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RESEARCH ARTICLE

DEVELOPMENT OF MATHEMATICAL MODEL TO PREDICT THE EFFECT OF *E.Coli* INFLUENCE BY POROSITY IN COASTAL AREA OF OKRIKA IN RIVERS STATE, NIGERIA

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ABSTRACT

This paper explain the level E.coli transport influenced by porosity, where by a mathematical model where developed including experimental analysis to generate a better solution to the rapid migration of microbes *E. coli* at Okrika mainland of Rivers state in the Niger Delta of Nigeria, from the result of both parameter it shows that the concentration of the microbes migrate in fluctuation form decrease at a distance of thirty meters has some high concentration of *E.coli* at thirty meters with a duration of hundred days, comparing it with the recommended value of World Health Organization (WHO), both result did not meet up the required water quality standard, therefore it is recommended that any ground water abstraction at the study location, should be design and those existing one that have cause a lots of water related diseases should undergo treatment analysis before it is allowed for human consumption , in other to stop the threat of human life.

INTRODUCTION

The study of porosity and its distribution is important since this yields information about the amount of fluid that can be stored and how it is being distributed within a formation (George *et al.,* 2011, Salem, 2001). Relationship between porosity and other measurable quantities in geophysical exploration, such as resistivity (and the associated formation factor), and other parameters, have long been the subject of much research and many publications. Information about the porosity of a formation is also important in oil exploration because of the relationship between effective porosity and permeability of the formation. (Archie, 1942) Effective porosity is the porosity that is available for the circulation of free fluid. It may be much lower than the total porosity which is the ratio of the volume of voids in the rock or soil to the total volume of the rock or soil. Porosity is a dimensionless quantity expressed as a decimal fraction or percentage (Niwas and Lima, 2003). Effective porosity can also be expressed as the ratio of the volume of water that can flow by gravity to the volume of the soil or rock. This definition accounts for the free circulation of fluid in porous consolidated and unconsolidated materials of rock and soil respectively (George *et al.,* 2011) the principles of fluid dynamics in porous media; explain that there are two type of porosity (Bear, 1972) which are: Volumetric porosity (n) and areal porosity (N_a).

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If the total unit volume VT of a soil or rock is divided into the volume of the solid portion Vs and the volume of the voids Vv, the volumetric porosity (n) can be expressed as in equation 1;*T.* Similarly, the areal porosity N_a, can be defined for any areal cross section through a unit volume, Where Av is the total area occupied by the voids and AT is the total area. Various cross sections within a given unit volume may exhibit differing areal porosities N_a1, N_a2….The volumetric porosity n, is an average of the various possible areal porosities where Av is the total area occupied by the voids and AT is the total area. Various cross sections within a given unit volume may exhibit differing areal porosities N_a1, N_a2….The volumetric porosity n, is an average of the various possible areal porosities, *NAi*. The porosity n can be an important controlling influence on hydraulic conductivity, K. In sampling programmes carried out within deposits of well-sorted sand or in fractured rock formations, samples with higher n generally also have higher k (Freeze and Cherry, 1979) unfortunately, the relationship does not hold on a regional basis across the spectrum of possible rock and soil types. Clay-rich soils, for example, usually have higher porosities than sandy or gravelly soils but lower Hydraulic conductivities (Sen and Goode, 1988). The porosity, n is closely related to the void ratio e, which is widely used in soil mechanics. The void ratio is defined as: In this paper an attempt is made to use laboratory analysis on core sample to estimate the porosity distribution of the aquifer in the area where Millennium Development Goals project for groundwater development is sited. This being the baseline study will help in evaluating other hydraulic parameters that characterize the existing aquifers in the zone since porosity is strongly related with other hydraulic parameters.(George *et al.,* 2011). A soil-water (moisture) characteristic curve (SWCC) shows the important relationship between water content of a soil to matric suction (Fredlund and Rahardjo, 1993, Ali and Rahardjo, 2004). In addition to describing the particular relationship between soil moisture and suction Fredlund and Xing (1994) have proposed that the SWCC can be used for the prediction of engineering properties, such as permeability, in soil in a partly saturated state as studies have shown a relationship between the soil-water characteristic curve of a partly saturated soil and its engineering properties.

