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RESEARCH ARTICLE

rwμ-COMPACTNESS AND rwμ-LINDEL¨OFNESS INGENERALIZED TOPOLOGICAL SPACES

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ABSTRACT

The purpose of the present paper is to introduce the concepts of rwμ -Lindel¨of spaces in generalized topological spaces and study someof their properties and characterizations.

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INTRODUCTION

In (`A. Cs`asz`ar, 1997; `A. Cs`asz`ar, 2000; `A. Cs`asz`ar, 2002; `A. Cs`asz`ar, 2003; `A. Cs`asz`ar, 2004; `A. Cs`asz`ar, 2005; `A. Cs`asz`ar, 2006; `A. Cs`asz`ar, 2007; `A. Cs`asz`ar, 2008; `A. Cs`asz`ar, 2008; `A. Cs`asz`ar, 2008; `A. Cs`asz`ar, 2009), `A. Cs`asz`ar introduced the concepts of generalized neighborhood systems and generalized topological spaces. He also introduced the concepts of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. In particular, he investigated characterizations for the generalized continuous function by using a closure operator defined on generalized neighborhood systems. In (Al-Omari and Noiri, 2012), A. Al-Omari and Noiri introduced the notions of contra - (μ, λ) - continuity, contra-(α,λ)-continuity, contra-(σ,λ) - continuity, contra - (π,λ) - continuity and contra-(β,λ) - continuity on generalized topological spaces. In this paper, we introduce the concepts of rw μ-Lindel¨of spaces ingeneralized topological spaces and study some of their properties and characterizations. We recall some basic definitions and notations. Let X be a set and denote exp X the power set of X. A subset $\mu$of exp X is said to be a generalized topology (`A. Cs`asz`ar, 2002) (briefly GT) on X if $\phi \in \mu$ and the arbitrary union of elements of μ belongs to. A set X with a GT μ on it is called a generalized topological space and is denoted by (X, μ). Let μ be a GT on X, the elements of μ are called μ-open sets and the complements of μ-open sets are called μ-closed sets. If A $\subseteq$X, then i_μ(A) denotes the union of all μ-open sets contained in A and c_μ(A) is the intersection of all μ-closed sets containing A (`A. Cs`asz`ar, 2005). According to (`A. Cs`asz`ar, 2007), for A $\subseteq$X and x $\in$X, we have x $\in c_\mu$(A) if and only if x $\in$M $\in \mu$ implies M $\cap$ A $\neq$.

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Definition 1.1.A subset A of a space (X, μ) is called

(i) regular open (Stone, 1937) if A = int(cl(A)).
(ii) regular closed (Stone, 1937) if A = cl(int(A)).
(iii) regular semiopen (Cameron, 1978) if there is a regular open set U such that U $\subseteq$A $\subseteq$cl(U).

Definition 1.2.A subset A of a space (X, μ) is said to be rwμ-closed (Vadivel, 2010) ifcl_μ(A) $\subseteq$U whenever A $\subseteq$U and U is regular semiopen in X. The complement of the above mentioned closed sets are respective opensets.

Definition 1.3. (`A. Cs`asz`ar, 2002) Let (X, μ) and (Y, μ') be generalized topological spaces. A function f : (X, μ) $\rightarrow$ (Y, μ') is said to be (μ, μ') - continuous if M$'\in\mu'$implies f^{1}(M$'$) $\in\mu$.

Definition 1.4. (Al-Omari and Noiri, 2012) Let (X, μ) and (Y, μ'') be generalized topological spaces. A function f : (X,μ_) → (Y, μ') is said to be contra- (μ, μ') -continuous if f [1](V) is rwμ-closed in X for each μ'-open set V of Y .

Definition 1.5. (Jyothis Thomas and Sunil Jacob John, 2012) Let (X, μ) be a GTS. A collection of subsets of X is said to be a μ-cover of X if the union of the elements of is equal to X .

