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RESEARCH ARTICLE

A BURIED VERTICAL RECTANGULAR FINITE FAULT IN AN ELASTIC LAYER OVER A VISCOELASTIC HALF-SPACE

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ABSTRACT

A rectangular buried vertical fault of finite length of strike-slip nature in an elastic layer over a viscoelastic half space representing the lithosphere-asthenosphere system has been considered here. Stresses and strain accumulate in the region due to various tectonic processes, such as mantle convection and plate movements etc, which ultimately leads to movements across the fault. In the present paper, a three-dimensional model of the system is considered and analytic expressions for displacements, stresses and strains in the model have been obtained using suitable mathematical techniques developed for this purpose. A suitable numerical technique is adapted for computer simulation. A detailed study of these expressions may give some ideas about the nature of stress-strain accumulation in the system, which in turn will be helpful in formulating an effective earthquake prediction programme.

INTRODUCTION

While the famous San Andreas fault is very long compared to it's depth there is a number of neighboring faults, e.g., Calaveras, Hayward, San Jacinto faults etc. which are not so long compared to its depth, we call them a finite fault. Regular observation in seismically active regions in recent years have revealed that during apparently quiet aseismic period, there are usually slow, quasi-static surface deformations indicating accumulations of stress and strain in the region during this aseismic period. In some cases, this accumulated stress may eventually lead to a sudden fault movement generating an earthquake while in other cases there may be continuous slow aseismic creeping movement across the fault. The effect of such aseismic creep on the accumulation and release of stress in the regions is of great interest in formulating an effective earthquake prediction programme. It is therefore seems to be an essential feature to identify the nature of the stress and strain accumulation in the vicinity of seismic faults situated in the region by studying the observed ground deformations during the aseismic period. A proper understanding of the mechanism of such aseismic quasi static deformation may give us some precursory

Information regarding the impending earthquakes

A pioneering work involving static ground deformation in elastic media were initiated by Steketee, J.A. (1958(a),(b)). Maruyama, T. (1964, 1966), Chinnery, M.A. (1961,1964,1965), Chinnery, M.A. and Dushan B. Jovanovich (1972) did a wonderful work in analyzing the displacement, stress and strain in the layered medium. Later some theoretical models in this direction have been formulated by a number of authors such as Rybicki. K. (1971, 1973), R. Sato (1972), M. Rosenman and S.J. Singh (1973), Spence, D.A and Turcotte, D.L(1976), Savage and Presscott (1976), Budianasky, B; Amazio, J.C. (1976), J.B. Rundle and D.D. Jackson (1977), T. Iwasaki and R. Sato (1979), Mukhopadhyay *et al.* (1979 a,b; 1980 a,b), Cohen(1980a), Sunita Rani and S.J. Singh (1991,1992), U. Ghosh and others (1992), Sen, S., Sarker S., Mukhopadhyay, A. (1993). Riad Hassani, Denis Jongmans and Zhen-Zing Yao (2003), G. Hillers and S.G. Wesnousky (2008), Paul Segall (2010) has discussed various aspects of fault movement in his book. Ghosh, U and Sen, S (2011) have discussed stress accumulation near buried fault in lithosphere-asthenosphere system.

In most of these works the medium were taken to be elastic and /or viscoelastic, layered or otherwise. In most of the cases the faults were taken to be too long compared to its depth, so that the problem reduced to a 2D model. Noting that there are several faults (As mentioned above) which are not so long compared to their depth, a 3D model is imminent. In the present paper a buried vertical rectangular finite strike-slip fault in an elastic layer over a viscoelastic half-space model representing the lithosphere-asthenosphere system is being considered. The medium is under the influence of tectonic forces due to mantle convection or some related phenomena. The fault undergoes a slipping movement when the stresses in the region exceed certain threshold values.

Formulation

We consider a buried vertical finite rectangular strike-slip fault F of length 2L (L-finite) and width D situated in an elastic layer over a viscoelastic half space of linear Maxwell type material. Let H be the thickness of the layer. A Cartesian co-ordinate system is used with the mid-point O of the fault as the origin, the strike of the fault along the Y_1 axis, Y_2 axis perpendicular to the fault and Y_3 axis pointing downwards so that the fault is given by F : $(-L \le y_1 \le L, y_2 = 0, d \le y_3 \le D)$ as shown in Fig1. Let (u^k_i), (τ^k_{ij}) and (e^k_{ij}) be the displacement, stress and strain components, i, j=1, 2, 3. And k=1 for the layer and k=2 for the half-space.

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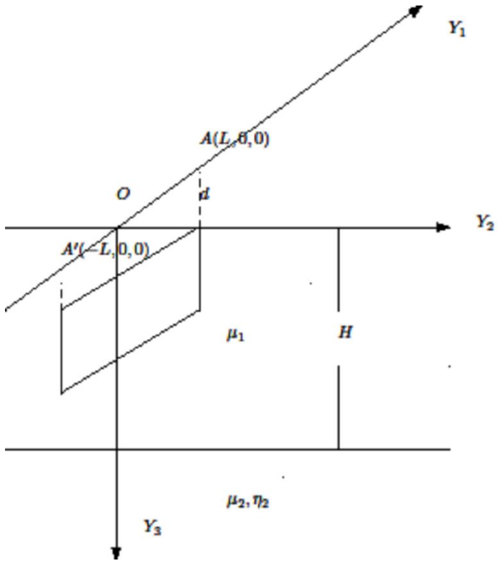

