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RESEARCH ARTICLE

INTEGRATED RELIABILITY MODEL FOR REDUNDANT SYSTEMS WITH MULTIPLE CONSTRAINTS

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ABSTRACT

The Integrated Reliability Models for Redundant Systems with multiple constraints for one mathematical function is established by applying Lagrangean approach, the related case problem is presented to find the component reliabilities (r_j), the number of components in each stage (x_j), stage reliability (R_j) and the System Reliability (R_s). The Lagrangean approach which is an effective procedure has got certain limitations in the sense that the number of components in each stage will be given only in real numbers, which is infeasible for implementation to real life problems. The purpose of this Paper is to optimize a class of Integrated Reliability Model for Redundant Systems with Cost and Weight as additional constraints apart from basic cost constraint. By taking the mathematical function $r_j = (c_j/b_j)^{1/d}{}_j$ the optimum component reliability, stage reliability, the number of components in each stage and the system reliability are determined after taking the pre-determined values of Cost, and Weight. In this work, an attempt is made to develop an integrated reliability redundant model for a Series – Parallel configuration subject to the multiple constraints. Generally reliability is treated as the function of Cost but in any given practical situation apart from cost, other constraint like Weight will have hidden impact on the reliability of the system. In this model the Lagrangean technique is implemented to determine the Cost and Weight as constraints. The model has yielded very encouraging results and it can be applied to any type of system, simple or complex. The advantage of this model is very flexible and requires little processing time.

INTRODUCTION

The development of science and technology and the needs of modern society are racing against each other. Industries are trying to introduce more and more automation in their industrial processes in order to meet the ever increasing demands of the society. The complexities of industrial system as well as their products are increasing day-by-day. The improvement in effectiveness of such complex systems has therefore acquired special importance in recent years. The effectiveness of a system is understood to mean the suitability of the system for the fulfillment of the intended tasks and the efficiency of utilizing the means put it in to it. The suitability of performing definite tasks is primarily determined by the "reliability" and "quality" of the system. Reliability is the probability of a device performing its purpose adequately for the period intended under the given operating conditions. It is increasingly necessary to design reliable systems as there is a great demand for products that offer quality and safety. Another way of improving the reliability of a system is to use redundant components or redundant sub-components. However an increase in the number of components and sub systems consequently results in project costs, weight and volume of the system and the design parameters increasing. Hence, it is necessary to use optimization techniques in order to obtain and optimum system with in the desired constraints. This Paper deals with reliability optimization using redundant component connected in Series – Parallel configuration. So far as the literature on maximization of system Reliability problems are concerned, the researchers opine that optimization problems can be handled with multiple constraints also, but to the best of the knowledge of the authors, the optimization of Integrated Reliability Models for Redundant systems with multiple constraints are not reported. In this scenario, the authors want to make an attempt to optimize the reliability of Integrated Reliability Model with multiple constraints.

To study and optimize, the Integrated Reliability Model for Redundant Systems with multiple constraints is considered with cost, weight and volume as constraints for the given known mathematical function $r_j = (c_j/b_j)^{1/d}{}_j$, To establish the results for the above specified mathematical function, Lagrangean Multiplier Method is applied to calculate the number of components in each stage, component reliability and corresponding stage reliability in real value numbers.

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Since, the number of components in each stage cannot be rounded off to nearest integer due to variation in cost and reliability. Finally the result of this work supports the researcher's statement of the problem concurring that these models are particularly of high application value for any Series – Parallel Redundant Systems with Multiple Constraints.

Statement of the problem

The problem considers the component reliabilities and the no. of components in each stage are unknowns for the given constraints to maximize the system reliability. The authors in this work make an attempt to negotiate the impact of the cost and weight as constraints in optimizing the redundant systems under consideration for the selected above mathematical function. Though Cost has direct relation in maximizing System Reliability, the indirect impact of Weight as additional constraints in optimizing the Reliability of a redundant system presents a novel beginning in the mentioned area of research. The Series – Parallel Systems are considered with Cost, and Weight as constraints to maximize the Reliability of a redundant system as its objective function.

Assumptions of the model

1. All the components in each stage are assumed to be identical.

2. The components are assumed to be statistically independent i.e. the failure of one component does not affect the performance of the other components in the system.

