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# **RESEARCH ARTICLE**

## A FUZZY LINEAR FRACTIONAL PROGRAMMING PROBLEM WITH FUZZY HOMOGENEOUS CONSTRAINTS IN TRAPEZOIDAL FUZZY NUMBERS

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ARTICLE INFO	ABSTRACT				
Article History: Received 23 <sup>rd</sup> April, 2015 Received in revised form 09 <sup>th</sup> May, 2015 Accepted 20 <sup>th</sup> June, 2015 Published online 31 <sup>st</sup> July, 2015	In this paper, we propose a new method for solving a Fuzzy Linear Fractional Programming Probl (FLFPP) when some of its constraints are fuzzy homogeneous in trapezoidal fuzzy numbers. Us these fuzzy homogeneous constraints a fuzzy transformation matrix $\tilde{T}$ is constructed. This transforms the given problem in to another FLFPP with fewer fuzzy constraints. A relations between these two problems, which ensure that the solution of the original problem can be recove from the solution of the transformed problem. A simple numerical example explains the procedure the proposed method.				
Key words:					
Trapezoidal fuzzy numbers, Fuzzy homogeneous constraints, Fuzzy transformation matrix, Fuzzy linear fractional programming problem.					

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## **INTRODUCTION**

Fuzzy Linear Fractional Programming Problem (FLFPP) is a special type of problem, in which all relations among the fuzzy variables are linear both in fuzzy constraints and the functions to be optimized. A fuzzy linear fractional functional program seeks to optimize a given ratio of two fuzzy linear functions of non-negative variables subject to fuzzy linear constraints. A system of fuzzy linear equations  $\widetilde{A} \widetilde{X} = \widetilde{b}$  is said to be fuzzy homogeneous constraint if  $\widetilde{b} = \widetilde{0}$  such a system always has the trivial solution  $\tilde{X} = \tilde{0}$ . Zimmermann (Zimmermann, 1996) presented a fuzzy approach to multi objective linear programming problem. He also studied the duality relations in fuzzy linear programming. Maleki, H. R. (Maleki et al., 2000) proposed a new method for solving linear programming problem with fuzzy variables. A new operations on triangular fuzzy numbers for solving fuzzy linear programming problem was introduced by Nagoor Gani, A. and Mohamed Asarudeen, S. N. (Nagoor Gani, 2012) Mohan S. and S. Sekar (Mohan and Sekar, 2014) proposed a new technique for solving a linear programming problem with homogeneous constraints in fuzzy environment. Also they introduced fuzzy linear programming problem with fuzzy homogeneous constraints (Mohan and Sekar, 2014). Nachammai, Al and Thangaraj P. are introduced Fuzzy Linear Fractional Programming Problem using metric Distance Ranking (Nachammai and Thangaraj, 2012). The intention here is to reduce the computing time of the optimization process when a constraint is fuzzy homogeneous. A fuzzy transformation matrix  $\tilde{T}$  which eliminates the fuzzy homogeneous constraints. In this paper, section 2 deals with some preliminary definitions and existing function principal operations are given. Development of a fuzzy transformation matrix and relationships between original FLFPP and transformed FLFPP explains in section 3. Numerical example and conclusion are given in last two sections.

## Preliminaries

In this section some fundamental definitions, operations and concepts of fuzzy set theory are given as in (Mohan and Sekar, 2014).

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#### **Definition 2.1**

Let X denote an universal set that is  $X = \{x\}$  then the characteristic function which assigns certain values or a membership grade to the elements of this universal set within a specified range[0,1] is known as the membership function and the set thus defined is call a fuzzy set. The membership grade correspond to the degree to which an element is compatible with the concept represented by the fuzzy set. If  $\mu_{\tilde{A}}$  is the membership function defining a fuzzy set  $\tilde{A}$  then  $\mu_{\tilde{A}} : x \rightarrow [0,1]$  where [0,1] denotes the interval of real numbers from 0 to 1.

