponding lengths are reported below.

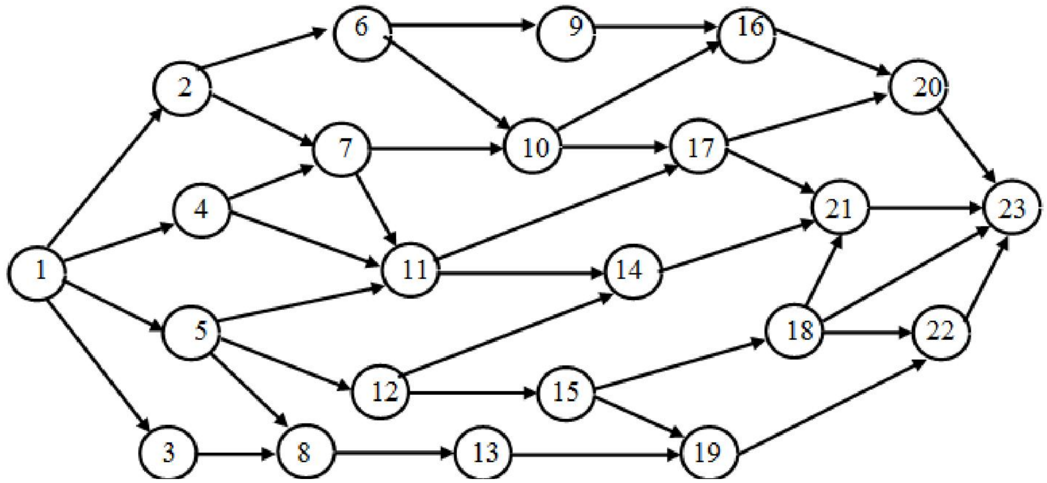

Figure 1. The network for example1.

Table 1. The arc lengths for example 1.

Arc	Lengths	Arc	Lengths	Arc	Lengths
(1,2)	$(\langle 8,9,11\rangle,\langle 10,12,14\rangle)$	(1,3)	$(\langle 9,10,11\rangle,\langle 11,13,15\rangle)$	(1,4)	$(\langle 5,7,9\rangle,\langle 8,10,11\rangle)$
(1,5)	$(\langle 4,5,6\rangle,\langle 7,8,9\rangle)$	(2,6)	$(\langle 8,9,10\rangle,\langle 10,12,13\rangle)$	(2,7)	$(\langle 9,11,13\rangle,\langle 12,14,15\rangle)$
(3,8)	$(\langle 10,11,12\rangle,\langle 12,13,14\rangle)$	(4,7)	$(\langle 12,14,15\rangle,\langle 15,17,18\rangle)$	(4,11)	$(\langle 9,10,12\rangle,\langle 11,13,14\rangle)$
(5,8)	$(\langle 8,9,12\rangle,\langle 10,13,14\rangle)$	(5,11)	$(\langle 9,10,13\rangle,\langle 11,14,15\rangle)$	(5,12)	$(\langle 10,11,12\rangle,\langle 12,15,16\rangle)$
(6,9)	$(\langle 7,8,11\rangle,\langle 10,12,15\rangle)$	(6,10)	$(\langle 10,11,14\rangle,\langle 12,15,16\rangle)$	(7,10)	$(\langle 9,10,11\rangle,\langle 12,14,15\rangle)$
(7,11)	$(\langle 6,7,10\rangle,\langle 9,11,14\rangle)$	(8,13)	$(\langle 4,7,10\rangle,\langle 8,11,15\rangle)$	(9,16)	$(\langle 6,7,11\rangle,\langle 9,11,15\rangle)$
(10,16)	$(\langle 12,13,17\rangle,\langle 15,18,21\rangle)$	(10,17)	$(\langle 15,16,20\rangle,\langle 18,21,24\rangle)$	(11,14)	$(\langle 8,9,13\rangle,\langle 11,14,17\rangle)$
(11,17)	$(\langle 7,9,13\rangle,\langle 9,14,17\rangle)$	(12,14)	$(\langle 12,14,18\rangle,\langle 14,19,22\rangle)$	(12,15)	$(\langle 12,13,17\rangle,\langle 15,18,21\rangle)$
(13,19)	$(\langle 15,16,20\rangle,\langle 18,21,24\rangle)$	(14,21)	$(\langle 10,11,15\rangle,\langle 14,16,19\rangle)$	(15,18)	$(\langle 7,8,12\rangle,\langle 10,13,16\rangle)$
(15,19)	$(\langle 14,15,18\rangle,\langle 16,19,22\rangle)$	(16,20)	$(\langle 9,12,16\rangle,\langle 13,15,18\rangle)$	(17,20)	$(\langle 7,10,14\rangle,\langle 11,13,16\rangle)$
(17,21)	$(\langle 5,8,12\rangle,\langle 9,13,14\rangle)$	(18,21)	$(\langle 14,16,20\rangle,\langle 17,19,22\rangle)$	(18,22)	$(\langle 2,6,11\rangle,\langle 6,12,13\rangle)$
(18,23)	$(\langle 4,8,13\rangle,\langle 8,14,15\rangle)$	(19,22)	$(\langle 14,15,18\rangle,\langle 17,19,21\rangle)$	(20,23)	$(\langle 12,13,16\rangle,\langle 15,17,19\rangle)$
(21,23)	$(\langle 11,14,17\rangle,\langle 14,18,20\rangle)$	(22,23)	$(\langle 3,6,11\rangle,\langle 7,12,13\rangle)$	-	-