3. A component is either in working condition or non-working condition.

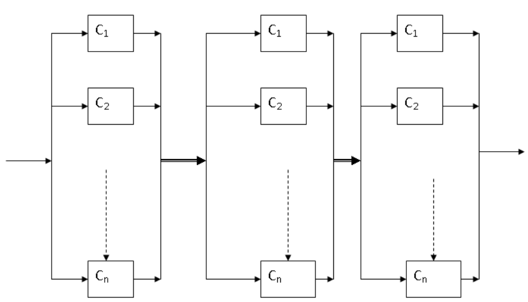

FIG: SERIES-PARALLEL CONFIGURATION

IV. NOMENCLATURE

R_s = System Reliability.
R_j = Stage Reliability, $0<R_j<1$
r_j = Reliability of each component in stage j, $0<r_j<1$.
x_j = No. of components in stage j.
c_j =Cost coefficient of each component in stage j
w_j = Weight coefficient of each component in stage j.
v_j = Volume coefficient of each component in stage j.
C_o = Maximum allowable System Cost.
W_o = Maximum allowable System Weight.

Mathematical model

Consider that there are 'n' statistically independent stages in Series with x_j statistically independent in each stage.

System Reliability for the given cost function

$$R_s = \prod_{j=1}^{n} R_j = \prod_{j=1}^{n} [\, 1-(1-r_j)^{x_j} \,] \quad (1)$$

Subjected to $\sum_{j=1}^{n} c_j.x_j \le C_o$ (2)

$\sum_{j=1} w_j.x_j \le W_o$ (3)

Non negativity restriction x_j is an integer and r_j, R_j >0

Mathematical function

Cost co efficient of each component in stage 'j' is derived from the following relationship between Cost and Reliability.

$$r_j = [c_j/b_j]^{(1/d_j)} \quad (4)$$

Where c_j is cost constraint and b_j, d_j are constants.

Problem formulation

System Reliability for the given cost function

$$R_s = \prod_{j=1}^{n} R_j \quad (5)$$

The equation (4) can be re written as

$$c_j = b_j.r_j^{d_j} \quad \text{.........Cost constraint} \quad (6)$$

$$w_j = p_j.r_j^{q_j} \quad \text{.........Weight constraint} \quad (7)$$

The number of components at each stage x_j is given through the relation

$$x_j = \frac{ln(1-R_j)}{ln(1-r_j)} \quad (8)$$

Maximize

$$R_s = \prod_{j=1}^{n} [1-(1-r_j)^{x_j}] \quad (9)$$

Subject to the constraints

$$\sum_{j=1}^{n}\left[\left(b_j.r_j^{d_j}\right).\frac{ln(1-R_j)}{ln(1-r_j)}\right] - C_0 \le 0 \quad (10)$$

$$\sum_{j=1}^{n}\left[\left(p_j.r_j^{q_j}\right).\frac{ln(1-R_j)}{ln(1-r_j)}\right] - W_0 \le 0 \quad (11)$$

Lagrangean method

The proposed formulation is solved by using Lagrangean method.

$$F = R_s + \lambda_1 \left[\sum_{j=1}^{n} \left\{ \left(b_j . r_j^{d_j} \right) \frac{ln(1-R_j)}{ln(1-r_j)} \right\} - C_0 \right] + \lambda_2 \left[\sum_{j=1}^{n} \left\{ \left(p_j . r_j^{q_j} \right) \frac{ln(1-R_j)}{ln(1-r_j)} \right\} - W_0 \right] = 0 \quad (12)$$

where λ_1 and λ_2 are lagrangean multipliers and F being Lagrangean function. The number of components in each stage (xj), optimum component reliability (rj), stage reliability (Rj) and the system reliability (Rs) are derived from the lagrangean method. The method provides real valued solution with reference to cost and weight. The stationary point can be obtained by differentiating the Lagrangean function with respect to R_j, r_j, λ_1, λ_2 and λ_3.