Porosity (n) is defined as the volume of the pores of a rock or soil sample (Vp) divided by the total volume (Vt) of both pores and solid materials, that is, $n = V\text{p}/V\text{t}$ (2.1) When a rock is first formed y precipitation, cooling from an igneous melt indurations from loose sediments; or when a soil is first formed by weathering of rock materials and possibly subsequent biological Action, the new entity will contain a certain inherent porosity known as *primary porosity*. This porosity may later be reduced by cementation from precipitates from circulating groundwater, or from compaction accompanying burial by later sediments. However, fractures or solution cavities formed in the rock, or Root tubes or animal burrows in soils may later form and are known as *secondary porosity*. Thus, the total porosity of a sample will be the sum of the primary and secondary porosities. Porosity of a consolidated sample can be determined quit e simply by first cutting the sample to a known dimension such as a cylinder or cube and measuring the total volume. Next, the sample is submerged in a known volume of water and allowed to saturate. After saturation, the volume of water displaced will be the volume of solids in the sample. The volume of voids is simply the difference between the total volume and the volume of solids, and/porosity can be calculated by the above formula. More accurate and more sophisticated methods for testing earth materials for various properties are given in various publications of the American Society for testing and materials (ASTM). If all the pores in a rock are not connected, only a certain fraction of the pores will allow the passage of water, and this fraction is known as the *effective porosity*. An example is pumice, a glassy volcanic ejects (a solidified froth) which may float on water because the total porosity is so high and it contains so much entrained gas that its bulk density is less than water. It may take some days to sink because the effective porosity is so low that water cannot easily pass through it. Coarse gravel may contain a porosity of only 25%, but that porosity will practically be effective. Thus, gravel is an excellent conductor of water. Porosity of a rock or soil is determined largely by the packing arrangement of its grains and the uniformity of the grain-size distribution. The greatest ideal porosity that could be attained in a material

Bulk density is a measure of the weight of the soil per unit volume (g/cc), usually given on an oven-dry (110° C) basis (Figure 1). Variation in bulk density is attributable to the relative proportion and specific gravity of solid organic and inorganic particles and to the porosity of the soil. Most mineral soils have bulk densities between 1.0 and 2.0. Although bulk densities are seldom measured, they are important in quantitative soil studies, and measurement should be encouraged. Such data are necessary, for example, in calculating soil moisture movement within a profile and rates of clay formation and carbonate accumulation. Even when two soils are compared qualitatively on the basis of their development for purposes of stratigraphic correlation, more accurate comparisons can be made on the basis of total weight of clay formed from 100 g of parent material than one percent of clay alone. To convert percent to weight per unit volume, multiply by bulk density (Birkeland, 1984). The determination usually consists of drying and weighing a soil sample, the volume of which is known (core method) or must be determined (clod method and excavation method). These methods differ in the way.

MATERIALS AND METHOD

The method of sample was in-situ method of sample collection from a bore hole drilling site at the study location, an aquifer materials were collected at sequence of three metres each, the method of drilling is manual method of drilling, with manual equipments

Column experiments

Column experiments were performed to monitor the level of transport of *E. coli* at different deposit of soil formation.

Experiment Set up

The column was set up; the height is 1 metre of 10mm diameter steel pipe, positioned at vertical level, including a funnel of 30cm that contains 4 litres of waste water. Each sample level of average of 2000mg/l of waste water containing *E. coli* was poured inside the column. While the flow was passing through the column, a stop watch was used to monitor the speed level, to determine the level of transport of each sample of aquifer materials. The effluent 1000mg/l from the column were collected and subjected to thorough analysis to determine the level of transport of *E. coli* in each of the aquifer material, which determines the level of transport to aquiferious zone

RESULTS AND DISCUSSION

Developed model equation to determination of *E. coli* concentration upon the influence of porosity of the soil stratum

Solve $$\frac{\partial cAf}{\partial z} - \frac{a(1-\varepsilon)}{Vo}\frac{\partial cs}{\partial t} = 0 \quad \ldots\ldots\ldots \quad (1)$$