Fig.1. Section of the model by the plane y_1=0

Constitutive equations

For the elastic layer: M1

$$\tau^1_{11} = \mu_1\left(\frac{\partial u^1_1}{\partial y_1}\right) \tag{1.1}$$

$$\tau^k_{12} = \left(\frac{1}{2}\right)\mu_1\left(\frac{\partial u^1_1}{\partial y_2} + \frac{\partial u^1_2}{\partial y_1}\right) \tag{1.2}$$

$$\tau^1_{13} = \left(\frac{1}{2}\right)\mu_1\left(\frac{\partial u^1_1}{\partial y_3} + \frac{\partial u^1_3}{\partial y_1}\right) \tag{1.3}$$

$$\tau^1_{22} = \mu_1 \frac{\partial u^1_2}{\partial y_2} \tag{1.4}$$

$$\tau^1_{23} = \frac{1}{2}\mu_1\left(\frac{\partial u^1_2}{\partial y_3} + \frac{\partial u^1_3}{\partial y_2}\right) \tag{1.5}$$

$$\tau^1_{33} = \mu_1 \frac{\partial u^1_3}{\partial y_3} \tag{1.6}$$

For a linear viscoelastic Maxwell type medium the constitutive equations have been taken a

$$\left(\frac{1}{\eta_2} + \frac{1}{\mu_2}\frac{\partial}{\partial t}\right)\tau^2_{11} = \frac{\partial}{\partial t}(e^2_{11}) = \frac{\partial}{\partial t}\left(\frac{\partial u^2_1}{\partial y_1}\right) \tag{1.7}$$

$$\left(\frac{1}{\eta_2} + \frac{1}{\mu_2}\frac{\partial}{\partial t}\right)\tau^2_{12} = \frac{\partial}{\partial t}(e^2_{12}) = \left(\frac{1}{2}\right)\frac{\partial}{\partial t}\left(\frac{\partial u^2_1}{\partial y_2} + \frac{\partial u^2_2}{\partial y_1}\right) \tag{1.8}$$

$$\left(\frac{1}{\eta_2} + \frac{1}{\mu_2}\frac{\partial}{\partial t}\right)\tau^2_{13} = \frac{\partial}{\partial t}(e^2_{13}) = \left(\frac{1}{2}\right)\frac{\partial}{\partial t}\left(\frac{\partial u^2_1}{\partial y_3} + \frac{\partial u^2_3}{\partial y_1}\right) \tag{1.9}$$

$$\left(\frac{1}{\eta_2} + \frac{1}{\mu_2}\frac{\partial}{\partial t}\right)\tau^2_{22} = \frac{\partial}{\partial t}(e^2_{22}) = \frac{\partial}{\partial t}\left(\frac{\partial u^2_2}{\partial y_2}\right) \tag{1.10}$$

$$\left(\frac{1}{\eta_2} + \frac{1}{\mu_2}\frac{\partial}{\partial t}\right)\tau^2_{23} = \frac{\partial}{\partial t}(e^2_{23}) = \left(\frac{1}{2}\right)\frac{\partial}{\partial t}\left(\frac{\partial u^2_2}{\partial y_3} + \frac{\partial u^2_3}{\partial y_2}\right) \tag{1.11}$$

$$\left(\frac{1}{\eta_2} + \frac{1}{\mu_2}\frac{\partial}{\partial t}\right)\tau^2_{33} = \frac{\partial}{\partial t}(e^2_{23}) = \frac{\partial}{\partial t}\left(\frac{\partial u^2_3}{\partial y_3}\right) \tag{1.12}$$

where η_2 is the effective viscosity and μ_1 , μ_2 are the effective rigidity of the material.The stresses satisfy the following equations (assuming quasistatic deformation for which the inertia terms are neglected); and body forces does not change during our consideration.

$$\frac{\partial}{\partial y_1}(\tau^k_{11}) + \frac{\partial}{\partial y_2}(\tau^k_{12}) + \frac{\partial}{\partial y_3}(\tau^k_{13}) = 0 \tag{1.13}$$

$$\frac{\partial}{\partial y_1}(\tau^k_{21}) + \frac{\partial}{\partial y_2}(\tau^k_{22}) + \frac{\partial}{\partial y_3}(\tau^k_{23}) = 0 \tag{1.14}$$

$$\frac{\partial}{\partial y_1}(\tau^k_{31}) + \frac{\partial}{\partial y_2}(\tau^k_{32}) + \frac{\partial}{\partial y_3}(\tau^k_{33}) = 0 \tag{1.15}$$

where $(-\infty<y_1<\infty, -\infty<y_2<\infty, 0\le y_3\le H, t\ge 0)$ for the layer k=1.
and $(-\infty<y_1<\infty, -\infty<y_2<\infty, y_3\ge H, t\ge 0)$ for the half-space k=2.