### **Definition 2.2**

A trapezoidal fuzzy number  $\tilde{A} = (a_1, a_2, a_3, a_4)$  is defined by the membership function

$$\mu_{\bar{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & ; if a_1 \leq x \leq a_2 \\ 1 & ; if a_2 \leq x \leq a_3 \\ \frac{x - a_3}{a_3 - a_4} & ; if a_3 \leq x \leq a_4 \\ 0 & ; otherwise \end{cases}$$

#### **Definition 2.3**

A ranking function R:  $F(R) \rightarrow R$  which maps each fuzzy number into the real line. F (R) denotes the set of all trapezoidal fuzzy number. If R be any linear ranking function, then  $R(\tilde{A}) = a_2 + a_3 + \frac{1}{2}[(a_4 + a_1) - (a_3 + a_2)].$ 

#### **Definition 2.4**

A System of fuzzy linear equations  $\tilde{A}\tilde{X} = \tilde{b}$  is said to be fuzzy homogeneous constraint, if  $\tilde{b} = \tilde{0}$ , such a system always has the trivial solution  $\tilde{X} = \tilde{0}$ .

#### Arithmetic operations on trapezoidal fuzzy numbers 2.5

If  $\tilde{A} = (a_1, a_2, a_3, a_4)$  and  $\tilde{B} = (b_1, b_2, b_3, b_4)$  are trapezoidal fuzzy numbers then the

- Image of  $\tilde{A} = (-a_4, -a_3, -a_2, -a_1)$
- $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
- $\widetilde{A} \widetilde{B} = (a_1 b_4, a_2 b_3, a_3 b_2, a_4 b_1)$
- If  $\lambda$  is any scalar then  $\lambda \tilde{A} = (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4)$ , if  $\lambda > 0$  and  $\lambda \tilde{A} = (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1)$ , if  $\lambda < 0$ .
- The multiplication of  $\tilde{A}$  and  $\tilde{B}$  is defined as

$$\tilde{A} \circ \tilde{B} = \left[\frac{a_1}{2}(b_1 + b_2 + b_3 + b_4), \frac{a_2}{2}(b_1 + b_2 + b_3 + b_4), \frac{a_3}{2}(b_1 + b_2 + b_3 + b_4), \frac{a_4}{2}(b_1 + b_2 + b_3 + b_4)\right],$$

if  $R(\tilde{B}) > \tilde{0}$  and

 $\tilde{A} \circ \tilde{B} = \left[\frac{a_4}{2}(b_1 + b_2 + b_3 + b_4), \frac{a_3}{2}(b_1 + b_2 + b_3 + b_4), \frac{a_2}{2}(b_1 + b_2 + b_3 + b_4), \frac{a_1}{2}(b_1 + b_2 + b_3 + b_4)\right],$ if  $R(\tilde{B}) < \tilde{0}$ 

•The division is defined as

$$\begin{split} \tilde{A} / \tilde{B} &= \left[\frac{2a_1}{b_1 + b_2 + b_3 + b_4}, \frac{2a_2}{b_1 + b_2 + b_3 + b_4}, \frac{2a_3}{b_1 + b_2 + b_3 + b_4}, \frac{2a_4}{b_1 + b_2 + b_3 + b_4}\right], & \text{if } R(\tilde{B}) > \tilde{0}, R(\tilde{B}) \neq \tilde{0} \text{ and} \\ \tilde{A} / \tilde{B} &= \left[\frac{2a_4}{b_1 + b_2 + b_3 + b_4}, \frac{2a_3}{b_1 + b_2 + b_3 + b_4}, \frac{2a_2}{b_1 + b_2 + b_3 + b_4}, \frac{2a_1}{b_1 + b_2 + b_3 + b_4}\right], & \text{if } R(\tilde{B}) < \tilde{0}, R(\tilde{B}) \neq \tilde{0}. \end{split}$$

#### Notations 2.6

Let us denote the zero fuzzy number  $\tilde{0}$  and unit fuzzy number  $\tilde{1}$  as follows  $\tilde{0} = (-2, -1, 1, 2)$ ,  $\tilde{1} = (-1, 0, 1, 2)$  and  $\tilde{I}_n$  denotes fuzzy identity matrix.

