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RESEARCH ARTICLE

MAXIMUM NETWORK FLOW APPROACH FOR FUZZY TRANSPORTATION PROBLEMS

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ABSTRACT

A new approach is proposed for finding an optimal solution for fuzzy transportation problem involving fuzzy trapezoidal numbers. This method is demonstrated with a numerical example.

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INTRODUCTION

Network flow problems are central problems in operations research and they arise in many real world applications. The concept of network is not new. It was implicit in the work of Quesnay (1758), Cournot(1838) and Pigou (1920). The latter studied a network system in the setting of a transportation network. The purpose of traditional transportation problems is to determine the optimal transportation pattern of a certain good from suppliers to demand customers so that the total transportation cost becomes minimum. It has been investigated by many researchers and is known as the Hitchcock Koopman transportation problem. Sometimes estimate of the supply and demand values may be imprecise. This imprecision may follow from the lack of exact information. So, fuzziness is introduced. The theory of fuzzy sets was introduced by Zadeh in 1965. The concept of Fuzzy transportation problem can be solved by several methods. Here we introduced the special structure of fuzzy transportation problems by applying the maximum network flow approach. The objective in a maximum network flow approach is to maximize the flow through the network from a single source to a single sink, while the arcs can only carry a limited flow. The basic idea behind the proposed approach is to systematically append label to the nodes of a network until the optimum solution is reached. The theory of network flows was discussed by Ravindra K. Ahuja and Andrew V. Goldberg.

Preliminaries

Fuzzy set

A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse $X$to the unit interval [0,1] $i.e., A = \{x,\ \mu_A(x)/x \epsilon X\}$. Here $\mu_A: X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \epsilon X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from 0 to1.

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Fuzzy number

A fuzzy number A is a fuzzy subset of the set of real numbers R which satisfies the following properties:

(i) A must be a normal fuzzy subset of R.
(ii) Each α-cut of A must be a closed interval for every $\alpha \in (0,1]$.
(iii)The support of A must be bounded.

Trapezoidal Fuzzy Number

A Trapezoidal fuzzy number $\tilde{a}$ is a fuzzy fully specified by 4-tuples (a_1, a_2, a_3, a_4) such that $a_1 \leq a_2 \leq a_3 \leq a_4$ with membership function defined as

$$\mu_{\tilde{a}} = \begin{cases} \frac{x - a_1}{a_2 - a_1} & if \ a_1 \leq x \leq a_x \\ 1 & if \ a_2 \leq x \leq a_3 \\ \frac{x - a_4}{a_3 - a_4} & if \ a_3 \leq x \leq a_4 \\ 0 & otherwise \end{cases}$$

Types of Trapezoidal Fuzzy Number

- *Non Negative Trapezoidal Fuzzy Number*
  Trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ is said to be non negative trapezoidal fuzzy number if and only if $a_1 - a_3 \geq 0$
- *Zero Trapezoidal Fuzzy Number*
  Trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ is said to be zero trapezoidal fuzzy number if and only if $a_1 = 0, a_2 = 0, a_3 = 0, a_4 = 0$
- *Equal Trapezodial Fuzzy Number*
  Two Trapezoidal fuzzy numbers $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ are said to be equal is., $\tilde{a} = \tilde{b}$, if and only if $a_1 = b_1, a_2 = b_2, a_3 = b_3, a_4 = b_4$

Magnitude of Trapezoidal Fuzzy Number

The magnitude of the trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ is given by
$$Mag(\tilde{a}) = \frac{a_1 + 5a_2 + 5a_3 + a_4}{12}$$

Fuzzy Transportation Problem

The fuzzy transportation is written in the following form:

Minimize $\tilde{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \, x_{ij}$
Subject to
$\sum_{j=1}^{n} x_{ij} \leq \tilde{s}_i \ , \quad i = 1, 2, \ . \ . \ . \ m$
$\sum_{i=1}^{m} x_{ij} \geq \tilde{d}_j \ , \quad j = 1, 2, \ . \ . \ . \ n$
$x_{ij} \geq 0 \ , \ \forall \, i, j$
where $\tilde{s}_i = (a_1, a_2, a_3, a_4)$, $\tilde{d}_j = (b_1, b_2, b_3, b_4)$ and $\tilde{c}_{ij} = (c_{ij}, c_{ij}, c_{ij}, c_{ij})$ representing the uncertain supply and demand for transportation problem.