Boundary conditions

The boundary conditions are taken as, with t=0 representing an instant when the medium is in aseismic state:

$$\lim_{y_1 \to L-} \tau^1{}_{11}(y_1, y_2, y_3, t) =$$

$$\lim_{y_1 \to L+} \tau^1{}_{11}(y_1, y_2, y_3, t) = \tau_L \text{ (say)}$$

For $y_2 = 0$, $d \le y_3 \le D$, $t \ge 0$ (1.16)

$$\lim_{y_1 \to -L-} \tau^1{}_{11}(y_1, y_2, y_3, t) =$$

$$\lim_{y_1 \to -L+} \tau^1{}_{11}(y_1, y_2, y_3, t) = \tau_L$$

For $y_2 = 0$, $d \le y_3 \le D$, $t \ge 0$ (1.17)

assuming that the stresses maintaining a constant value τ_L at the tip of the fault along Y_1 axis [the value of this constant stress is likely to be small enough so that no further extension is possible along the Y_1 axis].

$$\tau^1{}_{12}(y_1, y_2, y_3) \to \tau_\infty(t)$$

For ($-\infty < y_1 < \infty, -\infty < y_2 < \infty, 0 \le y_3 \le H, t \ge 0$)

(1.18)

On the free surface $y_3 = 0, (-\infty < y_1, y_2 < \infty, t \ge 0)$

$$\tau^1{}_{13}(y_1, y_2, y_3) = 0 \quad (1.16)$$

$$\tau^1{}_{23}(y_1, y_2, y_3) = 0 \quad (1.20)$$

$$\tau^1{}_{33}(y_1, y_2, y_3) = 0$$

(1.21)

Also as $y_3 \to \infty (-\infty < y_1, y_2 < \infty, t \ge 0)$

$$\tau^2{}_{13}(y_1, y_2, y_3, t) = 0 \quad (1.22)$$

$$\tau^2{}_{23}(y_1, y_2, y_3, t) = 0 \quad (1.23)$$

$$\tau^2{}_{33}(y_1, y_2, y_3, t) = 0 \quad (1.24)$$

$$\tau^2{}_{22}(y_1, y_2, y_3, t) = 0 \quad (1.25)$$

On the interface

$$y_3 = H, (-\infty < y_1, y_2 < \infty, t \ge 0)$$

$$\tau^1{}_{13}(y_1, y_2, y_3) = \tau^2{}_{13}(y_1, y_2, y_3, t) \quad (1.26)$$

$$\tau^1{}_{23}(y_1, y_2, y_3) = \tau^2{}_{23}(y_1, y_2, y_3, t) \quad (1.27)$$

$$\tau^1{}_{33}(y_1, y_2, y_3) = \tau^2{}_{33}(y_1, y_2, y_3, t) \quad (1.28)$$

$$u^1{}_3(y_1, y_2, y_3) = u^2{}_3(y_1, y_2, y_3, t) \quad (1.29)$$

[where $\tau_\infty(t)$ is the shear stress maintained by mantle convection and other tectonic phenomena].

The initial conditions are

Let $(u^1{}_i)_0$, $(\tau^1{}_{ij})_0$ and $(e^1{}_{ij})_0$ $i,j = 1,2,3$ be the value of $(u^1{}_i)$, $(\tau^1{}_{ij})$ and $(e^1{}_{ij})$ at time t=0 which are functions of y_1, y_2, y_3 and satisfy the relations (1.1)-(1.29).

(3) Solution before fault movement

(SEN.S and DEBNATH.S.K.2012),
(S.K.DEBNATH, 2013)

The boundary value problem given by (1.1)-(1.29), can be solved (as shown in the Appendix-I) by taking Laplace transformation with respect to time't' of all the constitutive equations and the boundary conditions. On taking the inverse Laplace transformation we get the solutions for displacement, stresses as:

$$u^1{}_1(y_1, y_2, y_3, t) = (u^1{}_1)_0 + \left(\frac{\tau_L}{\mu_1}\right) y_1 t + (y_2/\mu_1) \times$$
$$[\tau_\infty(t) - \tau_\infty(0)]$$
$$u^1{}_2(y_1, y_2, y_3, t) = (u^1{}_2)_0 + ((y_1 + y_2)/\mu_1)$$
$$\times [\tau_\infty(t) - \tau_\infty(0)]$$
$$u^1{}_3(y_1, y_2, y_3, t) = (u^1{}_3)_0$$
$$\tau^1{}_{11} = \left(\frac{\mu_1}{\eta_1}\right) \tau_L (1 - e^{-(\mu_1/\eta_1)t}) + (\tau^1{}_{11})_0 e^{-(\mu_1/\eta_1)t},$$
$$\tau^1{}_{12} = \tau_\infty(t) - [\tau_\infty(0) - (\tau^1{}_{12})_0] e^{-(\mu_1/\eta_1)t}$$
$$\tau^1{}_{13} = (\tau^1{}_{13})_0 e^{-(\mu_1/\eta_1)t} \quad \tau^1{}_{22} = (\tau^1{}_{22})_0 e^{-(\mu_1/\eta_1)t}$$
$$\tau^1{}_{23} = (\tau^1{}_{23})_0 e^{-(\mu_1/\eta_1)t}, \tau^1{}_{33} = (\tau^1{}_{33})_0 \quad \text{(A)}$$

From the above solution we find that $\tau^1{}_{12}$ increases with time and tends to $\tau_\infty(t)$ as t tends to ∞, while $\tau^1{}_{22}$, $\tau^1{}_{23}$ tends to zero, but $\tau^1{}_{33}$ retains the constant value $(\tau^1{}_{33})_0$. We assume that the geological conditions as well as the characteristic of the fault in such that when $\tau^1{}_{12}$ reaches some critical value, say $\tau_c < \tau_\infty(t)$ the fault F starts slipping. The magnitude of slip is expected to satisfy the following conditions:

(C_1) Its value will be maximum near the middle of the fault on the free surface.
(C_2) It will gradually decrease to zero at the tips of the fault ($y_1=\pm L$, y_2=0, $d \le y_3 \le D$) along its length.
(C_3) The magnitude of the slip will decrease with y_3 as we move downwards and ultimately tends to zero near the lower edge of the fault.
$(y_1 = \pm L, y_2 = 0, y_3 = D)$