#### Fuzzy linear fractional program with fuzzy homogeneous constraints in trapezoidal fuzzy numbers

In this section, we can discuss fuzzy linear fractional programming problem as in (Nachammai, Al. and Thangaraj, 2012), development of a transformation matrix and relationships between original problem and transformed problem as given in (Mohan and Sekar, 2014).

## Fuzzy linear fractional programming problem 3.1

Consider a FLFPP with homogeneous constraint is

Maximize 
$$\widetilde{Z} = \frac{\widetilde{\alpha} x + \alpha}{\widetilde{dx} + \beta}$$
 ..... (3.1.1)

.....(3.1.2)

Subject to  $\widetilde{A} \ \widetilde{x} = \widetilde{b}$ With  $\widetilde{a}i1\widetilde{x} 1 + \widetilde{a}i2\widetilde{x} 2 + \widetilde{a}i3\widetilde{x} 3 + ... + \widetilde{a}ik\widetilde{x}k + ... + \widetilde{a}il\widetilde{x}l + .... + \widetilde{a}in\widetilde{x}n = \widetilde{0}$ , for some i and  $\widetilde{x} \ge \widetilde{0}$ .

with the additional assumption that the denominator positive for possible solutions.

### Development of the fuzzy transformation matrix 3.2

We develop the fuzzy transformation matrix which is similar to [Mohan and Sekar, 2014].

From (3.1.1) –(3.1.2),  $\tilde{C} = (\tilde{c}_1, \tilde{c}_2, \tilde{c}_3, ..., \tilde{c}_n)$  is a row vector with **n** trapezoidal fuzzy numbers,  $\tilde{A} = (\tilde{a}_{ij})$  is a trapezoidal fuzzy matrix with **m** rows and **n** columns. Also  $\tilde{a}_{ij}, \tilde{c}_i$  and  $\tilde{b}_i$  are trapezoidal fuzzy numbers. We Partition trapezoidal fuzzy matrix as  $\tilde{A} = (\tilde{A}^0, \tilde{A}^+, \tilde{A}^-)$ .  $\tilde{A}^0$  is the set of all column of  $\tilde{A}$  whenever  $\tilde{a}_{ij} = \tilde{0}$ . Let **r** be the number of column  $\tilde{A}^0$ .  $\tilde{A}^+$  is the set of all column of  $\tilde{A}$  whenever  $\tilde{a}_{ij} < \tilde{0}$ . Let **q** be the number of column.  $\tilde{A}^-$  is the set of all column of  $\tilde{A}$  whenever  $\tilde{a}_{ij} < \tilde{0}$ . Let **q** be the number of column.  $\tilde{A}^-$  is the set of all column of  $\tilde{A}$  whenever  $\tilde{a}_{ij} < \tilde{0}$ . Let **q** be the number of column.  $\tilde{A}^-$  is the set of all column of  $\tilde{A}$  whenever  $\tilde{a}_{ij} < \tilde{0}$ . Let **q** be the number of column.  $\tilde{A}^-$  is the set of all column of  $\tilde{A}$  whenever  $\tilde{a}_{ij} < \tilde{0}$ . Let **q** be the number of column.  $\tilde{A}^-$  is the set of all column of  $\tilde{A}$  whenever  $\tilde{a}_{ij} < \tilde{0}$ . Let **q** be the number of column  $\tilde{A}^-$ . Thus **p** + **q** + **r** = **n** which is order of trapezoidal fuzzy identity matrix. It is denoted by  $\tilde{I}_n$ . We define trapezoidal fuzzy transformation matrix  $\tilde{T}$  as  $\tilde{T}_{n X pq+r}$  such that the i<sup>th</sup> equation of  $\tilde{A} \tilde{T}\tilde{W} = \tilde{b}$  will be  $\tilde{0}$ . Here  $\tilde{W}$  is a column vector with **pq**+**r** components. This is accomplished by defining variables  $\tilde{W}_{kl}$  for each pair (k,l) such that  $\tilde{A}_k \in \tilde{A}^+$  and  $\tilde{A}_l \in \tilde{A}^-$ . Now partition  $\tilde{T} = (\tilde{T}_1: \tilde{T}_2)$ , where  $\tilde{T}_1$  consist of unit trapezoidal fuzzy column vector  $\tilde{e}_j$  corresponding to  $w_{kl}$ . The trapezoidal fuzzy transformation matrix  $\tilde{T}$  can be represented as  $\tilde{T} = [(\tilde{e}_j), \forall j \in \tilde{a}_{ij} = \tilde{0}; (\tilde{t}_{kl}), \forall k \in \tilde{A}^+, \forall l \in \tilde{A}^-]$ . That is  $(\tilde{e}_j)$  is the j<sup>th</sup> column of trapezoidal fuzzy identity matrix  $\tilde{I}_n$  and  $\tilde{t}_{kl} = -\tilde{a}_{il}\tilde{e}_k + \tilde{a}_{ik}\tilde{e}_l$ .