Definition 1.6. (Jyothis Thomas and Sunil Jacob John, 2012) Let (X, μ) be a GTS. A μ-sub cover of a μ-cover is a sub collection G of which itself is a μ-cover.

Definition 1.7. (Uma Maheswari *et al.*, 2015) Let (X, μ) be a GTS. A μ-cover of a space X is said to be a rwμ-open cover if the elements of are rwμ-open subsets of X.

Definition 1.8. (Uma Maheswari *et al.*, 2015) Let (X, μ) be a GTS. Then X is said to be rwμ-compact space iff each rwμ-open cover of X has a finite rwμ-open subcover.

2 rwμ-Compact and rwμ-Lindel¨of Spaces

Definition 2.1. A generalized topological space (X, μ) is called rwμ-Lindel¨of if every rwμ-open cover of X has a countable subcover.

The proof of the following theorem is straightforward and thus omitted.

Theorem 2.1. If X is finite (resp. countable) then (X, μ) is rwμ-compact
(resp. rwμ-Lindel¨of) for any generalized topology μon X .

Definition 2.2.A subset B of a generalized topological space (X, μ) is said to be rwμ-compact (resp. rwμ-Lindel¨of) relative to X if, for every collection$\{U_\alpha : \alpha \in \quad\}$of rw$\mu$-open subsets of X such that B ⊆ $\{U_\alpha : \alpha \in \quad\}$, there exists a finite subset Δ0 of Δ such that B ⊆$\cup\{U_\alpha : \alpha \in \quad 0\}$. Notice that if (X, μ) is a generalized topological space and A ⊆X then μ_A= {U∩A: U ∈μ} is a generalized topology on A.(A, μ_A) is called a generalized subspace of (X, μ).

Definition 2.3. A subset B of a generalized topological space (X, μ) is said to be rwμ-compact (resp. rwμ-Lindel¨of) if B is rwμ-compact (resp.rwμ-Lindel¨of) as a generalized subspace of X. The proof of the following theorem is straightforward, and thus omitted.

Theorem 2.2. The finite (resp. countable) union of subsets of X which are rwμ-compact (resp. rwμ-Lindel¨of) relative to X is rwμ-compact (resp.rwμ-Lindel¨of) relative to X.

Theorem 2.3. Let A and B be two subsets of a generalized topological space X with A ⊆B. If A is rwμ-compact (resp. rwμ-Lindel¨of) relative X, then A is rwμ-compact (resp. rwμ-Lindel¨of) relative to B.

Proof. We will show the case when A is rwμ-compact relative to X, the other case is similar. Suppose that $\widetilde{U}$= $\{U_\alpha : \alpha \in \quad\}$is a cover of A by rw$\mu$-open sets in B. Then U$U_\alpha$= S_α∩ B for each α∈Δ, where $S_\alpha$is rwμ-open in X for each α∈Δ. Thus $\tilde{S}$= $\{S_\alpha : \alpha \in \quad\}$is a cover of A by rw$\mu$-open sets in X , but A is rwμ-compact relative X , so there exists a finite subset Δ0 of Δ such that A ⊆$\cup\{S_\alpha : \alpha \in \quad 0\}$, and thus A ⊆$\cup\{S_\alpha \cap$ B: $\alpha \in \Delta 0\}$ = $\cup\{U_\alpha : \alpha \in \Delta 0\}$. Hence A is rwμ-compact relative to B.

Corollary 2.1. Let A be a subset of a generalized topological space X. If A is rwμ-compact (resp. rwμ-Lindel¨of) relative to X , then A is rwμ-compact (resp. rwμ-Lindel¨of).

Theorem 2.4. Let A and B be two subsets of a generalized topological space X with A⊆B. Then A is rwμ-compact (resp. rwμ-Lindel¨of) relative X if and only if A is rwμ-compact (resp. rwμ-Lindel¨of) relative toB.