The function, f (y_1, y_2) satisfy the above conditions. [We call it creep function]

(4) Solutions after fault movement

(SEN.S and DEBNATH .S.K.2012),
(S.K.DEBNATH, 2013)

We assume that after a time T_1, the stress component $\tau^1{}_{12}$ (which is the main driving force for the strike-slip motion of the fault) exceeds the critical value τ_c, and the fault F starts slipping, characterized by a dislocation across the fault. We solve the resulting boundary value problem by modified Green's function method following Maruyama (1966), Rybicki (1971, 1973) and correspondence principle (As shown in the Appendix-2) and get the solution for displacements, stresses and strain as:

$$u^1{}_1(y_1, y_2, y_3, t) = (u^1{}_1)_0 + (\tau_L/\mu_1) y_1 t + (y_2/\mu_1) \times$$
$$[(\tau_\infty(t) + [(\tau_\infty(0)] +$$
$$H(t - T_1)/(2 \times \pi) \int_{-L}^{L} \int_d^D f(x_1, x_3)$$
$$[(y_2/[(y_1 - x_1)^2 + (y_2 - x_2)^2 + (y_3 - x_3)^2]^{3/2}$$
$$-(y_2/[(y_1 + x_1)^2 + (y_2 - x_2)^2 + (y_3 - x_3)^2]^{3/2}]$$

$-\sum_{1}^{\infty}\left(\frac{\alpha}{\beta}\right)^m \times A_m(t)\times$ [{(x2-y2)/[(y1-x1)^2+(x2–y2)^2+(x3-2mH–y3)^2]^{3/2}+(x2-y2)/[(y1-x1)^2+(x2–y2)^2+(x3-2mH+y3)^2]^{3/2}+(x2-y2)/[(y1-x1)^2+(x2–y2)^2+(x3+2mH+y3)^2]^{3/2}+(x2-y2)/[(y1-x1)^2+(x2–y2)^2+(x3+2mH-y3)^2]^{3/2}} dx_3dx_1]

$$u^1{}_2(y_1,y_2,y_3,t)=(u^1{}_2)_0+((y_1+y_2)/\mu)\times\left[(\tau_\infty(t)-\tau_\infty(0))\right]$$

$$u^1{}_3(y_1,y_2,y_3,t)=(u^1{}_3)_0$$

$$\tau^1{}_{11}(y_1,y_2,y_3,t)=(\mu_1/\eta)\tau_L(1-e^{(-\mu_1/\eta_1)t})+(\tau^1{}_{11})_0e^{(-\mu_1/\eta_1)t}+[H(t-T_1)/(2\times\pi)][U-\mu_1/\eta_1\int_0^t U(\tau)e^{(-\mu_1/\eta_1)(t-\tau)}d\tau\Big]\times$$

$$\frac{\partial}{\partial y_1}\Big(\int\int_F f_1(x_1,x_3)\times\Big[\frac{(y_2-x_2)}{\left[(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{\frac{3}{2}}}-\frac{(y_2-x_2)}{\left[(y_1+x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{3/2}}\Big]-(1/4\pi)$$

$\sum_{1}^{\infty}\left(\frac{\alpha}{\beta}\right)^m \times A_m(t)$ ×{(x2-y2)/[(y1-x1)^2+(x2–y2)^2+(x3-2mH–y3)^2]^{3/2}+(x2-y2)/[(y1-x1)^2+(x2–y2)^2+(x3-2mH+y3)^2]^{3/2}+(x2-y2)/[(y1-x1)^2+(x2–y2)^2+(x3+2mH+y3)^2]^{3/2}+(x2-y2)/[(y1-x1)^2+(x2–y2)^2+(x3+2mH-y3)^2]^{3/2}})dx_3dx_1) (4.12)

$$\tau^1{}_{12}(y_1,y_2,y_3,t)=\tau_\infty(t)-(\tau_\infty(0)-(\tau^1{}_{12})_0)e^{(-\mu_1/\eta_1)t}+[H(t-T_1)/(2\times\pi)][U-\mu_1/\eta_1\int_0^t U(\tau)e^{(-\mu_1/\eta_1)(t-\tau)}d\tau]\times$$

$$\frac{\partial}{\partial y_2}\Big(\int\int_F f_1(x_1,x_3)\times\Big[\frac{(y_2-x_2)}{\left[(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{\frac{3}{2}}}-\frac{(y_2-x_2)}{\left[(y_1+x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{3/2}}\Big]$$

$-(1/4\pi)\sum_{1}^{\infty}\left(\frac{\alpha}{\beta}\right)^m \times A_m(t)\times$ {(y2)/[(y1-x1)^2+(x2–y2)^2+(x3-2mH–y3)^2]^{3/2}+(y2)/[(y1-x1)^2+(x2–y2)^2+(x3-2mH+y3)^2]^{3/2}+(y2)/[(y1-x1)^2+(x2–y2)^2+(x3+2mH+y3)^2]^{3/2}+(y2)/[(y1-x1)^2+(x2–y2)^2+(x3+2mH-y3)^2]^{3/2}})dx_3dx_1)