## Transformed problem and relationships 3.3

we use the transformation  $\tilde{x} = \tilde{T}\tilde{w}$ , we define the transformed problem associated with the FLFPP (3.1.1) – (3.1.2).

Maximize 
$$\widetilde{Z} = \frac{\widetilde{c} \, \widetilde{T} \, \widetilde{w} + \alpha}{\widetilde{d} \, \widetilde{T} \, \widetilde{w} + \beta}$$
 .....(3.3.1)

Subject to  $\tilde{A} \ \tilde{T} \widetilde{w} = \tilde{b}$ and  $\widetilde{w} \ge \tilde{0}$ . (3.3.2)

**Relationship (i)** If  $\tilde{x}$  solves (3.1.2) then there exists a  $\tilde{w}$  ( $\tilde{x} = \tilde{T}\tilde{w}$ ) which solves (3.3.2).

**Relationship (ii)** If  $\tilde{x}^*$  solves the FLFPP (3.1.1) – (3.1.2) then  $\tilde{w}^* (\tilde{x}^* = \tilde{T}\tilde{w}^*)$  solves the FLFPP (3.3.1) – (3.3.2).

**Relationship (iii)** If  $\tilde{w}^*$  solves the FLFPP (3.3.1) – (3.3.2), then there exists  $\tilde{x}^* = \tilde{T}\tilde{w}^*$  which solves the FLFPP (3.1.1) – (3.1.2); and the extreme values of the two objective functions are equal.

#### Numerical example

Consider a FLFPP with fuzzy homogeneous constraint in trapezoidal fuzzy numbers as follows

Maximize 
$$\widetilde{Z} = \frac{(-2,1,2,3)\widetilde{x}_1 + (-9,4,7,10)\widetilde{x}_2}{(-1,0,1,2)\widetilde{x}_1 + (-1,0,1,2)\widetilde{x}_2 + (-1,0,1,2)} \dots (4.1)$$

Subject to 
$$(-1,0,1,2)\tilde{x}_1 + (-1,0,1,2)\tilde{x}_2 + (-1,0,1,2)\tilde{x}_3 = (-7,3,5,7)$$
 .....(4.2)

$$(-2,1,3,4)\tilde{x}_1 + (-1,0,1,2)\tilde{x}_2 + (-1,0,1,2)\tilde{x}_4 = (-9,4,7,10)$$
.....(4.3)

$$(-1,0,1,2)\tilde{x}_{1} + (-2,-1,0,1)\tilde{x}_{2} + (-2,-1,1,2)\tilde{x}_{3} + (-2,-1,1,2)\tilde{x}_{4} = (-2,-1,1,2)$$
  
and  $\tilde{x}_{1}, \tilde{x}_{2}, \tilde{x}_{3}, \tilde{x}_{4} \ge \tilde{0}.$  (4.4)