Considering when CAf = CS, thus equation (1) can be written as;

$$\frac{\partial c}{\partial z} - \frac{(1-\varepsilon)}{Vo}\frac{\partial c}{\partial t} = 0 \quad \ldots\ldots\ldots. \quad (2)$$

Where $C = C_{Af} = Cs$

Using separation of variables to resolve the above equation (2) by considering the coordinate point of Z in terms of time dependent, thus equation (2) can be expressed as

$$C = TZ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (3)$$

Therefore $\frac{\partial c}{\partial z} = TZ^1$ ……………….. (4)

$$\frac{\partial c}{\partial z} = T^1 Z \quad …………………. (5)$$

Substituting equation (3) and (4) into equation (2) becomes

$$TZ^1 - \frac{a(1-\varepsilon)}{Vo} T^1 Z = 0 ………………..……… (6)$$

Equation (6) can be expressed by considering both sides to be constant. In practice, it is convenient to write this real constant as either λ^2 $or -\lambda^2$. Therefore equation (6) can be written as

$$TZ^1 - \frac{a(1-\varepsilon)}{Vo} T^1 Z = 0 ……..……………… (7)$$

Equation (7) can also be written as

$$TZ^1 - \frac{a(1-\varepsilon)}{Vo} T^1 Z = \lambda^2 …………………… (8)$$

From equation (8)) we have

$$TZ^1 = \lambda^2 \quad ……………. . (9)$$

$$\frac{a(1-\varepsilon)}{Vo} T^1 Z = \lambda^2 \quad ……….. (10)$$

From equation (9) we have

$$TZ^1 = \lambda^2 \quad ……………… (11)$$

Divide left Hand side through equation (10) by *TZ*

$$\frac{TZ^1}{TZ} = \lambda^2$$

$$\frac{Z^1}{TZ} = \lambda^2$$

$$\boxed{Z^1 = \lambda^2 z = 0} \quad ……… (12)$$

Applying Laplace Transformation into equation (12) we have

$$Z^1 = Sz(s) - Z(o) \quad …………………. (13)$$

$$Z = Z(s) \quad …………………. (14)$$

Substituting equation (13) and (14) into (12) we have

$$Sz(s) - Z(o) - \lambda^2 z(s) = 0 ……… (15)$$

Considering the following boundary conditions at Z(0) = Z_0

SZ(s) – Zo - λ^2 Z(s) = 0 ……….. (16)

$$Sz(s) - \lambda^2 Z(s) = Zo \quad ……….. (17)$$

$$(S-\lambda^2)\, Z(s) = Zo \quad ………… (18)$$

$$Z(s) = \frac{Zo}{S-\lambda^2} \quad ……………. (19)$$

Converting equation (19) into inverse Laplace we have

$$S-\lambda^2 = 0$$

$S-\lambda^2$ Thus,

$$\boxed{Zt = Zo\ell\lambda^{2_t} t} \quad …….(20$$

$$Z(z) = Zo\ell\lambda^{2_z}$$

From equation (10) $\frac{a(1-\varepsilon)}{Vo} T^1 Z = \lambda^2$……. (21)

Dividing the left hand side of equation (21) by *TZ*

$$\frac{a(1-\varepsilon)}{Vo} \frac{T^1 Z}{TZ} = \lambda^2$$

$$\frac{a(1-\varepsilon)}{Vo} \frac{T^1}{T} = \lambda^2 \quad ………… (22)$$

Let $\beta = \frac{a(1-\varepsilon)}{Vo}$, therefore equation (22) can be written as