(4.13)

$$\tau^1{}_{23}=(\tau^1{}_{23})_0e^{-(\mu_1/\eta_1)t},\tau^1{}_{33}=(\tau^1{}_{33})_0$$

$$e^1{}_{12}(y_1,y_2,y_3,t)=\left(\frac{1}{2}\right)(e^1{}_{12})_0+(1/\mu_1)[\tau_\infty(t)-\tau_\infty(0)]+H(t-T_1)/(2\times\pi)\int_{-L}^{L}\int_d^D f(x_1,x_3)$$

$$[((y_2)(y_2-x_2))/[(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2]^{5/2}-(y_2)(y_2-x_2)/[(y_1+x_1)^2+(y_2-x_2)^2++(y_3-x_3)^2]^{5/2}]\times$$

$$\frac{\partial}{\partial y_2}\Big(\int\int_F f_1(x_1,x_3)\times\Big[\frac{(y_2-x_2)}{\left[(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{\frac{3}{2}}}-\frac{(y_2-x_2)}{\left[(y_1+x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{3/2}}\Big]$$

$-(1/4\pi)\sum_{1}^{\infty}\left(\frac{\alpha}{\beta}\right)^m \times A_m(t)\times$ {(y2)/[(y1-x1)^2+(x2–y2)^2+(x3-2mH–y3)^2]^{3/2}+(y2)/[(y1-x1)^2+(x2–y2)^2+(x3-2mH+y3)^2]^{3/2}+(y2)/[(y1-x1)^2+(x2–y2)^2+(x3+2mH+y3)^2]^{3/2}+(y2)/[(y1-x1)^2+(x2–y2)^2+(x3+2mH-y3)^2]^{3/2}})dx_3dx_1)

(B).

where, $A_m(t)=1+\sum_{1}^{m}\binom{m}{r}(2s/1-s)^r[1-e^{-a_1t}e_{r-1}(a_1t)]$

$$s=\frac{\mu_2}{\mu_1},\alpha=\frac{\mu_1}{\mu_2}-1,\beta=\frac{\mu_1}{\mu_2}+1,a_1=\frac{\mu_1\mu_2}{(\mu_1+\mu_2)\times\eta_2}$$

$$b_1=\frac{2\times\mu_1\mu_2^2}{(\mu_1^2-\mu_2^2)\times\eta_2},e_n(z)=1+\sum_{1}^{n}z^i,$$

$$e_0(z)=1,B_{rn}=\binom{n}{r},b_1^r,A_{rm}=\binom{m}{r}[(b_1/a_1)]^r$$

Numerical computations

Following Chathles, L.M. (1975), Aki, K. And Richards, P.G. (1980) and the recent studies on rheological behaviour of crust and upper middle by Peter Chift, Jian Lin, Udo Barcktiausen (2002), Shun-ichiro karato (July 2010) the values of the model parameters are taken as:

$$\mu_1=3\times10^{11}dyne/cm^2,$$

$$\mu_2=3.5\times10^{11}dyne/cm^2,\eta_2=3.2\times10^{21}poise$$

D=Depth of the fault=10km.,[noting that the depth of all major earthquake faults are in between 10-15 km]
d=10km.(say)
2L=Length of the fault=40km. (say).
$\tau_\infty(t)=2\times10^8$ dyne/cm^2 (200 bars), [post seismic observations reveal that stress released in major earthquake are of the order of 200 bars, in extreme cases it may be 400 bars.]
$(\tau^1{}_{12})_0=5\times10^7$ dyne/cm^2 (50 bars) and $\tau_\infty(0)=0$

We take the function

$f_1(x_1,x_3)=U\left(1-\frac{x_1^2}{L^2}\right)\left(1-\frac{3}{D_1^2}x_3^2+\frac{3}{D_1^3}x_3^3\right)$, with U = 1cm/year, satisfying the conditions stated in (C_1)–(C_3).

We now compute the following quantities:

$$U^1{}_1(y_1, y_2, y_3) = u^1{}_1(y_1, y_2, y_3) - [(u^1{}_1)_0 \tag{2.1}$$

$$\tau^1{}_{11}(y_1, y_2, y_3, t) = [(\mu_1/\eta_1)\tau_L(1 - e^{(-\mu_1/\eta_1)t}) + (\tau^1{}_{11})_0 e^{(-\mu_1/\eta_1)t}] \tag{2.2}$$

$$\tau^1{}_{12}(y_1, y_2, y_3, t) = [\tau_\infty(t) - ((\tau_\infty(0) - \tau^1{}_{12})_0) e^{(-\mu_1/\eta_1)t}] \tag{2.3}$$

Where $\tau^1{}_{11}, \tau^1{}_{12}$ and $e^1{}_{11}, e^1{}_{12}$ are given by (B).

RESULTS AND DISCUSSIONS

(A) Displacements on the free surface y_3=0.

We first consider the displacement $U^1{}_1$ due to the movement of the fault for y_3=0.The expression for $U^1{}_1$ is given in (2.1). Figure2: shows the variation of $U^1{}_1$ against y_2 for some selective values of y_1 representing the distance of the point along the strike of the fault. It is found that,

(i) $U^1{}_1$ is asymmetric with respect to $y_2 = 0$;

(ii) For comparatively large values of y_2 the magnitude of $U^1{}_1$ tends to zero at about $|y_2|$ =100km.

(iii) In each case for negative y_1, $U^1{}_1$ is the same for $y_1>0$.

(iv) $|U^1{}_1| \to 0$ as $|y_1|$ increases.

(v) $|U^1{}_1|$ always remains bounded. It attains it's extreme at points which gradually drift away from y_1=0 with increase in y_2. The maximum magnitude of $U^1{}_1$ is found to be of the order of 3 cm. one year after the commencement of the fault slip at points very close to the fault line on the free surface as is clear from the Fig.2.