$$\frac{\partial F}{\partial r_j} = \lambda_1 \left[\left\{ \sum_{j=1}^{n} b_j . \ln(1-R_j) \left(\frac{ \ln(1-r_j).d_j.r_j^{d_j-1} + \frac{r_j^{d_j}}{(1-r_j)} }{ [\ln(1-r_j)]^2 } \right) \right\} \right] + \lambda_2 \left[\left\{ \sum_{j=1}^{n} p_j . \ln(1-R_j) \left(\frac{ \ln(1-r_j).q_j.r_j^{q_j-1} + \frac{r_j^{q_j}}{(1-r_j)} }{ [\ln(1-r_j)]^2 } \right) \right\} \right] = 0 \quad (13)$$

$$\frac{\partial F}{\partial R_j} = 1 + \lambda_1 \left[\sum_{j=1}^{n} \frac{b_j . r_j^{d_j}}{ln(1-r_j)} . \frac{(-1)}{(1-R_j)} \right] + \lambda_2 \left[\sum_{j=1}^{n} \frac{p_j . r_j^{q_j}}{ln(1-r_j)} . \frac{(-1)}{(1-R_j)} \right] = 0 \quad (14)$$

$$\frac{\partial F}{\partial \lambda_1} = \sum_{j=1}^{n} \left[\left(b_j . r_j^{d_j} \right) \frac{ln(1-R_j)}{ln(1-r_j)} \right] - C_0 \leq 0 \quad (15)$$

$$\frac{\partial F}{\partial \lambda_2} = \sum_{j=1}^{n} \left[\left(p_j . r_j^{q_j} \right) \frac{ln(1-R_j)}{ln(1-r_j)} \right] - W_0 \leq 0 \quad (16)$$

RESULTS AND DISCUSSIONS

The following reliability design tables related to cost and weight are calculated by using the component reliabilities and the number of components derived from Lagrangean method.

Case Study

Consider the case of a Mechanical system with three stages for which the component Reliability is given by the equation

$r_j = (c_j/b_j)^{1/d_j}$

To determine the optimum component reliability, stage reliability, number of components in each stage and the System Reliability not to exceed the system cost Rs. 300.00, Weight of the system 450kgs. The component Reliabilities, Stage Reliabilities, Number of components in each stage and the System Reliability are determined by solving the above mathematical function by using MATLAB Version 7.10 and are presented in the following tables.

Cost and Weight as constraints

Reliability Design Without x_j rounding off

Table 1.Reliability design relating to Cost in (Rs)

Stage	r_j	R_j	x_j	c_j	$c_j.\ x_j$
01	0.8676	0.9997	2.26	81.00	183.06
02	0.8810	0.9161	1.16	40.50	46.98
03	0.8882	0.9160	1.13	44.55	50.35
Total Cost					280.39

Table 2.Reliability design relating to Weight (Rs)

Stage	r_j	R_j	X_j	W_j	$W_j.\ X_j$
01	0.8676	0.9997	2.26	75.27	170.1
02	0.8810	0.9161	1.16	116.42	135.0
03	0.8882	0.9160	1.13	122.22	138.1
Total Weight					443.2

System Reliability = 0.8388

Reliability Design with x_j rounding off

The reliability design is reestablished by considering the values of to be integers (by rounding off the value of to the nearest integer) and the relevant results relating to cost, weight and volume are presented in the following table, further giving the information by calculating the variation due to cost, weight, volume and system reliability (before and after rounding off).Table: 3Reliability design relating to Cost in Rs.

Table 3. Reliability design relating to Cost in (Rs)

Stage	r_j	R_j	c_j	x_j	$c_j.\ x_j$
01	0.8676	0.9976	81.00	2	243.0
02	0.8810	0.9858	40.50	2	81.00
03	0.8882	0.9875	44.55	2	89.10
Total Cost					413.0

Table 4. Reliability design relating to Weight (Rs)

Stage	r_j	R_j	x_j	c_j	$c_j.\ x_j$
01	0.8676	0.9976	3	75.27	225.81
02	0.8810	0.9858	2	116.42	232.84
03	0.8882	0.9875	2	122.22	244.44
Total Cost					703.09

System Reliability = 0.9711
Variation in total Cost = 47.33%
Variation in total Weight = 58.62%
Variation in System Reliability = 15.77%

CONCLUSION

All most all the models that are presented primarily considered Cost as the basic constraint. In this scenario, the authors proposed as a class of Integrated Reliability Models for Redundant Systems with multiple constraints as a novel beginning in the mentioned area of research and the optimizing the system reliability for the said model, and the results reported are highly useful for Reliability/Design Engineers. The Lagrangean approach has given the Reliability of three stage system is 0.8388, where the number of components are real. This model can also be further investigated for different mathematical functions of interest and also can be applied for Parallel – Series configuration systems, where the application of these models for such systems will be feasible only when the cost of the system is very low.

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