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RESEARCH ARTICLE

STEADY FLOWS IN PIPES OF RECTANGULAR CROSS-SECTION THROUGH POROUS MEDIUM WITH MAGNETIC FIELD

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ABSTRACT

In this paper we have investigated the steady flow in pipes of rectangular cross-section through porous medium with magnetic field. We have investigated the velocity, flux and vortex line.

INTRODUCTION

We have investigated the steady flow in pipes of rectangular cross-section through porous medium. Attempts have been made by several researchers i.e. **Givler and Altobelli (1994)** a determination of the effective viscosity for the Brinkman-Forchheimer flow model. **Goel and Agarwal (1998)** shears flow instability of visco- elastic fluid in a porous medium. **Goldstein (1930)** concerning some solutions of the boundary-layer equations in hydrodynamics.

Gopinath (1994) steady streaming due to small amplitude superposed oscillations of a sphere in a viscous fluid. **Gorski and Bernard (1995)** Vorticity Transport Analysis of turbulent flows. Journal of Fluids Engng. **Goyeau and Gobin (1996)** Numerical study of double-diffusive natural convection in a porous cavity using the Darcy-Brinkman formulation. **Goyon (1996)** high-Reynolds number solutions of Navier-stokes equations using incremental unknowns. **Grigoriev and Dargush (1999)** a poly-region boundary element method for incompressible viscous fluid flows.

Grosan, Revnic, **Pop and Ingham (2009)** Magnetic field and internal heat generation effects on the free convection in a rectangular cavity filled with a porous medium**. Gupta and Manohar (1979)** Boundary approximations and accuracy in viscous flow computations. **Gupta (1991)** high accuracy solutions of incompressible Navier-stokes equations. **Haddad and Corke (1998)** Boundary layer receptivity to free-stream sound on parabolic bodies. **Haddon and Riley (1985)** on flows with closed streamlines. **Haji-Sheikh and Vafai (2004)** analysis of flow and heat transfer in porous media imbedded inside various-shaped ducts. In this paper we have investigated the velocity, flux and vortex line.

FORMULATION OF THE PROBLEM

Let z-axis be taken the direction of flow along the axis of the pipe. Then u = 0, v = 0 for steady and incompressible fluid the velocity component is independent of **z**.

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The equation of continuity. $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ ....(1)

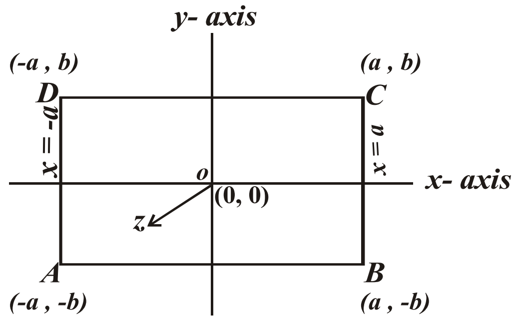
$$But \quad u=0,\ v=0,\ \frac{\partial w}{\partial z}=0 \Rightarrow w=w(x,y).......(2)$$

i.e. w is independent of **z**

The Navier-Stokes equations of motion in the absence of body forces.

$$-\frac{\partial P}{\partial x}=0 \ ....(3)$$

$$-\frac{\partial P}{\partial y}=0.........(4)$$



$$\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\left(\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\mu}\right)\mu w=0...........(5)$$

Figur-1

It is clear from (3) and (4) **P** is independent of **x** and **y** i.e. **p** is the Function of **z**

SOLUTION OF THE PROBLEM p = p (z) $\frac{\partial p}{\partial z}=\frac{dp}{dz}=\text{Constant}=-\text{P}$

$$let \ \left(\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\mu}\right)=B^2 \ , \ \mu\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-B^2 w\right]=\frac{dp}{dz} \Rightarrow \frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-B^2 w=-\frac{P}{\mu}.............(6)$$

$\left(D^2+D'^2-B^2\right)w=-\frac{P}{\mu} \ \therefore \ C.F=\sum a_n\, e^{h_n x+h'_n y}$ where h_n & h'_n are related by $h_n^{\,2}+h'^2_n-B^2=0$

and $P.I.=\frac{1}{D^2+D'^2-B^2}\left(-\frac{P}{\mu}\right)=\frac{P}{B^2\mu} \Rightarrow$ w (x, y) = $\sum_{n=1}^{\infty} a_n\, e^{h_n x+h'_n y}+\frac{1}{B^2\mu}P$ Where $h_n^2+h'^2_n=B^2$