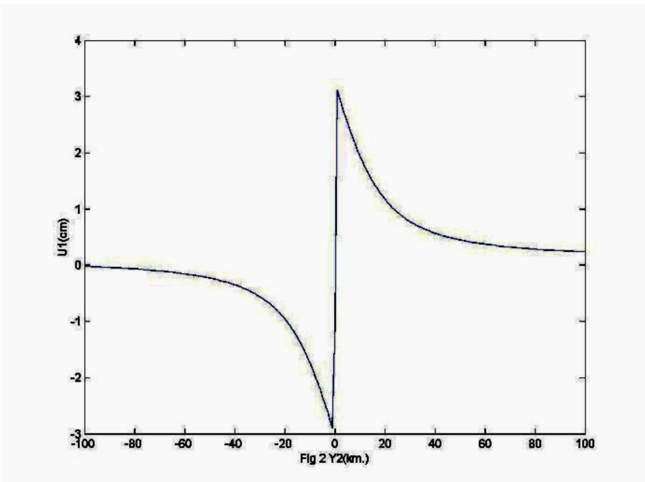

Fig.2. Variation of surface displacement $U^1{}_1$ with y_2 for y_3=0,y_1=10km and t_1=1 year due to fault movement

(B) Spacial variation of stresses due to fault movement with depth (with t_1=1 year)

(i) Variation of shear stress t_{12} with depth due to fault movement.

Numerical computational works carried out for computing the values of t_{12} at different points of the free surface. In the Fig.3 it is observed that as we go down along the line y_1=10km,y_2=10km for H=40km the accumulation of shear stress occurred with increasing depth with varying magnitude of accumulation. The magnitude of accumulation first increases up to a depth of about 14 km. attaining a maximum value 0.2 bar per year there and there after decreases sharply up to a depth of about 40 km and after that the magnitude of stress accumulation is found to die out gradually as depth increases. It is also observed that the accumulation of stress pattern is the same for y_1=±10km ,y_2=±10km. Also along the line far away from the fault, y_1=30km,y_2=30km the shear stress accumulation pattern is the same with less numerical value.

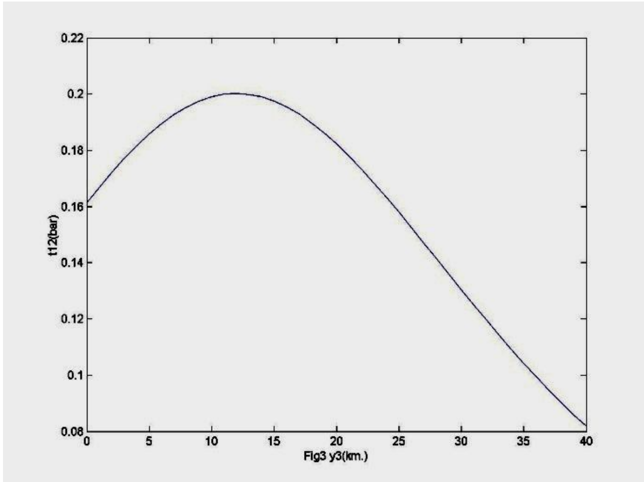

Fig.3. Variation of shear stress t_{12} with depth y_3 for y_1=10km.y_2=10km,H=100km and t_1=1year due to the fault movement

(C) Spacial variation of shear strain $e^1{}_{12}$ due to fault movement with depth (for t_1=1 year)

Fig.4 Shows the variation of surface shear strain E_{12} with y_2 for y_3=0km.y_1=10km and t_1=1 year. It is observed that the magnitude of this shear strain is of order 10^{-6} which is well matched with the observational value, the strain first increases attaining a maximum at about $|y_2|$=20km and then gradually decreases to zero as we move away from the fault.

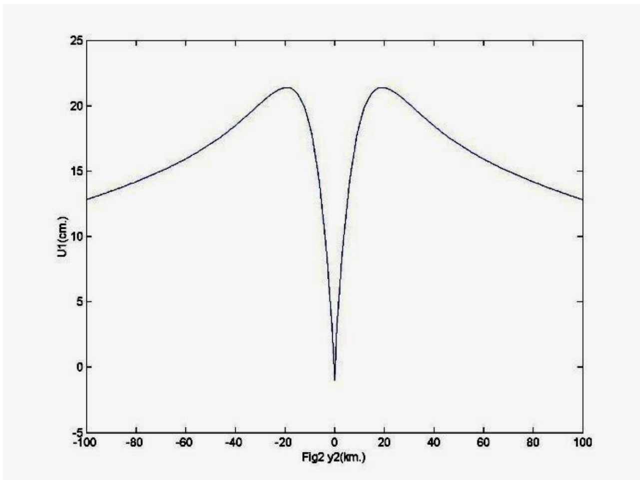

Fig.4. Varion of surface shear strain E_{12} for y_3=0km,y_1=10km ,t_1=1 year with y_2 due to fault movement

(D)Temporal variation of shear stress $\tau^1{}_{12}$

Figure 5. shows rate of shear stress $\tau^1{}_{12}$ accumulation/release at y_3=0km,y_1=10km and y_2=10km.It is observed that the rate of shear stress accumulation is linear, represented by a straight line which does not pass through the origin, which is justified as the material is assumed to be linear viscoelastic of Maxwell type which carries memory.