$$\beta \frac{T^1}{T} = \lambda^2$$

$$\beta T^1 = \lambda^2 T$$

$$\beta T^1 - \lambda^2 = 0$$

$$\beta[ST(s) - T(o)] - \lambda^2 T(s) = 0$$

Assuming $T(o) = To$

$$\beta[ST(s) - To] - \lambda^2 T(s) = 0$$

$$\beta ST(s) - \lambda^2 T(s) = \beta To$$

$$(\beta s - \lambda^2)T(s) = \beta To$$

$$T(s) = \frac{\beta To}{\beta s - \lambda^2} \quad ………… (23)$$

Converting equation (23) into inverse Laplace we have

$$\boxed{T(t) = \frac{(\beta To\ell\, \beta\lambda^2)^t}{\beta}} \quad … (24)$$

Substituting equation (20) and (22) into equation (3) we have

$$C = \beta To\ell\, \lambda^{2_t} \big/ \beta \,(Zo\ell\lambda^{2_t})$$

$$C = \beta ToZo\ell^{\left(\lambda^2/\beta + \lambda^2\right)^t}$$

$$C = \beta ToZo\ell^{\left(\lambda^2/\beta + \lambda^2\right)^t}$$

$$\boxed{C = \beta ToZo\, \ell\left(\frac{\lambda^2 + \beta\lambda^2}{\beta}\right)^t} \quad ……… (25)$$

Equation (25) defines the behavior of the system in terms of time dependent only but considering the change in concentration in respect to *Z* Coordinate as well as time dependent we have

$$C = \beta To\ell\, \lambda^2 \big/ \beta^t (Zo\ell\lambda^{2_z}) \quad …………………. (26)$$

$$C_{(t)} = \beta C_o\, \ell^{\left(\frac{\lambda^2 + \beta\lambda^2}{\beta}\right)^t} \quad …………………. (26b)$$

In terms of

$$C_{(v)} = \beta C_o\, \ell^{\left(\frac{\lambda^2 + \beta\lambda^2}{\beta}\right)\frac{d}{v}} \quad ……………… (27)$$

$$C_{(t)} = \beta T_o Z_o\, \ell^{\left(\frac{\lambda^2 t}{} + \lambda^2 \frac{v}{t}\right)} \quad ……………… (28)$$

Since $C = TZ$

$$C_t = \beta T_o Z_o\, \ell^{\left(\frac{\lambda^2 t}{} + \lambda^2 \frac{v}{t}\right)} \quad ……………… (29)$$

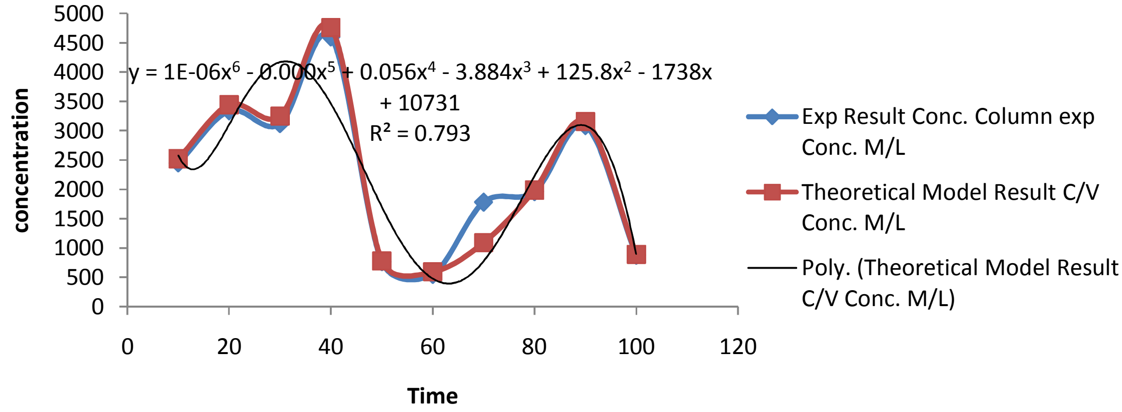

Fig. 1: Comparison of theoretical model value influence by porosity with column experiment Result versus Time