Network

A network is a schematic diagram, consisting of points which are connected by lines or arrows. The points are referred to as nodes and the lines are called arcs. A flow may occur between two nodes, via an arc. When the flow is restricted tone direction, then the arcs are pointed and the network is referred to as a directed network.

Flow-augmenting Path

A Flow augmented path is a directed path with flow from the source to the sink in the residual network such that every arc on this path has positive residual capacity.

Max Flow Theorem

The Maximum flow F in the network structure $\sum_{j=1}^{n} x_{ij} \leq \tilde{s}_i$, $i = 1,2, \ldots m$
$\sum_{i=1}^{m} x_{ij} \geq \tilde{d}_j$, $j = 1,2, \ldots n$ and $x_{ij} \geq 0$, $\forall\, i,j$ through $c_{ij} - u_i - v_j \geq 0$, $\forall\, i,j$ is equal to the maximum flow capacity relative to the source and sink.

Fuzzy Maximum Network Flow Approach

Step 1 Let us begin with any feasible flow. Certain constants u_i , $i = 1,2, \ldots n$ and v_j , $j = 1,2, \ldots n$ yielding nonnegative relative costs $c_{ij} - u_i - v_j \geq 0$, verify whether a feasible solution exists using only tracks with relative costs equal to zero. If so, stop, since the result is optimal; otherwise go to Step 2.

Step 2 Improve u_i and v_j such that at least one new track has relative cost equal to zero and also we should be careful not to destroy the equality $c_{ij} - u_i - v_j = 0$. A rule that achieves all these conditions is

(i) Add f to u_i if Node s_i is scanned,
(ii) Subtract f to v_j if Node d_j is scanned, where

$f =$ {Smallest relative cost for arcs between every scanned Node s_i and unlabeled Node d_j}

Step 3 Represent a network flow diagram comprised only of tracks with relative cost equal to zero of the revised relative costs $c_{ij} - u_i - v_j$.

Step 4 Starting at Node 0(source), Label Node 0 with the mark $*$ and put a $(+)$ on each arc $(0, j)$ without flow and label Node j with a check mark $*$. Label Node 0 with the mark $\boxed{*}$.

Step 5 Consider any Node j that is labeled $*$. Put a $(+)$ on every flowless outward arc (j, k) if Node k is not labeled, and label Node k with $*$. Then put a $(-)$ on every inward arc (k, j) with flow if Node k is not labeled, and label Node k with $*$. Finally, label Node j with the mark $\boxed{*}$ that the Node also has been scanned.

Step 6 Continuing in this way until the sink Node is labeled or all labeled Nodes have been scanned. A breakthrough occurs as soon as the sink Node is labeled, because a flow-augmenting path has been discovered from source Node to sink Node. Add some required flow on each arc with a $(+)$ and remove the flow from each arc with a $(-)$. Return to Step 4 erasing all the previous labels ($*$, $\boxed{*}$) and signs $(+, -)$. If however, sink Node remains unlabeled at the termination of Step 5, then the current solution is maximal.

Step 7 (Stopping Rule & Optimality)
Every arc from a scanned node to an unlabeled node is at its full capacity and every arc from an unlabeled node to a scanned node is at zero flow and if the flow augmented value is equal to the sum of suppliers(or sum of demands) then, the solution is optimal.

Numerical Example

Consider a fuzzy transportation problem where all the parameters are trapezoidal fuzzy numbers

Table 1. Fuzzy Transportation Problem

	D_1	D_2	D_3	D_4	Supply
S_1	(0,1,3,4)	(9,10,11,12)	(6,12,18,24)	(0,0,0,0)	(1,3,5,7)
S_2	(4,8,12,16)	(7,15,22,28)	(8,16,24,32)	(3,7,11,15)	(6,12,18,24)
S_3	(6,12,18,24)	(9,19,29,39)	(11,21,31,41)	(9,10,11,12)	(0,1,3,4)
S_4	(9,10,11,12)	(10,20,30,40)	(8,16,24,32)	(2,6,10,14)	(3,6,9,12)
Demand	(3,7,11,15)	(10,11,12,13)	(2,4,6,8)	(2,3,4,5)	

Now we find the magnitude of each cell, supplies and demands as

Table 2.