To compute the minimum path from the node 1 to each of the other nodes in the network

Now, N_1 = {2,4,5,3} and using step 1 to step 3 of the modified algorithm, we have the following

Destination Node	Minimum label from the node 1	Shortest path between The node 1 & the Destination node	Minimum path between the node 1 & the destination node
2	$[(\langle 8,9,11\rangle,\langle 10,12,14\rangle);1-2]$	1-2	1-2
4	$[(\langle 5,7,9\rangle,\langle 8,10,11\rangle);1-4]$	1-4	1-4
5	$[(\langle 4,5,6\rangle,\langle 7,8,9\rangle);1-5]$	1-5	1-5
3	$[(\langle 9,10,11\rangle,\langle 11,13,15\rangle);1-3]$	1-3	1-3

$N_2 = \{6,7,11,12,8\}$ and using the step 1 to the step 3 of the modified algorithm, we have the following:

Destination Node	Minimum label from the node 1	Shortest path between The node 1 & the Destination node	Minimum path between the node 1 & the destination node
6	$\left[\left(\langle 16,18,21\rangle,\langle 20,24,27\rangle\right);1-2-6\right]$	1-2-6	1-2-6
7	$\left[\left(\langle 17,20,24\rangle,\langle 22,26,29\rangle\right);1-2-7\right]$ $\left[\left(\langle 17,21,24\rangle,\langle 23,27,29\rangle\right);1-4-7\right]$	1-2-7 1-4-7	1-2-7
11	$\left[\left(\langle 14,17,21\rangle,\langle 19,23,25\rangle\right);1-4-11\right]$ $\left[\left(\langle 13,15,19\rangle,\langle 18,22,24\rangle\right);1-5-11\right]$	1-4-11 1-5-11	1-5-11
12	$\left[\left(\langle 14,16,18\rangle,\langle 19,23,25\rangle\right);1-5-12\right]$	1-5-12	1-5-12
8	$\left[\left(\langle 12,14,18\rangle,\langle 17,21,23\rangle\right);1-5-8\right]$ $\left[\left(\langle 19,21,23\rangle,\langle 23,26,29\rangle\right);1-3-8\right]$	1-5-8 1-3-8	1-5-8

Now, $N_3 = \{9,10,17,14,15,13\}$ and using step 1 to the step 3 of the modified algorithm, we have the following table:

