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## RESEARCH ARTICLE

### INFLUENCE OF THE SIMULTANEOUS TRANSFER OF HEAT AND MASS ON THE MEASURE OF MATERIALS THERMAL CONDUCTIVITY

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#### ABSTRACT

The present work gives the current situation of the methods developed to measure the thermal conductivity of building materials used in Benin, country of Western Africa. Modus operandi, basic theories and experimental results of the case of cement mortar were explicitly reminded. The thermal conductivity of the cement mortar is obtained on average at the value of  $1.15\text{Wm}^{-1}\text{K}^{-1}$ . Then, the study focused on the influence of the simultaneous heat and mass transfer on these measures. Indeed, the methods of measurements participate, during their implementation, to a discrete but real drying of the material.

To go further, a model of answer to one level perturbation was then proposed and has predicted the value of thermal conductivity of the cement mortar about  $0,799\text{Wm}^{-1}\text{K}^{-1}$ . Therefore the double transport of heat and mass during the measurement of the thermal conductivity has an important influence about 30 %. The final analysis of this paper has justified relationships between properties and performance of materials.

#### INTRODUCTION

The study of the properties of mass transfer presents some difficulties due to their heterogeneous and porous structure and to the relative but unavoidable simultaneous transfer of heat and mass. For a better use of the porous materials in construction specially, it is essential to have a perfect knowledge of their physical properties, in particular the thermal conductivity which governs their insulation power. The experimental measurement of the thermal conductivity is made by several methods such as "the method of the maintained hot plate", "the method of boxes", "the method of comparison or standard materials in steady state", "the method of hot thread", "the method of movie warming plan", etc. For example, (Laaroussi *et al.*, 2013) conducted an experimental study in order to characterize the thermal properties of a sample coming from the Moroccan Slaoui's factory used to manufacture the bricks. The estimate error on the temperature due to the measurement was found less than 3% and a good consistency has been obtained between the experimental measurements and the model proposed by the authors. Concerning both local construction materials from Benin that

are the composite cement-wood and the cement mortar, campaigns of measurements of physical properties realized in Ecole Polytechnique, Abomey-Calavi, must be exploit enough to make prediction. This prediction, to be precise, has to take into account the fact that, during the measurement of conductivity, a transfer of mass comes to overlap the transfer of heat, because materials are absorbent. It exists in the literature numerous models interpreting and predicting the mechanisms of simultaneous transfer of heat and moisture. These models vary according to the factors which govern the phenomena of transfer and also according to the type of materials. It's thus convenient to find a corrective model able to predict and quantify the influence of the variation of moisture on the profile of thermal conductivity for the composite cement-wood and cement mortar which are porous and hydrophilic materials. It is this important lack which the present work tries to fill.

#### MATERIALS AND METHODS

**The thermal conductivity: a physical property of major interest**

Thermo physic parameter characterizing the behavior of materials during the thermal transfer by conduction, the thermal conductivity of building materials is object of

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development research. Indeed, the research for a minimum of thermal comfort inside houses with a lower cost imposes more and more precise methods of measurement of the thermal conductivity of construction materials (Laaroussi *et al.*, 2013). The thermal conductivity  $\lambda(T)$ , is function, among others, of the temperature  $T$  of the material. The thermal conduction being a spontaneous thermal transfer from a higher temperature region towards a lower temperature region, it obeys one of the phenomenological laws of transport, the Fourier' law, established mathematically by Jean-Baptiste Biot in 1804 and there after experimentally by Fourier in 1822. This law precise that the density of flow of heat is proportional to the gradient of temperature,  $\varphi = -\bar{\lambda} \text{grad } T$  with:  $\bar{\lambda}$  tensor of thermal conductivity in  $\text{W.m}^{-1}.\text{K}^{-1}$ ,  $\varphi$  vector of heat flowdensity in  $\text{W.m}^{-2}$  and  $T$  temperature, in K. The thermal conductivity thus represents the quantity of heat transferred by unit area and during a unit of time under a gradient of temperature equal to one degree per meter. It depends on the type of the medium (porosity, hygrophilous, composition, etc.) and on its temperature.