Proof. Necessity: Follows from Theorem 2.3. Sufficiency: We will show the case when A is rwμ-compact relative to B, the other case is similar. Suppose that $\tilde{S}$= $\{S_\alpha : \alpha \in \quad\}$ is a cover of A by rwμ-open sets in X. Then $\widetilde{U}$= $\{S_\alpha \cap$ B :α∈Δ$\}$is a cover of A. Since$S_\alpha$is rwμ-open in X for each α∈Δ, it follows that S_α∩ B is rwμ-open in B for each α∈Δ, but A is rwμ-compact relative B , so there exists a finite subset Δ0 of Δ such that A ⊆$\cup\{S_\alpha \cap$B : α∈Δ0$\}$ ⊆$\cup\{S_\alpha$: α∈Δ0$\}$. Hence A is rwμ-compact relative to X.

Corollary 2.2. A subset A of a generalized topological space X is rwμ-compact (resp. rwμ-Lindel¨of) if and only if A is rwμ-compact (resp. rwμ-Lindel¨of) relative to X.

Theorem 2.5. If a subset A of X is rwμ-compact (resp. rwμ-Lindel¨of) relative to X and B is a rwμ-closed subset of X, then A ∩ B is rwμ-compact (resp. rwμ-Lindel¨of) relative to X. In particular, a rwμ-closed subset of a rwμ-compact (resp. rwμ-Lindel¨of) space X is rwμ-compact (resp. rwμ-Lindel¨of) relative to X.

Proof. We will show the case when A is rwμ-compact relative to X , the other case is similar. Let $\widetilde{U}$= $\{U_\alpha$: α∈Δ$\}$ be a cover of A ∩ B by rwμ-open subsets of X. Then $\widetilde{U}$ = $\widetilde{U}$∪ {X B} is a cover of A by rwμ-open sets in X, but A is rwμ-compact relative to X, so there exists a finite subset Δ0 of Δ such that A ⊆($\cup\{U_\alpha$: α∈Δ0$\}$) ∪(X B). Thus A∩B ⊆$\cup\{U_\alpha \cap$B : α∈Δ0$\}$ ⊆$\cup\{U_\alpha$: α∈Δ0$\}$. Hence A∩B is rwμ-compact relative to X.

Definition 2.4. A subset F of a space X is called rwμ- F_σ-set if F =∪{Fi : i = 1, 2, . . .} where Fi is a rwμ-closed subset of X for eachi = 1, 2, ...

Theorem 2.6.A rwμ- F_σ-subset F of a rwμ-Lindel¨of space X is rwμ-Lindel¨of relative to X

Proof. Let F = ∪{Fi : i = 1, 2, . . .} where Fi is a rwμ-closed subset of X for each i = 1, 2, ... Let $\widetilde{U}$ be a cover of F by rwμ-open sets in X, then $\widetilde{U}$is a cover of Fi, i = 1, 2, . . . by rwμ-open subsets of X . Since Fi is rwμ-Lindel¨of relative to X, $\widetilde{U}$ has a countable subcover $\widetilde{U}$i = {$\widetilde{U}$i1 , $\widetilde{U}$i2 , . . .} for Fi for each i = 1, 2, ...

Now $\widetilde{U}$ = ∪{ $\widetilde{U}$i : i = 1, 2, . . .} = {Ui_n: i, n = 1, 2, . . .}is a countable subcover of $\widetilde{U}$ for F . So F is rwμ-Lindel¨of relative to X.

Theorem 2.7. Every generalized subspace of a generalized topological space (X, μ) is rwμ-Lindel¨of relative to X if and only if every rwμ-opengeneralized subspace of X is rwμ-Lindel¨of relative to X .

Proof. ⇒Is clear.