Destination Node	Minimum label from the node 1	Shortest path between The node 1 & the Destination node	Minimum path between the node 1 & the destination node
9	$\left[\left(\langle 23,26,32\rangle,\langle 30,36,42\rangle\right);1-2-6-9\right]$	1-2-6-9	1-2-6-9
10	$\left[\left(\langle 26,29,35\rangle,\langle 32,39,43\rangle\right);1-2-6-10\right]$ $\left[\left(\langle 26,31,35\rangle,\langle 35,41,44\rangle\right);1-4-7-10\right]$	1-2-6-10 1-4-7-10	1-2-6-10
17	$\left[\left(\langle 21,26,34\rangle,\langle 28,37,42\rangle\right);1-4-11-17\right]$ $\left[\left(\langle 21,24,32\rangle,\langle 27,36,41\rangle\right);1-5-11-17\right]$	1-4-11-17 1-5-11-17	1-5-11-17
14	$\left[\left(\langle 22,26,34\rangle,\langle 30,37,42\rangle\right);1-4-11-14\right]$ $\left[\left(\langle 26,30,36\rangle,\langle 33,42,47\rangle\right);1-5-12-14\right]$	1-4-11-14 1-5-12-14	1-4-11-14
15	$\left[\left(\langle 26,29,35\rangle,\langle 34,41,46\rangle\right);1-5-12-15\right]$	1-5-12-15	1-5-12-15
13	$\left[\left(\langle 16,21,28\rangle,\langle 25,32,38\rangle\right);1-5-8-13\right]$ $\left[\left(\langle 13,28,23\rangle,\langle 31,37,44\rangle\right);1-3-8-13\right]$	1-5-8-13 1-3-8-13	1-5-8-13

Now, $N_4 = \{16,20,21,18,19\}$ and using step 1 to the step 3 of the modified algorithm, we have the following table:

Destination Node	Minimum label from the node 1	Shortest path between The node 1 & the Destination node	Minimum path between the node 1 & the destination node
16	$\left[\left(\langle 38,44,52\rangle,\langle 50,59,65\rangle\right);1-4-7-10-16\right]$ $\left[\left(\langle 29,33,43\rangle,\langle 39,47,57\rangle\right);1-2-6-9-16\right]$	1-4-7-10-16 1-2-6-9-16	1-2-6-9-16
20	$\left[\left(\langle 38,45,59\rangle,\langle 52,62,75\rangle\right);1-2-6-9-16-2$	1-2-6-9-16-20 1-4-11-17-20	1-4-11-17-20

	$\left[\left(\langle 28,36,48\rangle,\langle 39,50,58\rangle\right);1-4-11-17-20\right]$		
21	$\left[\left(\langle 26,34,46\rangle,\langle 37,50,56\rangle\right);1-4-11-17-21\right]$ $\left[\left(\langle 32,37,49\rangle,\langle 44,53,61\rangle\right);1-4-11-14-21\right]$	1-4-11-17-21 1-4-11-14-21	1-4-11-17-21
18	$\left[\left(\langle 33,37,47\rangle,\langle 44,54,62\rangle\right);1-5-12-15-18\right]$	1-5-12-15-18	1-5-12-15-18
19	$\left[\left(\langle 40,44,53\rangle,\langle 50,60,68\rangle\right);1-5-12-15-19\right]$	1-5-12-15-19	1-5-12-15-19

Now, N_5 = {23,22} and using step 1 to the step 3 of the modified algorithm, we have the following table:

Destination Node	Minimum label from the node 1	Shortest path between The node 1 & the Destination node	Minimum path between the node 1 & the destination node
23	$\left[\left(\langle 37,48,63\rangle,\langle 51,68,76\rangle\right);1-4-11-17-21-23\right]$ $\left[\left(\langle 43,51,66\rangle,\langle 58,71,81\rangle\right);1-4-11-14-21-23\right]$ $\left[\left(\langle 37,45,60\rangle,\langle 52,68,77\rangle\right);1-5-12-15-18-23\right]$	1-4-11-17-21-23 1-4-11-14-21-23 1-5-12-15-18-23	1-5-12-15-18-23
22	$\left[\left(\langle 35,43,58\rangle,\langle 50,66,75\rangle\right);1-5-12-15-18-22\right]$	1-5-12-15-18-22	1-5-12-15-18-22

Now N_6 = { }. We stop the computation. By the modified method, the shortest path and the minimum path from the node 1 to each other nodes is given below using the modified method.