It's standard form is

Maximize $\widetilde{Z} = \frac{(-2,1,2,3)\tilde{x}_1 + (-9,4,7,10)\tilde{x}_2}{(-1,0,1,2)\tilde{x}_1 + (-1,0,1,2)\tilde{x}_2 + (-1,0,1,2)}$	(4.5)
Subject to $(-1,0,1,2)\tilde{x}_1 + (-1,0,1,2)\tilde{x}_2 + (-1,0,1,2)\tilde{x}_3 = (-7,3,5,7)$	(4.6)
$(-2,1,3,4)\tilde{x}_1 + (-1,0,1,2)\tilde{x}_2 + (-1,0,1,2)\tilde{x}_4 = (-9,4,7,10)$	(4.7)
$(-1,0,1,2)\tilde{x}_1 + (-2,-1,0,1)\tilde{x}_2 + (-2,-1,1,2)\tilde{x}_3 + (-2,-1,1,2)\tilde{x}_4 = (-2,-1,1,2)$	(4.8)
and $ ilde{x}_1,  ilde{x}_2,  ilde{x}_3,  ilde{x}_4 \geq  ilde{0}.$	

Here (4.8) is a homogeneous constraint. Since according to section 3.2, we have

 $\tilde{A} = [(\tilde{0}, \tilde{0}): (\tilde{1}), (\tilde{1})]$ , where p=1, q=1 and r=2. This implies n=p+q+r=4.

$$\begin{split} \tilde{I}_{4} &= \begin{bmatrix} (-1,0,1,2) & (-2,-1,1,2) & (-2,-1,1,2) & (-2,-1,1,2) \\ (-2,-1,1,2) & (-1,0,1,2) & (-2,-1,1,2) & (-2,-1,1,2) \\ (-2,-1,1,2) & (-2,-1,1,2) & (-1,0,1,2) & (-2,-1,1,2) \\ (-2,-1,1,2) & (-2,-1,1,2) & (-2,-1,1,2) & (-1,0,1,2) \end{bmatrix} \\ \tilde{t}_{kl} &= -(-2,-1,0,1) \begin{bmatrix} (-1,0,1,2) \\ (-2,-1,1,2) \\ (-2,-1,1,2) \\ (-2,-1,1,2) \\ (-2,-1,1,2) \end{bmatrix} + (-1,0,1,2) \begin{bmatrix} (-2,-1,1,2) \\ (-1,0,1,2) \\ (-2,-1,1,2) \\ (-2,-1,1,2) \\ (-2,-1,1,2) \end{bmatrix} = \begin{bmatrix} (-1,0,1,2) \\ (-1,0,1,2) \\ (0,0,0) \\ (0,0,0,0) \end{bmatrix} \\ \text{From these} \quad \tilde{T} = \begin{bmatrix} (-2,-1,1,2) & (-2,-1,1,2) & (-1,0,1,2) \\ (-2,-1,1,2) & (-2,-1,1,2) & (-1,0,1,2) \\ (-2,-1,1,2) & (-2,-1,1,2) & (-1,0,1,2) \\ (-1,0,1,2) & (-2,-1,1,2) & (0,0,0,0) \\ (-2,-1,1,2) & (-2,-1,1,2) & (0,0,0,0) \end{bmatrix} \end{split}$$