#### The two local construction materials tested: composite cement-wood and cement mortar

The composite cement-wood is a manufactured product obtained from wood particles or wood fibers to which is added cement as the binding material. Composites cement-wood are obtained by pressing of the mixture. The composite cement-wood used in this work (Allognon, 2007) is in proportion 8,8kg of sawdust, 10kg of cement and 15kg of water. The cement mortar with sand is a mixture of cement, mineral grains of sand and water for mixing. The cement mortar is very resistant, sets and quickly hardens. The volume tric proportion of cement and sand is generally of 1 for 3 and the ratio water by cement is approximately 0.35. Furthermore, a good dosage in cement makes the mortar practically waterproof.

#### The so-called method of comparison for the measure of the thermal conductivity in steady state

The experimental methods of determination of the thermal conductivity are diverse as the regime is permanent or transient. In the practice, the measurement in steady state is the one which holds our attention. Indeed, except for some rare abrupt changes of temperature and humidity conditions, the walls of buildings made in composite cement-wood or in cement mortar cement remain enough for a long time exposed to the ambient conditions that we can validly accept measure in steady state as significant one for the reality: the cement-wood or cement mortar medium is in thermal equilibrium and it is submitted to a permanent flow according to time. So, using the Fourier' law and the boundary conditions, the thermal conductivity is determined by certain methods in steady state such as "the method of the maintained hot plate", "the method of boxes", "the method of movie warming plan", "the method of comparison or standard materials in steady state", "the method of hot thread", etc. The method of "comparison or standard materials in steady state" was used. It is founded upon the determination of the thermal conductivity of a material knowing that of another reference material. We impose on the free side of the studied material a constant temperature  $T_2$

whereas the free side of the reference material is at a temperature  $T_1$ , both materials being attached together by resin.

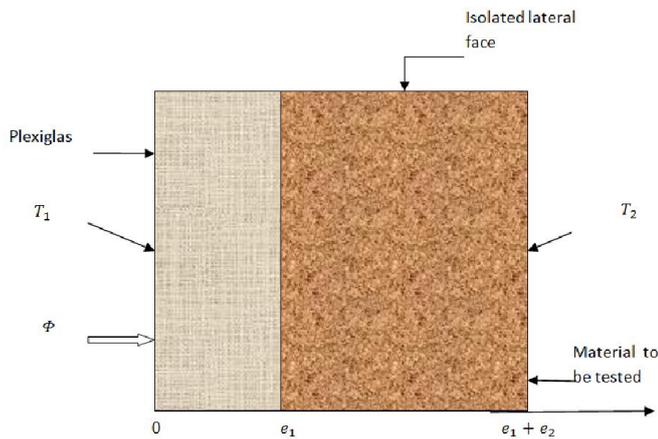


Fig. 1. The complete instrumentation of measurement of thermal conductivity by the comparison method (Allognon, 2007)



Fig 2. Tested samples (Houngan, 2008)

The measurement of the temperatures on the free sides and in the interface in steady state, allows computing the thermal conductivity of the material. The method allows making measurements of conductivity of all the solid materials used in the building industry, except metals. It requires among others things a low experiment time. It happens that we reach the steady state after 6 hours. See Fig 1 and Fig 2. Referring to the Fig.3, both materials have the same base area and respective thickness  $e_1$ ,  $e_2$ .



**Fig. 3. Scheme of the principle of the experimental instrumentation of the comparison method**

The assumptions formulated for this method are:

- The transfer of heat is supposed to be unidirectional,
- The temperatures on the faces ( $x=0$ ) and ( $x_c = e_1 + e_2$ ) are known,
- The sides of the sample are isolated,
- Both materials are isotropic and homogeneous,
- The thermal resistance due to the contacts is negligible.