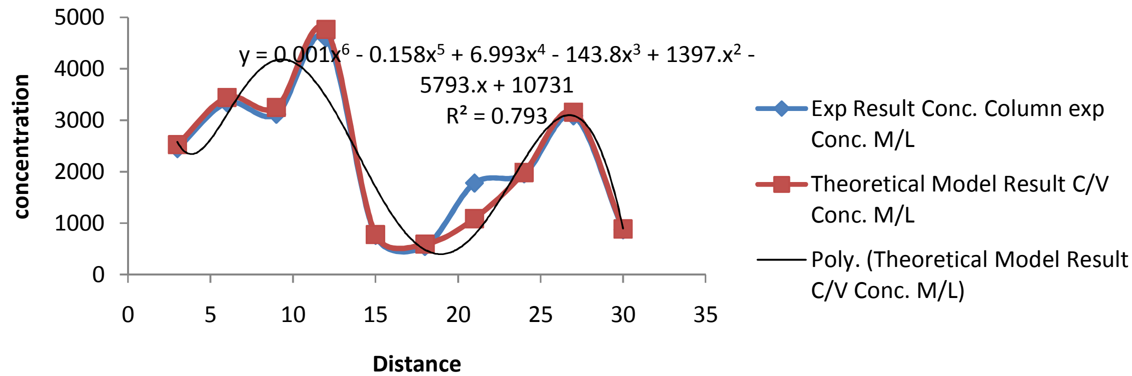

Fig. 2: Comparison of theoretical model value influence by porosity with column experiment result versus Distance

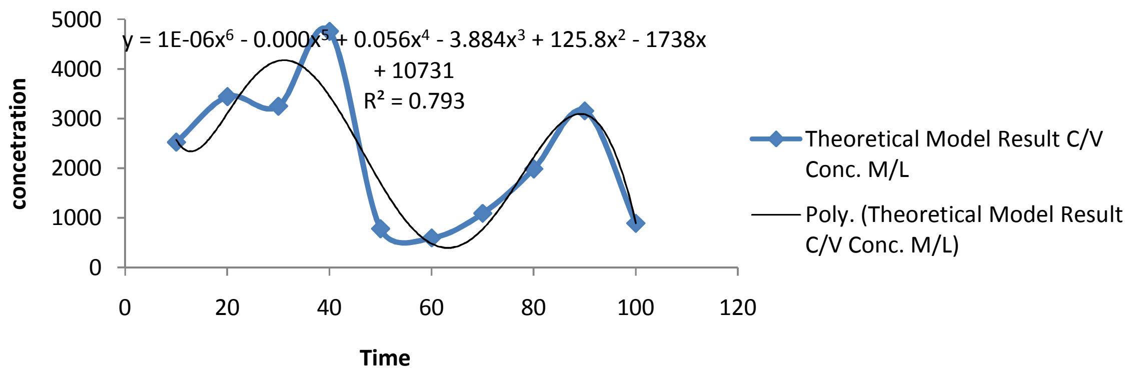

Fig. 3. Concentration of E. coli at various Times

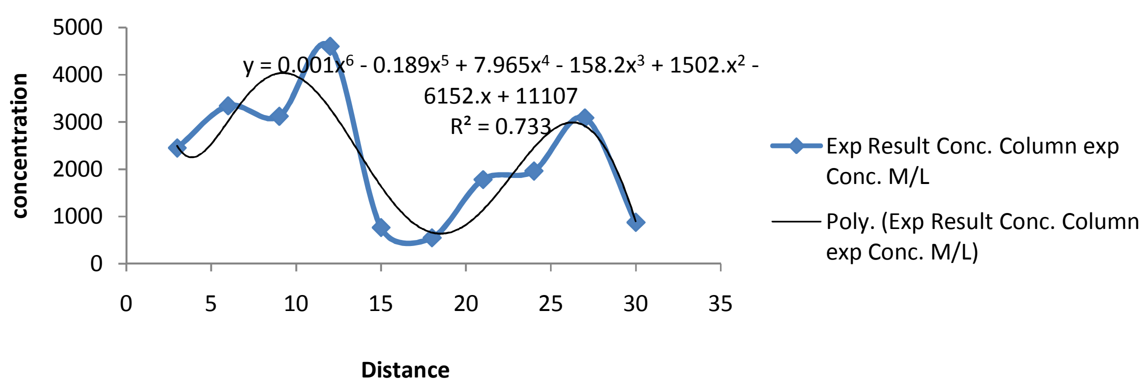

Fig. 4: Concentration of E. coli at various Distances

Table 1. Concentration of column at various distances

Time	Exp Result Conc. Column exp Conc. M/L
10	2449
20	3340
30	3118
40	4600
50	765
60	548
70	1780
80	1960
90	3085
100	875