The above problem (4.5) - (4.8) transformed in to the problem below with fewer (two) constraints

$$\begin{array}{ll} \text{Maximize } \widetilde{Z} &= \frac{(-11,5,9,13)\widetilde{w}_3}{(-2,0,2,4)\widetilde{w}_3 + (-1,0,1,2)} & \dots \dots (4.9) \\ \text{Subject to } (-1,0,1,2)\widetilde{w}_1 + (-2,-1,1,2)\widetilde{w}_2 + (-2,0,2,4)\widetilde{w}_3 = (-7,3,5,7) & \dots \dots (4.10) \\ (-2,-1,1,2)\widetilde{w}_1 + (-1,0,1,2)\widetilde{w}_2 + (-3,1,4,6)\widetilde{w}_3 = (-9,4,7,10) & \dots \dots (4.11) \end{array}$$

 $\widetilde{w}_1$  ,  $\widetilde{w}_2$  , $\widetilde{w}_3 \geq \widetilde{0}$ .

## **Iteration Table 1**

			<i>C</i> <sub>i</sub>	(-2,-1, 1,2)	(-2,-1, 1,2)	(-11,5,9,13)	Minimum $\tilde{\theta}$
			$\widetilde{d}_{i}$	(-2,-1, 1,2)	(-2,-1, 1,2)	(-2,0,2,4)	
$\widetilde{C}_{\widetilde{B}}$	$\widetilde{d}_{\widetilde{B}}$	$\widetilde{y}_{\tilde{B}}$	$\widetilde{X}_{\widetilde{B}}$	$\widetilde{W}_1$	$\widetilde{W}_2$	$\widetilde{W}_3$	
(-2,-1, 1,2)	(-2,-1, 1,2)	$\widetilde{W}_1$	(-7,3,5,7)	(-1,0,1,2)	(-2,-1, 1,2)	(-2,0,2,4)	(-7/2,3/2,5/2,7/2)
(-2,-1, 1,2)	(-2,-1, 1,2)	$\widetilde{w}_2$	(-9,4,7,10)	(-2,-1, 1,2)	(-1,0,1,2)	(-3,1,4,6)	(-9/4,1,7/4,5/2)
$\tilde{z}^{(1)} = 0$	$\tilde{z}^{(2)} = (-1,0,1,2)$		$\widetilde{Z}_{j}$ - $\widetilde{C}_{j}$	(-2,-1,1,2)	(-2,-1,1,2)	(-13,-9,-5,11)	
			$ ilde{Z}_{ ext{j-}} ilde{d}_{ ext{j}}$	(-2,-1,1,2)	(-2,-1,1,2)	(-4,-2,0,2)	
			$\Delta_i$	(-2,-1,1,2)	(-2,-1,1,2)	(-13,-9,-5,11)	

Since there is one  $\Delta_j \leq \tilde{0}$ . Therefore, we go to next iteration. Here  $\tilde{w}_2$  leaves from the basis and  $\tilde{w}_3$  enters in to the basis.

## **Iteration Table 2**

			$\widetilde{\mathcal{C}}_{\mathrm{i}}$	(-2,-1, 1,2)	(-2,-1, 1,2)	(-11,5,9,13)
			$\widetilde{d}_{j}$	(-2,-1, 1,2)	(-2,-1, 1,2)	(-2,0,2,4)
$\widetilde{C}_{\widetilde{B}}$	$\widetilde{d}_{\widetilde{B}}$	$\widetilde{y}_{\widetilde{B}}$	$\widetilde{X}_{\widetilde{B}}$	$\widetilde{W}_1$	$\widetilde{W}_2$	$\widetilde{W}_3$
(-2,-1, 1,2)	(-2,-1, 1,2)	$\widetilde{w}_1$	$(-12,\frac{-1}{2},3,\frac{23}{2})$	$(-2,\frac{-1}{2},\frac{3}{2},3)$	$(-3, \frac{-3}{2}, 1, \frac{5}{2})$	$(-5, -2, \frac{3}{2}, , \frac{11}{2})$
(-11,5,9,13)	(-2,0,2,4)	$\widetilde{W}_3$	(-9/4,1,7/4, 10/4)	(-1/2,-1/4, 1/4,1/2)	(-1/4,0,1/4,1/2)	(-3/4,1/4,1,3/2)
$\tilde{z}^{(1)} = \left(-\frac{33}{2}, \frac{15}{2}, \frac{27}{2}, \frac{39}{2}\right)$	$\tilde{z}^{(2)} = (-4,0,4,8)$		$ ilde{Z}_{\mathrm{j}}$ - $ ilde{\mathcal{C}}_{\mathrm{j}}$	(-2,-1,1,2)	(-11/4,5/4,9/4,13/4)	(-22,-4,4,22)
			$\widetilde{Z}_{ ext{j-}}\widetilde{d}_{ ext{j}}$	(0,0,0,0)	(-1/2,0,1/2,1)	(-6,-2,2,6)
			$\Delta_j$	(0,0,0,0)	(-71/4,-27/4, 17/4, 81/4)	(0,0,0,0)