By using the Fourier's law when the second order tensor  $\bar{\lambda}$  is reduced to a scalar  $\lambda$  for an isotropic medium, we obtain successively:

$$\vec{\varphi} = -\lambda \text{grad } T \quad (1)$$

and the equation of the heat, where P is the term which express the production of power, is:

$$\frac{\rho c}{\lambda} \frac{\partial T}{\partial t} - \Delta T = \frac{P}{\lambda} \quad (2)$$

As there is no power production within the material we have:

$$\frac{\rho c}{\lambda} \frac{\partial T}{\partial t} - \Delta T = 0 \quad (3)$$

In steady state, equation (3) becomes:

$$\Delta T = 0 \quad (4)$$

We find as solution:

$$\text{Control material (Plexiglas)}: \frac{\partial^2 T}{\partial x^2} = 0; T(0, t) = T_1 \quad (5)$$

$$\text{Composite cement-wood}: \frac{\partial^2 T'}{\partial x^2} = 0; T'(e_1 + e_2, t) = T_2 \quad (6)$$

By taking into account boundary conditions, the resolution of these equations gives:

$$T(x) = -\frac{\varphi}{\lambda_1} x + T_1 \text{ pour } 0 \leq x \leq e_1 \quad (7)$$

$$T'(x) = -\frac{\varphi}{\lambda_2} [x - (e_1 + e_2)] + T_2 \text{ for } e_1 \leq x \leq e_1 + e_2 \quad (8)$$

If we call  $T_i$  the temperature on the interface of both plates ( $x = e_1$ ), we can write:

$$(T_1 - T_i) \frac{\lambda_1}{e_1} = (T_i - T_2) \frac{\lambda_2}{e_2} \quad (9)$$

Relation (9) allows computing  $\lambda_2$  the thermal conductivity of the composite of cement-wood. In a general way if we can determine the gradients of temperature in every plate, respectively  $(\text{grad}T)_1$  et  $(\text{grad}T)_2$  we can write the condition of preservation of the flow in steady state:

$$\lambda_1 (\text{grad}T)_1 = \lambda_2 (\text{grad}T)_2 \quad (10)$$

From which we get:

$$\lambda_2 = \frac{\lambda_1 (\text{grad}T)_1}{(\text{grad}T)_2} \quad (11)$$

So, we proceed to temperature readings in two points of every sample and distant respectively of  $d_1$  and  $d_2$ . The values of  $(\text{grad}T)_1$  and  $(\text{grad}T)_2$  are approximated by  $\frac{(\Delta T)_1}{d_1}$  and  $\frac{(\Delta T)_2}{d_2}$ .

#### A few experimental results in the case of the cement mortar

The measurements made on the cement mortar, with the Plexiglas as the reference material (density =  $1190 \text{ kg.m}^{-3}$ ) and thermal conductivity =  $0.184 \text{ W.m}^{-1}$ , are in the Table 1. Let us remind that the thermal conductivity depends on the temperature. It is thus relevant to take into account the inevitable temperature variation during the measurement of conductivity, so weak is this variation. In fact, it means taking into account the simultaneous heat and mass transfer. According to (Bal *et al.*, 2003), the apparent density of samples varying between  $1.950 \text{ kg.m}^{-3}$  for the pure laterite dry block and  $1.180 \text{ kg.m}^{-3}$  for the dry block with a maximum millet mass content  $0.122 \text{ kg/kg}$ , the corresponding values of the thermal conductivities varied between  $1.4$  and  $0.29 \text{ W.m}^{-1}.\text{K}^{-1}$ . This result also demonstrates the interest of adding millet waste for example to lower the thermal conductivity of some materials.

#### Analysis

Influence of the simultaneous transfer of heat and mass on the measure of the thermal conductivity, physical fundamentals. From a physical point of view, the modeling of a simultaneous heat and mass transferred to a system of strongly non linear and coupled equations. The transfer of heat by conduction is formulated by the relation  $\vec{\varphi} = -\bar{\lambda} \text{grad } T$  where the second order tensor  $\bar{\lambda}$  is the thermal conductivity of the medium.