Case-1 $w\ (x,\ y)\ =\ 0\ at\ (a,\ b)\ w\ (x,\ y)\ =\ 0\ at\ (a,-b)$

$$\sum_{n=1}^{\infty} a_n\ e^{h_n a+h'_n b}+\frac{P}{\mu B^2}=0 \qquad and \qquad \sum_{n=1}^{\infty} a_n\ e^{h_n a-h'_n b}+\frac{P}{\mu B^2}=0$$

$$\Rightarrow -\frac{P}{\mu B^2}=\sum_{n=1}^{\infty} a_n\ e^{h_n a+h'_n b}.........(a) \ \& \ -\frac{P}{\mu B^2}=\sum_{n=1}^{\infty} a_n\ e^{h_n a-h'_n b}..........(b)$$

$$\text{on solving } h_n' = 0 \;\Rightarrow\; h_n = -B \;\Rightarrow\; -\frac{P}{\mu B^2} = e^{-aB}\sum_{n=1}^{\infty} a_n \;\Rightarrow \sum_{n=1}^{\infty} a_n = e^{-\frac{P}{\mu B^2}} e^{-aB}$$

$$w_1(x,y) = -\frac{P}{\mu B^2}\; e^{aB}\; e^{-xB} + \frac{P}{\mu B^2} \;\Rightarrow\; -\frac{P}{\mu B^2} = e^{-aB}\sum_{n=1}^{\infty} a_n \;\Rightarrow \sum_{n=1}^{\infty} a_n = e^{-\frac{P}{\mu B^2}} e^{-aB}$$

Case -2 $w\,(x,\, y) = 0 \quad at\ (-a,\, b)\ \&\ (-a,\, -b)$

$$w_2(x,y) = -\frac{P}{\mu B^2}\; e^{aB}\; e^{xB} + \frac{P}{\mu B^2} = -\frac{P}{\mu B^2}\; e^{B(x+a)} + \frac{P}{\mu B^2}$$

Case - 3

$$w\,(x,\, y) = 0 \text{ at } (-a,\, b)\ \&\ (a,\, b) \Rightarrow w_3(x,y) = -\frac{P}{\mu B^2}\; e^{bB} e^{-yB} + \frac{P}{\mu B^2} = -\frac{P}{\mu B^2}\; e^{B(-y+b)} + \frac{P}{\mu B^2}$$

Case - 4 $w\,(x,\, y) = 0 \text{ at } (-a,\, -b)\ \&\ (a,\, -b) \Rightarrow w_4(x,y) = -\frac{P}{\mu B^2}\; e^{B(y+b)} + \frac{P}{\mu B^2}$

$$w(x,y) = \frac{P}{\mu B^2}\left[1 - 2\; e^{aB}\; Cosh\, x B - 2\, e^{bB}\; Cosh\, y B\right] \text{.............. (7)}$$

In particular case: In the case of square i.e $.a = b$

$$w(x,y) = \frac{P}{\mu B^2}\left[1 - 2\; e^{aB}\left(Cosh\, x B + Cosh\, y B\right)\right] \text{............. (8)}$$

Flux Q of the fluid over an area of rectangular cross-section:

$$Q = \int_{x=-a}^{a}\int_{y=-b}^{b} w(x,y)\; dx\; dy = \int_{-a}^{a}\int_{-b}^{b} \frac{P}{\mu B^2}\left\{1 - 2\; e^{aB}\; Cosh\, x B - 2\; e^{bB} Cosh\, y B\right\} dx\; dy$$

$$= \frac{2P}{\mu B^2}\int_{-a}^{a}\int_{0}^{b}\left\{1 - 2\; e^{aB} Cosh\, x B - 2\; e^{bB} Cosh\, y B\right\} dy\; dx = \frac{2P}{\mu B^2}\int_{-a}^{a}\left\{\left(1 - 2\; e^{aB} Cosh\, x B\right) b - 2\; e^{bB}\left(\frac{1}{B}\; Sinh\, b B\right)\right\} dx$$

$$= \frac{4P}{\mu B^2}\int_{0}^{a}\left\{b\left(1 - 2\; e^{aB}\; Cosh\, x B\right) - \frac{2}{B}\; e^{bB}\; Sinh\, b B\right\} dx = \frac{4P}{\mu B^2}\left[b\left\{x - \frac{2}{B}\; e^{aB}\; Sinh\, x B\right\}_0^a - \frac{2a}{B}\; e^{bB}\; Sinh\, b B\right]$$

$$= \frac{4P}{\mu B^2}\left[b\left\{a - \frac{2}{B}\; e^{aB}\; Sinh\, a B\right\} - \frac{2a}{B}\; e^{bB}\; Sinh\, b B\right]$$

$$Q = \frac{4P}{\mu B^2}\left[ab - \frac{2}{B}\left(b\; e^{aB}\; Sinh\, a B + a\; e^{bB}\; Sinh\, b B\right)\right] \text{ .... (9)}$$

