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RESEARCHARTICLE

AN INDUSTRIAL APPLICATION OF HOTELLING T-SQUARE CONTROL CHART FOR INDIVIDUAL OBSERVATIONS

*Ibrahim Mu'awiyya Idris

Umaru Musa Yar'adua University katsina, Nigeria

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ABSTRACT

Hotelling T-square control chart is an extension of univariateshewhart control chart where two or more related quality variables can be monitored simultaneously. Univariateshewhart control chart provides a good approach for monitoring quality variables separately, but most often in industry, some degree of correlation due exist between quality variables and which makes the so-called univariateshewhart chart in-appropriate for monitoring quality in production processes. However, when the quality variables are correlated then the most suitable approach for multivariate process monitoring is the Hotelling T-square control chart. In this research, a multivariate data at the early stage of production consisting of individual observations obtained from Dana Steel Company Ltd katsina is analyzed for quality, and the result of the retrospective analysis shows that at the initial stage of production the data is in statistical control.

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INTRODUCTION

Industrial application of multivariate quality control (MQC) have attracted attention in the recent decades Timothy and Paul (1999); Theodora (2005), this is because quality control in industries involves monitoring a vector of related quality variables. The main significance of process monitoring is the early detection of an out-of-control signal followed by the identification of the quality attribute responsible for the out-of-control signal, often in industrial settings, there exist variations in the raw materials and production process all of which are due the natural and assignable causes. Natural causes arose due to random forces which are uncontrollable while assignable causes are those which occurred as a result of wrong industrial settings, raw materials variations and staff personnel, in addition assignable causes can be managed, monitored and controlled. Hotelling who is famous in the development of multivariate quality control first developed a T-square test procedure for multivariate population and later extend it to control charts Hotelling (1931), Hotelling (1947). An extensive literature review for multivariate quality control (MQC) which among others includes multivariate cumulative sum (MCUSUM) and multivariate exponentially weighted moving average (MEWMA) are discussed in Lowry and Montgomery (1995); Alt (1985); Bersimis *et al*. (2007).

Corresponding author: Ibrahim Mu'awiyyaIdris
Umaru Musa Yar'adua University katsina, Nigeria.

This research investigates whether quality control monitoring are applied from the initial stage of production process when only data in individual observations are available. The data used in this research are secondary data comprising of five variables (Weight KG/M, Area MMSQ, Breaking force KGF/MMSQ, Tensile strength N/MMSQ and Yield strength N/MMSQ) of hundred individual observations each and a simple random sampling technique was employed for drawing the sampled data.

MATERIALS AND METHODS

Hotelling T-square control chart for individual observations is used when the subgroup size, the test statistics is given as

$$T^2 = (X - \bar{X})' S^{-1} (X - \bar{X}) \quad (1)$$

Where X are the individual observations for all the five quality variables and $\bar{X}$ are the respective means. $\bar{X}$ is an unbiased estimator for the unknown population mean μ and S^{-1} is the inverse of the covariance matrix which also an unbiased estimator of the population variance. The upper and lower control limits for the retrospective analysis used for monitoring the in-control condition are computed by

$$UC = \frac{p(m+1)(m-1)}{m^2 - mp} F_{\alpha(p,mm-p)} \quad (2)$$

$$LCL = 0 \quad (3)$$

Retrospective Analysis for the Individual Observations

The means $\bar{X}$ for each of the $p = 5$ quality variables are computed as given by the vector below

$$\bar{X} = (0.86158\ \ 108.71000\ \ 7453.35000\ \ 679.56000\ \ 437.06000)$$

We set the matrices $(X - \bar{X})$ and transpose it $(X - \bar{X})'$ for the hundred vectors of observations by first obtaining the following P_i matrices for which $(i = 1,2, \ldots ,100)$

$$P_1 = (0.850\ \ 108\ \ 9650\ \ 894\ \ 431) - \bar{X}$$

$$P_2 = (0.849\ \ 108\ \ 7800\ \ 722\ \ 431) - \bar{X}$$

$$\ldots \ldots \ldots$$

$$P_{100} = (0.899\ \ 114\ \ 8500\ \ 749\ \ 466) - \bar{X}$$

We then have $(X - \bar{X}) = \sum P_i$ and we have the sample covariance matrix$S$ computed as

$$\begin{bmatrix} 0.000290607 & 0.02945273 & 3.653101e-01 & -0.1930756 & -0.0735604 \\ 0.0294527273 & 102.16757576 & -8.250354e+01 & -28.1187879 & -9.4167677 \\ 0.3653101010 & -82.50353535 & 3.757909e+05 & 34419.3171717 & 4134.2919192 \\ -0.1930755556 & -28.1187878 & 83.441932e+04 & 3364.4509091 & 443.9357576 \\ -0.0735604040 & -9.41676768 & 4.134292e+03 & 443.9357576 & 388.1175758 \end{bmatrix}$$