$M(\tilde{C}_{11}) = 2$	$M(\tilde{C}_{12}) = 10$	$M(\tilde{C}_{13}) = 15$	$M(\tilde{C}_{14}) = 0$
$M(\tilde{C}_{21}) = 10$	$M(\tilde{C}_{22}) = 18$	$M(\tilde{C}_{23}) = 20$	$M(\tilde{C}_{24}) = 9$
$M(\tilde{C}_{31}) = 15$	$M(\tilde{C}_{32}) = 24$	$M(\tilde{C}_{33}) = 26$	$M(\tilde{C}_{34}) = 10$
$M(\tilde{C}_{41}) = 10$	$M(\tilde{C}_{42}) = 25$	$M(\tilde{C}_{43}) = 20$	$M(\tilde{C}_{44}) = 8$
$M(S_1) = 4$	$M(S_2) = 15$	$M(S_3) = 2$	$M(S_4) = 7$
$M(D_1) = 9$	$M(D_2) = 11$	$M(D_3) = 5$	$M(D_4) = 3$

Now, we replace these values for their corresponding cell in which result in a convenient balanced transportation,

Table 3. Crisp Transportation Problem

	D_1	D_2	D_3	D_4	Supply
S_1	2	10	15	0	4
S_2	10	18	20	9	15
S_3	15	24	26	10	2
S_4	10	25	20	8	7
Demand	9	11	5	3	

The above crisp transportation problem can be uttered in the form of network chart below,

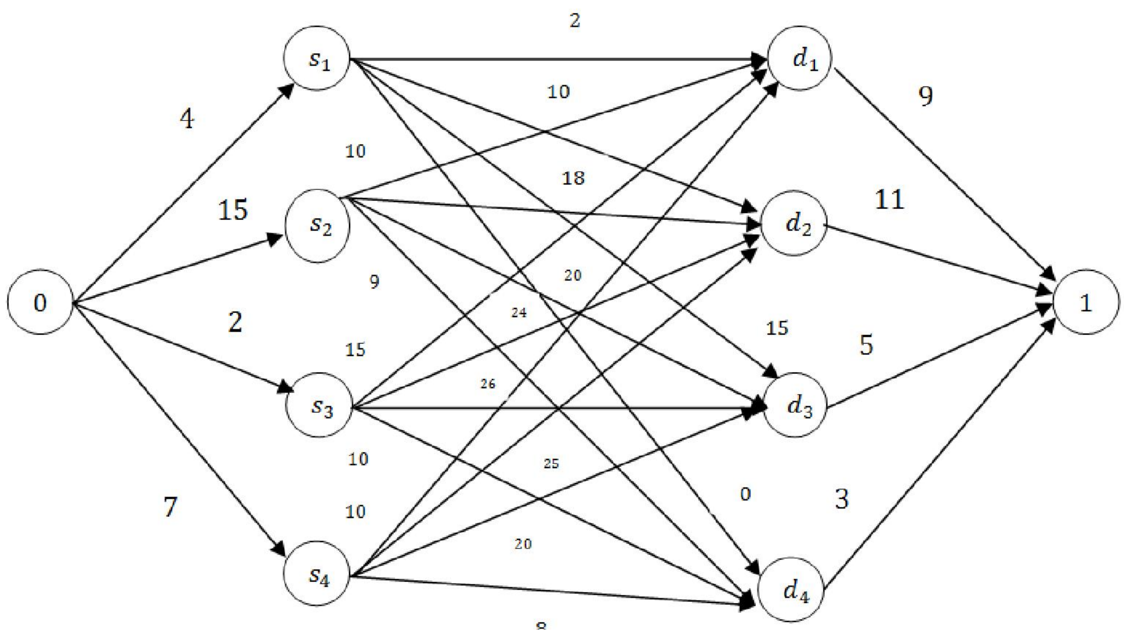

Fig. 1. Network Model

Trial values for u_i and v_j and the corresponding relative costs $c_{ij} - u_i - v_j$ are shown in Table 4.