Let Y be a generalized subspace of X and let $\widetilde{U}$= {U_α: α∈Δ} be a cover of Y by rwμ-open sets in X . Now, let V = ∪$\widetilde{U}$ then V is a rwμ-open subset of X, so it is rwμ-Lindel¨of relative to X. But $\widetilde{U}$is acover of V so $\widetilde{U}$has a countable subcover $\widetilde{U}$ for V. Then V ⊆∪$\widetilde{U}$ and therefore Y ⊆V ⊆ ∪$\widetilde{U}$. So $\widetilde{U}$ is a countable sub cover of $\widetilde{U}$ for Y. Then Y is rwμ-Lindel¨of relative to X. The proofs of the following two theorems are straightforward, and thus omitted.

Theorem 2.8. A generalized topological space (X, μ) is rwμ-compact if and only if every rwμ-closed family of subsets of X with empty intersection, has a finite subfamily with empty intersection.

Theorem 2.9. A generalized topological space (X, μ) is rwμ-compact if andonly if every rwμ-closed family of subsets of X having the finite intersection property, has a nonempty intersection.

Theorem 2.10. Let f : (X, μ) → (Y, μ') be a (μ, μ')-continuous function. Then, if A is rwμ-compact (resp.rwμ-Lindel¨of) relative to X, then f(A) is rwμ-compact (resp. rwμ-Lindel¨of) relative to Y.

Proof. We will show the case when A is rwμ-compact relative to X, the other case is similar. Suppose that $\widetilde{U}$= {U_α: α∈Δ} is a cover of f(A) by rwμ-open subsets of Y. Then $\widetilde{U}$ = {f 1(U_α) : α∈Δ} is a cover of A by rwμ-open subsets of X. Since A is rwμ-compact relative to X, there exists a finite subset Δ0 of Δ such that A ⊆∪{f 1(U_α) : α∈Δ0}. Thus f(A) ⊆∪{f(f 1(U_α)) : α∈Δ0} ⊆∪{U_α: α∈Δ0}. Hence f(A) is rwμ-compact relative to X .

Theorem 2.11. For a function f : (X, μ) → (Y, μ') , the following are equivalent:

(a) f is contra- (μ, μ') -continuous.
(b) For every μ'-closed subset F of Y, f 1(F) is rwμ-open in X.
(c) For each x ∈X and each μ'-closed subset F of Y with f(x) ∈F, there exists a rwμ-open subset U of X with x ∈U such that f(U) ⊆F.

Proof. The implications (a) ⇔(b) and (b) ⇒(c) are obvious.
(c) ⇒(b). Let F be any μ'-closed subset of Y. If x ∈f 1(F) then f(x) ∈F and there exists a rwμ-open subset Ux of X with x ∈Ux such that f(Ux) ⊆F. Therefore, we obtain f 1(F) = ∪{Ux: x ∈f 1(F)}. Therefore, f 1(F) is rwμ-open.

Definition 2.5. A generalized topological space (X, μ) is said to be strongly rwμ-closed if every rwμ-closed cover of X has a finite subcover.

Theorem 2.12. If f : (X, μ) → (Y, μ') is contra- (μ, μ') - continuous and K is rwμ-compact relative to X, then f(K) is strongly rwμ'-closed in Y.

Proof. Let {C_α: α∈Δ} be any cover of f(K) by rwμ'-closed subsets of f(K). For each α∈Δ, there exists a rwμ'-closed set $F_\alpha$of Y such that C_α= F_α∩f(K). For each x ∈K, there exists αx ∈Δ such that f(x) ∈F_αx. Now by Theorem, there exists a rwμ-open set Ux of X with x ∈Ux such that f(Ux) ⊆F_x. Since the family {Ux:x ∈K} is a rwμ-open cover of K by sets rwμ-open in X, there exists a finite subset K0 of K such that K ⊆∪{Ux: x ∈K0}. Therefore we obtain f(K) ⊆∪{f(Ux) : x ∈K0} which is a subset of ∪{F_αx: x ∈K0}. Thus f(K) ⊆∪{C_αx: x ∈K0} and hence f(K) is strongly rwμ-closed.

Corollary 2.3. If f : (X, μ) → (Y, μ') is contra- (μ,μ') – continuous surjection and X is rwμ-compact, then Y is strongly rwμ'-closed.

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