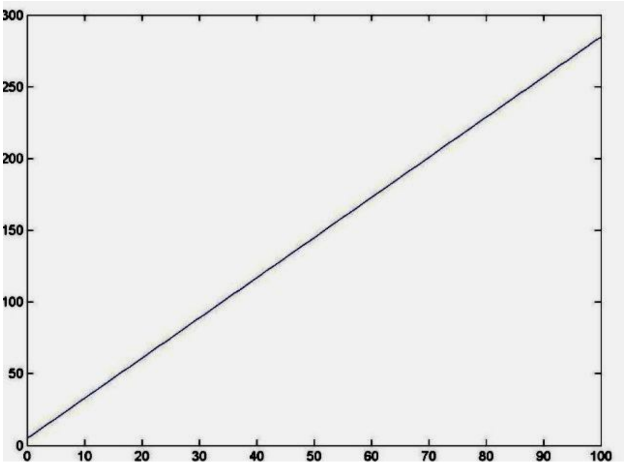

Fig.5. Variation of shear stress τ^1_{12} with time t

Appendix-II

Solutions after the fault movement

We assume that after a time T_1 the stress component τ^1_{12} (which is the main driving force for the strike-slip motion of the fault) exceeds the critical value τ_c, the fault F starts slipping. Then we have an additional condition characterizing the dislocation in u_1 due to the creeping movement as:

$$\left[(u^1{}_1)\right]_F = Uf_1(y_1, y_3)H(t_1) \qquad (4.1)$$

where $\left[(u^1{}_1)\right]_F$ = The discontinuity of u^1_1 across F given by

$$\left[(u^1{}_1)\right]_F = \lim_{(y_2\to 0+)}(u^1{}_1) - \lim_{(y_2\to 0-)}(u^1{}_1)$$
$$(-L \le y_1 \le L, d \le y_3 \le D) \qquad (4.2)$$

where $H(t_1)$ is the Heaviside function.

Taking Laplace transformation in (4.1), we get,

$$\left[\overline{(u^1{}_1)}\right]_F = (U/p)\, f_1(y_1, y_3) \qquad (4.3)$$

The fault slip commences across F after time T_1,

Clearly, $\left[(u^1{}_1)\right]_F = 0$

for $t_1 \le 0$, where $t_1 = t - T_1$, F is located in the region $(-L \le y_1 \le L, y_2 = 0, d \le y_3 \le D)$.

We try to find the solution as :

$$u^1{}_1 = (u^1{}_1)_1 + (u^1{}_1)_2,\ u^1{}_2 = (u^1{}_2)_1 + (u^1{}_2)_2,$$
$$u^1{}_3 = (u^1{}_3)_1 + (u^1{}_3)_2,$$
$$\tau^1{}_{11} = (\tau^1{}_{11})_1 + (\tau^1{}_{11})_2,\ \tau^1{}_{12} = (\tau^1{}_{12})_1 + (\tau^1{}_{12})_2,$$
$$\tau^1{}_{13} = (\tau^1{}_{13})_1 + (\tau^1{}_{13})_2,\ \tau^1{}_{22} = (\tau^1{}_{22})_1 + (\tau^1{}_{22})_2,$$
$$\tau^1{}_{23} = (\tau^1{}_{23})_1 + (\tau^1{}_{23})_2,\ \tau^1{}_{33} = (\tau^1{}_{33})_1 + (\tau^1{}_{33})_2 \qquad (4.4)$$

where $(u^1{}_i)_1$, $(\tau^1{}_{ij})_1$, are continuous everywhere in the model and are given by (A), i,j=1,2,3. While the second part $(u^1{}_i)_2$, $(\tau^1{}_{ij})_2$ are obtained by solving modified boundary value problem as stated below. We note that $(u^1{}_2)_2$, $(u^1{}_3)_2$, are both continuous even after the fault creep, so that $[(u^1{}_2)]_2 = 0, [(u^1{}_3)]_2 = 0$, while $(u^1{}_1)_2$ satisfies the dislocation condition given by (4.2).

The resulting boundary value problem can now be stated as: $(u^1{}_1)_2$ satisfies 3D Laplace equation as

$$\nabla^2 \overline{(u^1{}_1)}_2 = 0 \qquad (4.5)$$

where $\overline{(u^1{}_1)}_2$ is the Laplace transformation of $(u^1{}_1)_2$ with respect to t, with the modified boundary condition. $\overline{\tau^1{}_{12}}(y_1, y_2, y_3, p) = 0$ as $[y_2]\to\infty$, $-\infty<y_1<\infty$, $y_3 \ge 0$. (4.6)

the other boundary conditions are same as before. We solve the above boundary value problem by modified Green's function method following Maruyama (1966), Rybicki (1971), and the correspondence principle.

Let Q (y_1, y_2, y_3) be any point in the field and $P(x_1, x_2, x_3)$ be any point on the fault, then we have,

$$\overline{(u^1{}_1)}_2(Q) = \int\int_F [(u^1{}_1)_2(P)]G(P,Q)\,dx_3\,dx_1$$
$$= \int\int_F Uf_1(x_1, x_3)G(P,Q)\,dx_3\,dx_1 \qquad (4.7)$$

where G is the Green's function satisfying the above boundary value problem and

$$G(P,Q) = \frac{\partial}{\partial x_2}G_1(P,Q) \qquad (4.8)$$

Where,

$$G_1(P,Q) = \frac{1}{\left[(y_1-x_1)^2 + (y_2-x_2)^2 + (y_3-x_3)^2\right]^{\frac{1}{2}}} - \frac{1}{\left[(y_1+x_1)^2 + (y_2-x_2)^2 + (y_3-x_3)^2\right]^{\frac{1}{2}}}$$

$$-(1/4\pi\mu_1)\sum_{1}^{\infty}\left[(\mu_1-\bar{\mu}_2)/(\mu_1+\bar{\mu}_2)\right]^m$$
$$\{1/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3-2mH-y_3)^2]^{1/2}+1/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3-2mH+y_3)^2]^{1/2}+1/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3+2mH+y_3)^2]^{1/2}+1/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3+2mH-y_3)^2]^{1/2}\},\ \text{where},\ 0\le y_3\le H. \qquad (4.9)$$