Node	Shortest Path	Weight of the Shortest Path	Minimum path
2	1-2	$\left(\langle 8,9,11\rangle,\langle 10,12,14\rangle\right)$	1-2
3	1-3	$\left(\langle 9,10,11\rangle,\langle 11,13,15\rangle\right)$	1-3
4	1-4	$\left(\langle 5,7,9\rangle,\langle 8,10,11\rangle\right)$	1-4
5	1-5	$\left(\langle 4,5,6\rangle,\langle 7,8,9\rangle\right)$	1-5
6	1-2-6	$\left(\langle 16,18,21\rangle,\langle 20,24,27\rangle\right)$	1-2-6
7	1-2-7 1-4-7	$\left(\langle 17,20,24\rangle,\langle 22,26,29\rangle\right)$ $\left(\langle 17,21,24\rangle,\langle 23,27,29\rangle\right)$	1-2-7
8	1-5-8 1-3-8	$\left(\langle 12,14,18\rangle,\langle 17,21,23\rangle\right)$ $\left(\langle 19,21,23\rangle,\langle 23,26,29\rangle\right)$	1-5-8
9	1-2-6-9	$\left(\langle 23,26,32\rangle,\langle 30,36,42\rangle\right)$	1-2-6-9
10	1-2-6-10 1-4-7-10	$\left(\langle 26,29,35\rangle,\langle 32,39,43\rangle\right)$ $\left(\langle 26,31,35\rangle,\langle 35,41,44\rangle\right)$	1-2-6-10
11	1-4-11 1-5-11	$\left(\langle 14,17,21\rangle,\langle 19,23,25\rangle\right)$ $\left(\langle 13,15,19\rangle,\langle 18,22,24\rangle\right)$	1-5-11

12	1-5-12	$(\langle 14,16,18\rangle,\langle 19,23,25\rangle)$	1-5-12
13	1-5-8-13 1-3-8-13	$(\langle 16,21,28\rangle,\langle 25,32,38\rangle)$ $(\langle 13,28,33\rangle,\langle 31,37,44\rangle)$	1-5-8-13
14	1-4-11-14 1-5-12-14	$(\langle 22,26,34\rangle,\langle 30,37,42\rangle)$ $(\langle 26,30,36\rangle,\langle 33,42,47\rangle)$	1-4-11-14
15	1-5-12-15	$(\langle 26,29,35\rangle,\langle 34,41,46\rangle)$	1-5-12-15
16	1-4-7-10-16 1-2-6-9-16	$(\langle 38,44,52\rangle,\langle 50,59,65\rangle)$ $(\langle 29,33,43\rangle,\langle 39,47,57\rangle)$	1-2-6-9-16
17	1-4-11-17 1-5-11-17	$(\langle 21,26,34\rangle,\langle 28,37,42\rangle)$ $(\langle 21,24,32\rangle,\langle 27,36,41\rangle)$	1-5-11-17
18	1-5-12-15-18	$(\langle 33,37,47\rangle,\langle 44,54,62\rangle)$	1-5-12-15-18
19	1-5-12-15-19	$(\langle 40,44,53\rangle,\langle 50,60,68\rangle)$	1-5-12-15-19
20	1-2-6-9-16-20 1-4-11-17-20	$(\langle 38,45,59\rangle,\langle 52,62,75\rangle)$ $(\langle 28,36,48\rangle,\langle 39,50,58\rangle)$	1-4-11-17-20
21	1-4-11-17-21 1-4-11-14-21	$(\langle 26,34,46\rangle,\langle 37,50,56\rangle)$ $(\langle 32,37,49\rangle,\langle 44,53,61\rangle)$	1-4-11-17-21
22	1-5-12-15-18-22	$(\langle 35,43,58\rangle,\langle 50,66,75\rangle)$	1-5-12-15-18-22
23	1-4-11-17-21-23 1-4-11-14-21-23 1-5-12-15-18-23	$(\langle 37,48,63\rangle,\langle 51,68,76\rangle)$ $(\langle 43,51,66\rangle,\langle 58,71,81\rangle)$ $(\langle 37,45,60\rangle,\langle 52,68,77\rangle)$	1-5-12-15-18-23

Conclusion

In this paper, a new algorithm namely, minimum path labelling algorithm for solving SPP as well as minimum path problems on a network is provided. In the modified method, we are able to obtain all non- dominated paths from the specified node to every other node and any two specified nodes.

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