The transfer of mass, as for him, is formulated by Fick's law  $J_m = -\rho \bar{D} \text{grad } X$  where the second order tensor is  $\bar{D}$  the diffusivity of humidity in the material and X the water content of the medium. In dry season or in wet season, the local materials of construction studied in this paper may see their humidity varying seriously. It is thus necessary to look for the influence of this humidity.

The equation of energy conservation combined with mass conservation is proposed by (Bakkas *et al.*, 2001):

$$A(T) \frac{\partial T}{\partial t} = \lambda \Delta T + B(T) \quad (12)$$

where A (T) and B (T) are terms dependent in particular of the density of the material, the humidity in the material and the porosity.

## RESULTS

**Influence of the simultaneous transfer of heat and mass on the measure of thermal conductivity, predictive model of correction.** The analogy between these two physical phenomena is obvious. Indeed, let us consider the one dimensional case in Fig.4 (Perré, 1993). The disturbance of an initial state (value 1) caused by changing abruptly at time  $t=0$  the value of the temperature on a frontier of the medium (value 2) is represented for various dates on the Fig.4. The disturbance propagates in the medium and the profile evolves to tighten towards the equilibrium profile that is the straight line connecting two given values. In the case of heat transfer, the value 1 is the temperature  $T_1$  and the value 2 is the temperature  $T_2$  evoked in the Fig.3.

## DISCUSSION

In unsteady-state, the equation (3) becomes, by taking into account (12):

$$A(T) \frac{\partial T}{\partial t} = \bar{\lambda} \Delta T + B(T) \quad (12)$$

In this equation  $\bar{\lambda}$  is the second order tensor of the thermal conductivity of the medium. Supposing that the medium is isotropic, this tensor is reduced to a scalar  $\lambda$ . Then, the equation (12) becomes:

$$\frac{\partial T}{\partial t} = \frac{B(T)}{A(T) - \rho c} \quad (13)$$

The resolution of this equation with Mathematica application, supplies solutions which depend on the terms  $\rho$ ,  $c$ ,  $B(T)$  et  $A(T)$ , the latter being independent from the time  $t$ . The shape of the curves of Fig.4b shows a logarithmic aspect. The transition from the unsteady state to the steady one, thus, is not too fast, what justifies once again the aim of the present study. Let us move closer to the results of slopes of Fig.4b and Fig.4c.

Table 1. Thermal conductivity of the cement mortar

| Material<br>(dimensions of samples<br>$14 \times 14 \times 10 \text{ cm}^3$ ) | Temperature maintained on<br>the free faces ( $^{\circ}\text{C}$ ) | Gradient P ( $^{\circ}\text{Cm}^{-1}$ ) | Thermal conductivity,<br>average of 4 results<br>( $\text{Wm}^{-1} \cdot ^{\circ}\text{C}^{-1}$ ) |
|---|--|---|---|
| Cement mortar with sea sand   | $T_1 = 62$<br>$T_2 = 42.5$   | -0.1024                                 | 1.16  |
| Cement mortar with<br>sand of Badjoudè (North of Benin)                       | $T_1 = 60$<br>$T_2 = 40$   | -0.2283                                 | 1.14  |

Source : ALLOGNON [2]