Therefore,

$$G(P,Q) = \frac{(y_2-x_2)}{\left[(y_1-x_1)^2 + (y_2-x_2)^2 + (y_3-x_3)^2\right]^{\frac{3}{2}}} - \frac{(y_2-x_2)}{\left[(y_1+x_1)^2 + (y_2-x_2)^2 + (y_3-x_3)^2\right]^{\frac{3}{2}}}$$

$$-(1/4\pi\mu_1)\sum_{1}^{\infty}\left[(\mu_1-\bar{\mu}_2)/(\mu_1+\bar{\mu}_2)\right]^m$$
$$\{(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3-2mH-y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3-2mH+y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3+2mH+y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3+2mH-y_3)^2]^{3/2}\}$$

$$\overline{(u^1{}_1)}_2(Q) = \int\int_F U(P)\, f_1(x_1, x_3)\times$$
$$\left[\frac{(y_2-x_2)}{\left[(y_1-x_1)^2 + (y_2-x_2)^2 + (y_3-x_3)^2\right]^{\frac{3}{2}}}\right.$$

$$-\frac{(y_2-x_2)}{\left[(y_1+x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{\frac{3}{2}}}]$$

$-(1/4\pi\mu_1)\sum_{1}^{\infty} [(\mu_1-\bar{\mu}_2)/(\mu_1+\bar{\mu}_2)]^m\{(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3-2mH-y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3-2mH+y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3+2mH+y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3+2mH-y_3)^2]^{3/2}\}dx_3dx_1]$

$= U\ \phi(y_1,y_2,y_3)$ (say) (4.10)

where,

$$\phi(y_1,y_2,y_3)=\int\int_F f_1(x_1,x_3)\times$$

$$[\frac{(y_2-x_2)}{\left[(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{\frac{3}{2}}}$$

$$-\frac{(y_2-x_2)}{\left[(y_1+x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{\frac{3}{2}}}]$$

$-(1/4\pi\mu_1)\sum_{1}^{\infty} [(\mu_1-\bar{\mu}_2)/(\mu_1+\bar{\mu}_2)]^m$
$\{(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3-2mH-y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3-2mH+y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3+2mH+y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3+2mH-y_3)^2]^{3/2}\}dx_3dx_1]$

(4.11)

Taking inverse Laplace transformation,

$$(u^1{}_1)_2(Q)=U\phi(y_1,y_2,y_3)H(t_1)$$

where H (t_1) is the Heaviside step function, which gives the displacement at any points $Q\ (y_1,y_2,y_3)$.

We also have,

$$\overline{(\tau^1{}_{11})}_2=\mu_1\left(\frac{\partial\overline{(u^1{}_1)}_2}{\partial y_1}\right) \qquad (4.12)$$

and similar other equations.

$$\text{Now, } \frac{\partial\overline{(u^1{}_1)}_2}{\partial y_1}=U\frac{\partial(\phi)}{\partial y_1}$$

$$=U\phi_1 \text{ (say)}$$

where,

$$\phi_1(y_1,y_2,y_3)=\frac{\partial}{\partial y_1}\left(\int\int_F f_1(x_1,x_3)\right.$$

$$\times[\frac{(y_2-x_2)}{\left[(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{\frac{3}{2}}}$$

$$-\frac{(y_2-x_2)}{\left[(y_1+x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{3/2}}]$$

$-(1/4\pi\mu_1)\sum_{1}^{\infty} [(\mu_1-\bar{\mu}_2)/(\mu_1+\bar{\mu}_2)]^m$

$\{(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3-2mH-y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3-2mH+y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3+2mH+y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3+2mH-y_3)^2]^{3/2}\}$
$dx_3dx_1)$

(4.13)

Using (4.13) and taking inverse Laplace transformation, we get

$$(\tau^1{}_{11})_2=H(t-T_1)/(2\times\pi)$$

$$[U-\mu_1/\eta_1\int_0^t U(\tau)e^{(-\mu_1/\eta_1)(t-\tau)}d\tau]$$

$$\times\frac{\partial}{\partial y_1}\left(\int\int_F f_1(x_1,x_3)\right.$$

$$\times[\frac{(y_2-x_2)}{\left[(y_1-x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{\frac{3}{2}}}$$

$$-\frac{(y_2-x_2)}{\left[(y_1+x_1)^2+(y_2-x_2)^2+(y_3-x_3)^2\right]^{3/2}}]$$

$-(1/4\pi\mu_1)\sum_{1}^{\infty} [(\mu_1-\bar{\mu}_2)/(\mu_1+\bar{\mu}_2)]^m$

$\{(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3-2mH-y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3-2mH+y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3+2mH+y_3)^2]^{3/2}+(x_2-y_2)/[(y_1-x_1)^2+(x_2-y_2)^2+(x_3+2mH-y_3)^2]^{3/2}\})dx_3dx_1)$

(4.14)

Similarly the other components of the displacements, stresses and strains can be found out. These are given in (B).

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