Taking the inverse of the covariance matrix S we've accordingly S^{-1} as given by the matrix below

$$\begin{bmatrix} 3441.041405 & 33.952713130 & 2.737400e+00 & -5.179320e+00 & -1.359427e+01 \\ 33.952713 & 0.00978841 & -1.212069e-02 & -3.556341e-02 & -1.061936e-01 \\ 2.737400 & -0.012120693 & 2.661054e-06 & 2.905345e-05 & 2.418794e-04 \\ -5.179320 & -0.035563411 & 2.905345e-05 & 2.972253e-04 & 2.252578e-03 \\ -13.594270 & -0.106193551 & 2.418794-04 & 2.252578e-03 & 2.576539e-03 \end{bmatrix}$$

We then compute the hotelling T-square statistics in equation (1) which is given by the matrix below

$$\begin{bmatrix} 1.297005e+073.191327e+051.464672e+052.787350e+055.599418e+06 \\ 5.035877e+062.829021e+062.617245e+069.365840e+058.797032e+04 \\ 2.074730e+068.255810e+055.348613e+033.832786e+053.284522e+05 \\ 2.405042e+052.438649e+053.294202e+053.608359e+053.619117e+05 \\ 6.048847e+046.048848e+041.212585e+061.207063e+063.741404e+06 \\ 5.332431e+061.545291e+054.650003e+052.575047e+06\ 2.444707e+06 \\ 8.730559e+058.730558e+051.061979e+061.965558e+06-1.403396e+03 \\ 8.165858e+025.558189e+054.431347e+055.823463e+04\ 5.823517e+04 \\ 2.771817e+06\ 2.453692e+064.929578e+042.253497e+034.064216e+04 \\ -4.352088e+032.392607e+051.222710e+082.396547e+052.450705e+05 \\ 1.274932e+051.971752e+054.855521e+055.380248e+052.812360e+04 \\ 5.836439e+048.953244e+054.220552e+041.168220e+042.094617e+05 \\ 5.604187e+065.800888e+067.101456e+035.010495e+051.288447e+06 \\ -1.609642e+042.652029e+03-2.328576e+036.606169e+054.574405e+05 \\ 1.139665e+06\ 1.230331e+06\ 3.644380e+05\ 5.352923e+052.933568e+04 \\ 1.399218e+046.316056e+045.422341e+047.550415e+059.157097e+05 \\ 8.149607e+031.410969e+052.017491e+052.781218e+052.055255e+05 \\ 3.159301e+048.760737e+039.709250e+042.058398e+069.624342e+05 \\ 3.749206e+041.129519e+053.233680e+052.288177e+056.591797e+05 \\ 5.979219e+051.244571e+051.669361e+032.815675e+062.965463e+06 \end{bmatrix}$$

Using $\alpha = 0.01$ in equation (2) for the upper control limit. The corresponding control chart for the retrospective analysis is given in Figure 1 below, where $T^2 = Yz$ values are plotted against the individual observations Xz.

DISCUSSION AND CONCLUSION

From the retrospective analysis as shown in Figure 1, all points fall within the control region and this indicates that the sampled

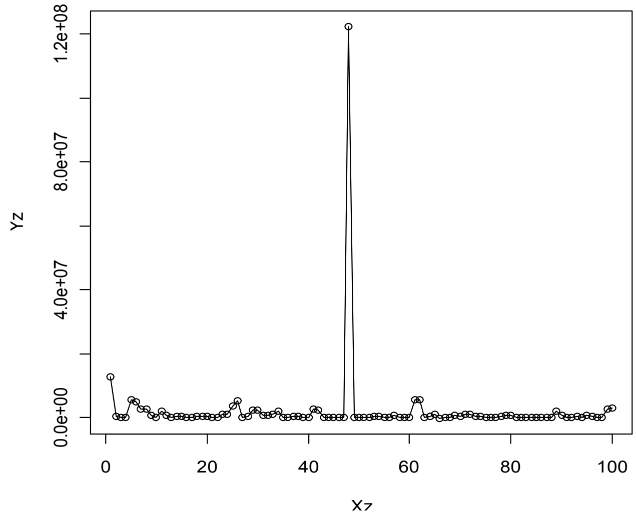

Fig 1. Control Chart of Individual Observations

data was drawn from a statistically monitored and controlled production process, even though one point is seen to be very close to the out-of-control region, this may be as a result of random causes and which can be ignored. Nowadays production companies are competing to produce best quality items that meet the demand of the 21st century while low quality goods are increasingly neglected in the market, multivariate statistical process control is employed to achieve the desired objective of monitoring and increasing quality in production processes, especially when several related variables that measures quality are involved. We have in this research analyze a multivariate data of individual observations drawn from an early stage of production in Dana Steel Company Limited katsina Nigeria, and the result of the retrospective analysis based onhotelling's T-square control chart for individual observations shows that at the initial stage of production the data is in statistical control.

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