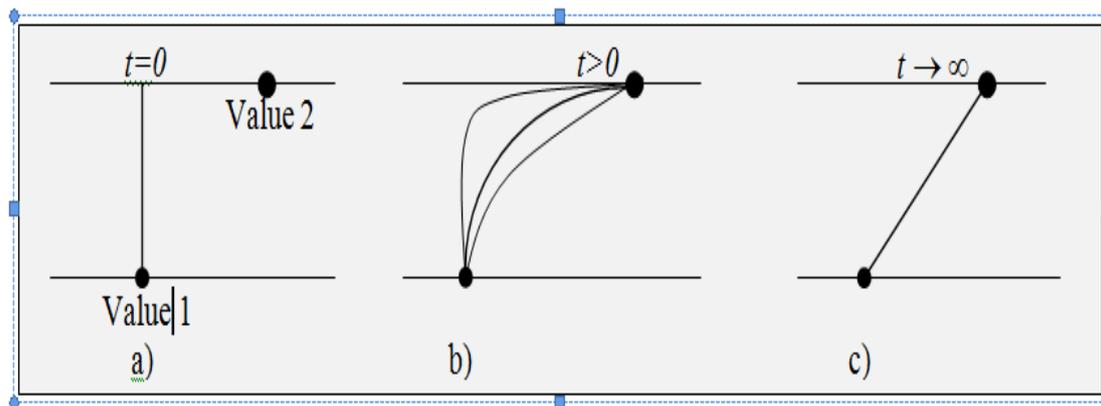


Fig 4. Evolution of a thermal or mass disturbance (6)

In the case of mass transfer, the value 1 is the humidity  $X_1$  and the value 2 is the humidity  $X_2$ . The “method of comparison in steady state” for the measurement of the thermal conductivity is established on the exploitation of slopes (gradT)<sub>1</sub> and (gradT)<sub>2</sub> computed by  $\frac{(\Delta T)_1}{d_1}$  and  $\frac{(\Delta T)_2}{d_2}$  of the line T(x). After all, the influence of the humidity on the measure of the thermal conductivity is translated by the variation of slopes to be determined, the steady state being preceded by a transit regime (See Fig.4).

This method allowed showing the distance between the raw experimental results and the corrected results of conductivity.

The slope in the logarithmic curve (Fig.4b) is 0.290 while the slope of the right (Fig.4c) is 0.207. To these slopes correspond respectively:

For the tangent in the logarithmic curve in Fig. 4b

$$\lambda = \lambda_0 \frac{\frac{(\Delta T)_1}{d_1}}{\frac{(\Delta T)_2}{d_2}} = 0.5704 \text{Wm}^{-1}\text{K}^{-1} \quad (14)$$

For the slope of the line in Fig.4c:  $\lambda = \lambda_0 \frac{\frac{(\Delta T)_1}{d_1}}{\frac{(\Delta T)_2}{d_2}} = 0.799 \text{Wm}^{-1}\text{K}^{-1} \quad (15)$

Comparing these values with the experimental result of Table 1 which gives an average of  $1.15 \text{Wm}^{-1}\text{K}^{-1}$  for the cement mortar, we notice clearly that the simultaneous transfer of heat and mass has a significant influence on the conductivity. The thermal conductivity of a homogeneous porous material already depends on numerous parameters such as the rate of porosity, the size, aspect and distribution of pores. It depends then so more or less on the simultaneous transfer of heat and mass. To be precise, the conductivity computed by taking into account this simultaneous transfer of heat and mass is lower than that calculated by neglecting the simultaneous transfer of heat and mass. The relative difference is about 30 %, which is very important. Normally, the cement mortar being a porous material, it is thus constituted by solid matrix in which are distributed the open and closed pores, the sizes of which are variable. These pores containing air, we expect that the influence of the double transfer of heat and mass would be weak, but it is not the case. After all, the influence of the coupling of heat and mass during the measure of the thermal conductivity is important and the research for an effective thermal conductivity  $\lambda_{eff}$  must be gone deeper into and we rather would write the phenomenological equation as:

$$\vec{\varphi} = -\lambda_{eff} \vec{\text{grad}T} \quad (16)$$

## CONCLUSION

The present work showed the important influence of the simultaneous transfer of heat and mass on the measure of building materials thermal conductivity as the cement mortar. The method of measure of thermal conductivity, called comparison method, established on the principle of answer to a level of disturbance, induced a predictive model of thermal conductivity.

The results calculated by this model which takes into account this superposition of movement of heat and migration of mass are lower about 30 % than the uncorrected experimental results. The cement mortar seems thus more weakly conductive than reality. This report encourages going further by later works in order to highlight the adjustments to be made on the physical experiment of measure of thermal conductivity, on one hand, and uphold better the influence of the simultaneous transfer of heat and mass on the measure of thermal conductivity, on the other hand.

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