In particular case: In the case of square $a = b$

$$Q = \frac{4P}{\mu B^2}\left[a^2 - \frac{4a}{B}\ e^{aB}\ Sinh\, a\, B\right] \ .... \ (10)$$

The equation of vortex line: $\frac{dx}{\Omega_x} = \frac{dy}{\Omega_y} = \frac{dz}{\Omega_z}$ **here** $\Omega_x,\ \Omega_y\ \&\ \Omega_z$ **are vorticity components**

$$where\ \ \bar{q} = ui + vj + wk\ = \frac{P}{\mu B^2}\left[1 - 2\ e^{aB}\ Cosh\, x\, B - 2\ e^{bB}\ Cosh\, y\, B\right]\hat{k}$$

$$\Omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = \frac{P}{\mu B^2}\left[-2B\ e^{bB}\ Sinh\, y\, B\right]\ = -\frac{2P}{\mu B}\ e^{bB}\ Sinh\, y\, B$$

$$\Omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = -\frac{P}{\mu B^2}\left[-2B\ \ e^{aB}\ Sinh\, x\, B\right] = \frac{2P}{\mu B}\ \ e^{aB}\ Sinh\, x\, B \qquad \Omega_z = 0$$

$$\Rightarrow\ \ \frac{dx}{-\frac{2P}{\mu B}\ e^{bB}\ Sinh\, y\, B} = \frac{dy}{\frac{2P}{\mu B}\ e^{aB}\ Sinh\, x\, B} = \frac{dz}{0} \Rightarrow dz = 0 \quad \Rightarrow \quad z = B$$

$$\frac{dx}{-e^{bB}\ Sinh\, y\, B} = \frac{dy}{e^{aB}\ Sinh\, x\, B} \Rightarrow\ e^{aB}\int Sinh\, x\, B\, dx\ +\ e^{bB}\int Sinh\, y\, B\, dy = C_1$$

$$\frac{1}{B}\ e^{aB}\ Cosh\, x\, B + \frac{1}{B}\ e^{bB}\ Cosh\, y\, B = C_1 \quad \text{or} \quad e^{aB}\ Cosh\, x\, B + e^{bB}\ Cosh\, y\, B = C_1\, B = A$$

$\therefore$ **Vortex lines:** $e^{aB}\ Cosh\, x\, B +\ e^{bB}\ Cosh\, y\, B = A \qquad \& \qquad Z = B$ .... (11)

Clearly the flow is Rotational in pipe.

I
n particular case: In the case of square $a = b$

$$e^{aB}\ \left[Cosh\, x\, B +\ Cosh\, y\, B\right]\ = A \qquad \& \qquad Z = B \ .... \ (12)$$

Tables for velocity: Case- 1

$$Let\ \ P = \frac{1}{4},\ \ \mu = .5,\ \ a = b = 1,\ \frac{1}{\sqrt{\sigma K}} = \sqrt{\frac{\sigma B_0^2}{\rho\mu}} = \frac{1}{2} \Rightarrow B = \sqrt{\frac{1}{\sigma K} + \frac{\sigma B_0^2}{\rho\mu}} = \frac{1}{\sqrt{2}}$$

Table 1 (for velocity)

	(x,y)	(.1, .1)	(.2, .3)	(.3, .4)	(.4, .5)	(.5, .6)	(.6, .7)	(.7, .8)
$\frac{1}{\sqrt{\rho K}}=\frac{1}{2}$	$w(x,y)$	**-11.21**	**-11.297**	**-11.396**	**-11.529**	**-11.696**	**-11.897**	**-12.133**
$\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{2}$	$w(x,y)$	**-11.21**	**-11.297**	**-11.396**	**-11.529**	**-11.696**	**-11.897**	**-12.133**
$\sqrt{\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{\sqrt{2}}$	$w(x,y)$	**-7.133**	**-7.245**	**-7.367**	**-7.532**	**-7.739**	**-7.99**	**-8.286**

Case- 2

$$Let \quad P=\frac{1}{4}, \quad \mu=.5, \quad a=b=1, \quad \frac{1}{\sqrt{\sigma K}}<\sqrt{\frac{\sigma B_0^2}{\rho\mu}} \quad Let \quad \frac{1}{\sqrt{\sigma K}}=\frac{1}{2} \; and \; \sqrt{\frac{\sigma B_0^2}{\rho\mu}}=1 \;\Rightarrow\; B=\sqrt{\frac{1}{\sigma K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{\sqrt{5}}{2}$$