Table 4.

					u_i
	0	0	3	0	0
	0	0	0	1	8
	3	4	4	0	10
	0	7	0	0	8
v_j	2	10	12	0	

A network flow chart comprised only of the tracks with relative costs equal to zero and a trial flow 25 units is also indicated as,

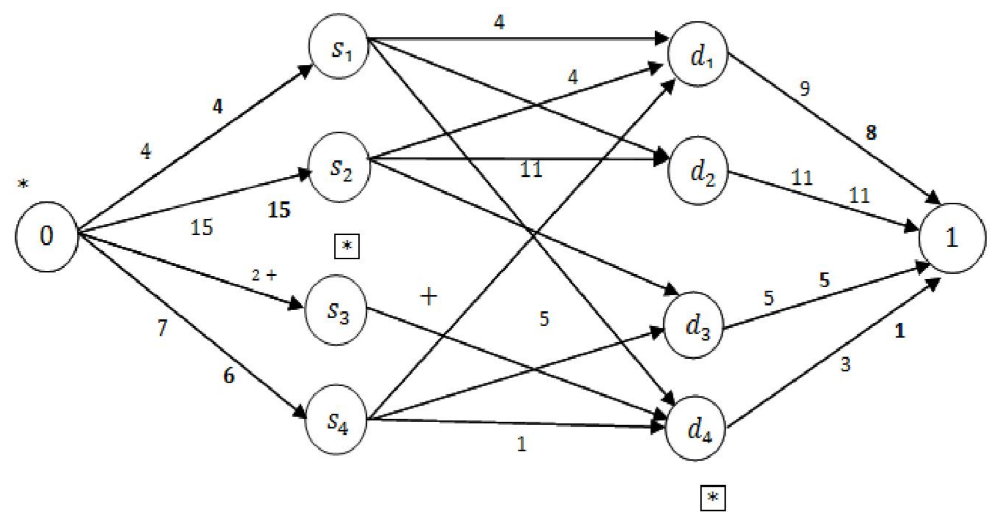

Fig. 2. Initial Flow 25

The total flow is only 25 units. Since, the trial solution is not feasible in the transportation problem; we must carry out in Step 1 the maximum flow algorithm to determine whether additional flow can be routed to the network. This process labels Nodes 0, s_3 and d_4 does not result in flow augmented path, for that reason we must go to step 2.

By means of Step 2, improved u_i and v_j and the corresponding new relative costs $c_{ij} - u_i - v_j$ are shown below.

Table 5.

				u_i
	0	0	3	3	0
	0	0	0	4	8
	0	1	1	0	13
	0	7	0	3	8
v_j 2	10	12	-3	

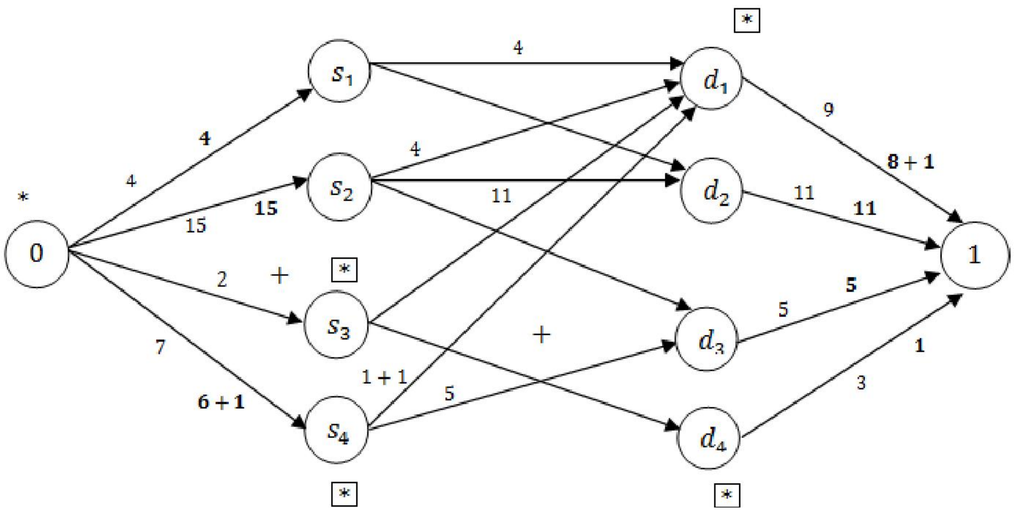

Fig. 2. Flow 26

At this juncture arc (s_3, d_1) has relative cost equal to zero, but arc (s_1, d_4) and (s_4, d_4) comprise the positive relative costs. The integrated network chart with scanning development as exposed below.