Table 2. Concentration of column at various distances

Distance	Exp Result Conc. Column exp Conc. M/L
3	2449
6	3340
9	3118
12	4600
15	765
18	548
21	1780
24	1960
27	3085
30	875

Table 3. Concentration of column at various distances

Distance	Exp Result Conc. Column exp Conc. M/L	Theoretical Model Result C/V Conc. M/L
3	2449	2524.5
6	3340	3443.4
9	3118	3250.28
12	4600	4760.03
15	765	780.64
18	548	595.5
21	1780	1091.08
24	1960	1985.76
27	3085	3156.7
30	875	890.2

Table 4. Concentration of column at various distances

Time	Exp Result Conc. Column exp Conc. M/L	Theoretical Model Result C/V Conc. M/L
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50	765	780.64
60	548	595.5
70	1780	1091.08
80	1960	1985.76
90	3085	3156.7
100	875	890.2

From the Figure 1 presented it shows that the concentration of *E.coli* in an oscillation form increase with time to a point where an optimum value were recorded at fifty days, suddenly a decrease with time was observed at sixty and hundred days , while that of the column experiment maintained the same form of concentration it obtained its optimum value at the same fifty days, suddenly decrease at the same time like that of the theoretical value this explained the of level influence with respect to duration the microbes can migrate to ground water aquiferious zone, the fast migration of this microbes explained the influence of porosity, base on the geological formation on the deltaic environment at the study area, the contaminants deposited a high level of concentration, this shows that the study area deposited some high level of microbes *E.coli which* has generated a lots of pollution and cause a lots death, both parameters produce the best fit line equation displayed on the graph.

The Figure 2 shows that the concentration increase with increase in distance to a point where an optimum value were recorded at fifteen metres, suddenly a decrease in distance were observed at thirty metres but with some level of deposition of microbes, while that of column experiment maintained the same level of concentration where it obtained its optimum level at fifteen metres suddenly a decrease in distance were also observed at thirty metres depositing some level microbes like that of the theoretical value, the comparison with the experimental result fit in as it is presented in the figure, both result produced the same best fit line equation displayed on the graph. Figure 3 presented shows that the concentration increase in a fluctuation and obtained it optimum value at fifty days and suddenly drops down with increase in time at thirty metres , this can be attributed to the level of geological deposition as for seen in the figure, it can also be attributed to environmental factors base on the level of indiscriminate dumping of waste dump,the dumping of biological waste at the study area may have accumulated and generate high deposition of microbes migration influenced by high level of porosity, the parameters produced the best fit line equation displayed on the graph. The Figure 4 shows the concentration increase with distance in fluctuation form to a point where an optimum value where recorded at fifteen metres suddenly, a decreased with increase in distance where observed at thirty metres with some level concentration of microbes, this explained the variation of deposition of stratum influence by the micro pores variation, including the degree of saturation influence by high level of rain intensities in deltaic location from the study area. It produced the best fit line equation displayed on the graph.

Conclusions

From every point of indication, the influence of porosity has a maximum influence in the transport of E coli at the study area, as for seen from the developed model and the experimental result presented in the figure, the concentration with respect to time and distance in the developed theoretical value and experimental result has shows the level of the microbes migration at short distance of thirty metres and short duration of hundred days, it has proof the rapid migration of microbes at the study area influence by high variation of porosity, including the geological deposition. The experimental result explained the rapid migration of the microbes in a physical process and from the figure presented, it has shows the variation level of concentration at different aquifer at various distances which has detailed the dynamic migration with respect to distance and time, since the study location from the findings has generated this explained result. It will be a benchmark for the abstractions of ground water development and design parameter, including mathematical model to develop for design of high quality ground water for human consumption at the study area. Finally the result has shown that for ground water to be abstracted in the study area must be design, following this model and result finding in the study area, in other to stop the rapid increase of water related diseases and threat of life.

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