Since all  $\Delta_i \ge \tilde{0}$ . Therefore, we reached the optimum solution.

Therefore Maximum  $\widetilde{Z} = \left(-\frac{33}{8}, \frac{15}{8}, \frac{27}{8}, \frac{39}{8}\right)$  when  $\widetilde{w}_{1=}(-12, \frac{-1}{2}, 3, \frac{23}{2})$  and  $\widetilde{w}_{3=}(-9/4, 1, 7/4, 10/4)$ .

The solution of the original problem, ( $\widetilde{x} = \widetilde{T} \widetilde{w}$ ), is

$$\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \tilde{x}_4 \end{bmatrix} = \begin{bmatrix} (-2, -1, 1, 2) & (-2, -1, 1, 2) & (-1, 0, 1, 2) \\ (-2, -1, 1, 2) & (-2, -1, 1, 2) & (-1, 0, 1, 2) \\ (-1, 0, 1, 2) & (-2, -1, 1, 2) & (0, 0, 0, 0) \\ (-2, -1, 1, 2) & (-1, 0, 1, 2) & (0, 0, 0, 0) \end{bmatrix} \begin{bmatrix} \left(-12, \frac{-1}{2}, 3, \frac{23}{2}\right) \\ (0, 0, 0, 0) \\ (-9/4, 1, 7/4, 10/4) \end{bmatrix} = \begin{bmatrix} \left(-9/4, 1, 7/4, 10/4\right) \\ \left(-9/4, 1, 7/4, 10/4\right) \\ \left(-12, \frac{-1}{2}, 3, \frac{23}{2}\right) \\ (0, 0, 0, 0, 0) \end{bmatrix}$$

Therefore, the solution of the original problem is Maximum  $\tilde{Z} = \left(-\frac{33}{8}, \frac{15}{8}, \frac{27}{8}, \frac{39}{8}\right)$  when  $\tilde{x}_{1=(-9/4, 1, 7/4, 10/4), \tilde{x}_{2=}(-9/4, 1, 7/4, 10/4), \tilde{x}_{3} = \left(-12, \frac{-1}{2}, 3, \frac{23}{2}\right)$  and  $\tilde{x}_{4} = (0, 0, 0, 0)$ .

#### Conclusion

The process, described in section 3, can be extended to define  $\tilde{T}$  if  $\tilde{A} \ \tilde{x} = \tilde{b}$  has more than one homogeneous constraint. In case there are s homogeneous constraints, we define s transformation matrices  $\tilde{T}(1), \tilde{T}(2), \tilde{T}(3), \dots, \tilde{T}(s), \tilde{T}(2)$  is determined once  $\tilde{A} \ \tilde{T}(1)$  has been computed. In general  $\tilde{T}(s)$  is determined only when  $\tilde{A} \ \tilde{T}(1) \ \tilde{T}(2) \ \tilde{T}(3), \dots, \tilde{T}(s-1)$  has been computed. This method reduces the number of constraints.

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