Still, the total network flow is only 26. We must make another attempt in the direction of increase the flow. The course of action labels Nodes $s_3, d_1, s_1, d_2, s_2, d_3, s_4, d_4$ and the sink. The flow augmenting path allows 2 units of additional flow, thereby causing the total flow equal to 28. The final optimal steering is shown below.

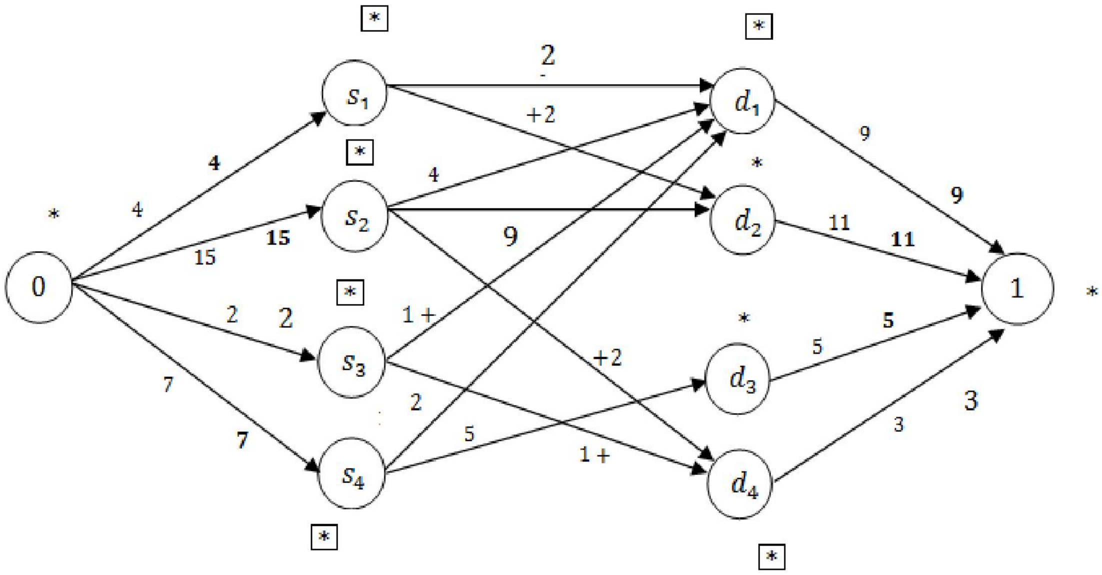

Fig. 3. Flow 28 (Optimal Solution)

Conclusion

Fuzzy transportation problem is much more natural to originate them in terms of nodes and arcs, taking advantage of the special structure of the problem. The core objective in a maximum network flow technique is that seek to maximize the flow through a flow network from a single source to a single sink, while minimizing the cost of that flow. As in the MODI method, the maximum flow is evenly distributed to entire cells in our proposed method. This proposed algorithm often reaches an optimal solution much faster than the linear programming solvers. In future, many optimized algorithms can be developed to solve the network flow problems in more efficient manner.

REFERENCES

"Maximum Flow Problem" http://en.wikipedia.org/wiki/Maximum_flow_Problem.

"Network Flow Programming" www.sce.carleton.ca/faculty/chinneck/po/Chapter 10.pdf.

"Recent Developments in Maximum Flow Algorithms", www.springerlink.com/index/tfdyr7n9xyer80p7.pdf.

Aguado, J.S. 2009. Fixed charge Transportation Problem: A new heuristic approach based on Lagrangian relaxation and the solving of core problems. Annals of OR 172(1), 45-69.

Andrew V .Goldberg, Eva Tardos, and Robert E.Tarjan. "Network flow algorithms. Paths, Flows and VLSI-Layas", Springer-Verlag, 1990, Pages 101-164.

Bazarra.M.S, John. J.Jarvis, Hanif, D. and Sherali, 2005. Linear Programming and networks flows

Ford, L.R.Jr and D-R. Fulkerson. Flows in networks. Princeton University Press, 1962

Harvey M.Wagner, "Principles of Operations Research", PHI Learning Private Limited, New Delhi-110001, 2010

Makani Das and Hemanta K. Baruah, Solution of the TP in Fuzzified form, The Journal of fuzzy mathematics Vol.15, No.1.2007.Los Angeles

Ravindra K.Ahuja, Thomas L. Magnanti and James B. Orlin 1993. Network Flows: Theory, Algorithms and Applications. Prentice Hall.

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