Table- 2 (for velocity)

	(x,y)	(.1, .1)	(.2, .3)	(.3, .4)	(.4, .5)	(.5, .6)	(.6, .7)	(.7, .8)
$\frac{1}{\sqrt{\rho K}}=\frac{1}{2}$	$w(x,y)$	**-11.21**	**-11.297**	**-11.396**	**-11.529**	**-11.696**	**-11.897**	**-12.133**
$\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=1$	$w(x,y)$	**-4.964**	**-5.114**	**-5.28**	**-5.504**	**-5.788**	**-6.134**	**-6.547**
$\sqrt{\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{\sqrt{5}}{2}$	$w(x,y)$	**-4.525**	**-4.695**	**-4.882**	**-5.135**	**-5.458**	**-5.854**	**-6.328**

Case –3

$$Let \quad P=\frac{1}{4}, \quad \mu=.5, \quad a=b=1, \quad \frac{1}{\sqrt{\sigma K}}>\sqrt{\frac{\sigma B_0^2}{\rho\mu}} \quad Let \quad \frac{1}{\sqrt{\sigma K}}=1 \; and \; \sqrt{\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{2} \;\Rightarrow\; B=\sqrt{\frac{1}{\sigma K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{\sqrt{5}}{2}$$

Table- 3 (for velocity)

	(x,y)	(.1, .1)	(.2, .3)	(.3, .4)	(.4, .5)	(.5, .6)	(.6, .7)	(.7, .8)
$\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{2}$	$w(x,y)$	**-4.964**	**-5.114**	**-5.28**	**-5.504**	**-5.788**	**-6.134**	**-6.547**
$\frac{1}{\sqrt{\rho K}}=1$	$w(x,y)$	**-11.21**	**-11.297**	**-11.396**	**-11.529**	**-11.696**	**-11.897**	**-12.133**
$\sqrt{\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{\sqrt{5}}{2}$	$w(x,y)$	**-4.525**	**-4.695**	**-4.882**	**-5.135**	**-5.458**	**-5.854**	**-6.328**

CONCLUSION AND DISCUSSION

In this paper we have investigated the velocity by the **Table-1** of equations (7) between velocities and point (x, y) it is clear that paths of velocity are approximately parallel increases uniformly with negative sign in porous medium, magnetic field and porous medium with magnetic field in the interval $(.1,.1) \leq (x, y) \leq (.7,.8)$ but the value of velocity in porous medium and magnetic field at $\frac{1}{\sqrt{\rho K}} = \sqrt{\frac{\sigma B_0^2}{\rho \mu}} = \frac{1}{2}$ is greater than the corresponding value of velocity in porous medium with magnetic field at $\sqrt{\frac{1}{\rho K} + \frac{\sigma B_0^2}{\rho \mu}} = \frac{1}{\sqrt{2}}$ in the interval $(.1,.1) \leq (x, y) \leq (.7,.8)$.

Again by the **Table-2** of equations (7) between velocities and point (x, y) it is clear that paths of velocity are approximately parallel increases uniformly with negative sign in porous medium, magnetic field and porous medium with magnetic field in the interval $(.1,.1) \leq (x, y) \leq (.7,.8)$ but the value of velocity in porous medium at $\frac{1}{\sqrt{\rho K}} = \frac{1}{2}$ is greater (negatively) then corresponding value of velocity in magnetic field at $\sqrt{\frac{\sigma B_0^2}{\rho \mu}} = 1$ and also is greater than the corresponding value of velocity in porous medium with magnetic field at $\sqrt{\frac{1}{\rho K} + \frac{\sigma B_0^2}{\rho \mu}} = \frac{\sqrt{5}}{2}$ respectively.

Again by the **Table-3** of equations (7) between velocities and point (x, y) it is clear that paths of velocity are approximately parallel increases uniformly with negative sign in porous medium, magnetic field and porous medium with magnetic field in the interval $(.1,.1) \leq (x, y) \leq (.7,.8)$ but the value of velocity in magnetic field at $\sqrt{\frac{\sigma B_0^2}{\rho \mu}} = \frac{1}{2}$ is greater (negatively) then corresponding value of velocity in porous medium at $\frac{1}{\sqrt{\rho K}} = 1$ and also is greater than the corresponding value of velocity in porous medium with magnetic field at $\sqrt{\frac{1}{\rho K} + \frac{\sigma B_0^2}{\rho \mu}} = \frac{\sqrt{5}}{2}$ respectively. We have investigated the vortex lines and the volumetric flow of elliptic and circle given by the equations (8), (9), (10), (11) and (12) respectively.

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