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RESEARCH ARTICLE

OPTIMIZATION OF FUZZY EPQ MODEL UNDER SAFETY STOCK & REORDER LEVEL

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ABSTRACT

This paper studies the production manager's preference for economic production quantity of his single item inventory to avoid the escalating prices of the different cost like setup cost, holding costs and ordering costs. We also introduce a fuzzy inventory model under safety stock based on fuzzy total annual safety stock cost and annual holding cost of safety stock out and fuzzy total stock out cost. We apply the function principle and Graded Mean Integration representation method for computing the optimal production quantity and fuzzy total annual inventory cost. A numerical example is illustrated to derive the optimal solutions for fuzzy annual costs for both regular and safety stock model.

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INTRODUCTION

Inventory control system in classical inventory models usually assumes that demand and lead time are uniform and known. Once we embark on a path to reality, we must accept the fact that demand is not certain but it takes place with some probability.Fuzzy inventory model with uncertain parameters and fuzzy variables have been discussed in the real world recently. An interesting approach for aggregate inventory planning was propounded by Kaeprazyk and Staniewski. Park (1987) applied fuzzy set theory concept to tackle the inventory problem with fuzzy inventory cost under arithmetic operations of Extension Principle. Chen et al. (1996) propounded backorder fuzzy inventory model under Function Principle. He deliberated as fuzzy inventory model for crisp order quantity. Chang (1999) exhibited a membership function of the fuzzy aggregate cost of production inventory model and appliedExtension Principle and Centroid method to get an estimate of total cost and economic production quantity. Here our Inventory model also takes into account risk factors to manage the possibility of stockout and also maintain proper reorder level and optimum annual cost including holding costs, setup costs, stockout costs, ordering costs of spare parts of goods to be produced. The object of the safety stock is to hedge the random variations in demand and lead time. The safety stock is not intended to hedge 100% of the variations during that period.The extent of variations coveredby the safety stock depends on the desired stockout risk or the customer service level. In the event of stockout costs being not available a common surrogate is the customer service level, which eventually refers to the possibility that a demand or a combinations of demands are fulfilled. In this study the unit service level by the production manager is also taken as an option in service levels. It is applied to decide what should be the safety stock and reorder point during uncertain demand and lead time. The unit service level can give us the precise quantities of units of customer demand met during any given period of time and in view of this it is apt for plethora of customer satisfaction applications.

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Basic Definitions and Principles

Mean of Trapezoidal Fuzzy Number: A fuzzy number can be considered a generalized of the interval of confidence. However, it is not a random variable. A random variable is defined in terms of the theory of probability, which has evolved from theory of measurement. A random variable is an objective datum, whereas a fuzzy number is a subjective datum. It is a valuation, not a measure.In this paper, we utilize the concept of Trapezoidal fuzzy number as the type of all fuzzy parameters.Suppose A is a trapezoidal fuzzy number as shown in figure 1. It is described as any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

(1) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval[0,1],

(2) $\mu_{\tilde{A}}(x) = 0, \infty < x \le a_1$,

(3) $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on [a_1, a_2],

(4) $\mu_{\tilde{A}}(x) = 1, a_2 \le x \le a_1$,

(5) $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on [a_3, a_4],

(6) $\mu_{\tilde{A}}(x) = 0, a_4 \le x < \infty$,

Where a_1, a_2, a_3 and a_4 are real numbers. Also this type of trapezoidal fuzzy number be denoted as $A = (a_1, a_2, a_3, a_4)$,

In addition, Chen and Hsieh introduced Graded Mean Integration Representation method based on the integral value of graded mean h-level of generalized fuzzy number. This method is reasonable that to adopts grade as the important degree of each point of support set of generalized fuzzy number, and to discuss the grade of each point of support set of fuzzy number for representing fuzzy number. Here, we first obtain the representation of trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4)$, P_k (A), by using the method of Chen et al. under w=1 and k-preference by manager as follows.

$$P_k(A) = \frac{k(a_1+2a_2)+(1-k)(2a_3+a_4)}{3} \quad (1)$$

Also, the mean of trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4)$, $M_{\tilde{A}}$, by using the method of Chen et al. under w=1 and k=0.5 is defined as follows, But we put k=0.5,since it is does not bias to left or right.

$$M_{\tilde{A}} = \frac{a_1+2a_2+2a_3+a_4}{6} \quad (2)$$

For example, suppose $A = (1,2,3,7)$ is a trapezoidal fuzzy number, then the representation of A and the mean of A can be calculated by Formula(1) and (2) respectively, as follows.

$$P_k(A) = \frac{5k+13(1-k)}{3} = \frac{13-8k}{3}$$

$$M_{\tilde{A}} = \frac{1+4+6+7}{6} = 3$$

The Fuzzy Arithmetical Operation under Function Principle: In this paper we use the Function Principle to simplify the calculation. Function Principle in fuzzy theory is used as the computational model avoiding the complications which can be caused by the operations using Extension Principle. We describe some fuzzy arithmetical operations under Function Principle as follows.

Suppose $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers.

Then,

(1)The addition of A and B is

$$A \oplus B = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$$

where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

(2)The multiplication of A and B is

$$A \otimes B = (c_1, c_2, c_3, c_4)$$

Where $T = \{a_1b_1, a_1b_4, a_4b_1, a_4b_4\}$, $T_1 = \{a_2b_2, a_2b_3, a_3b_2, a_3b_3\}$, $c_1 = \min T$, $c_2 = \min T_1$ $c_3 = \max T_1$, $c_4 = \max T$.

Also, if $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all non zero positive real numbers, then

$$A \otimes B = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$$

Where $A \otimes B$ is a trapezoidal fuzzy number.

(3) $-B = (-b_4, -b_3, -b_2, -b_1)$,then the subtraction of A and B is

$$A \Theta B = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

Where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

(4) $1/B = B^{-1} = (1/b_4, 1/b_3, 1/b_2, 1/b_1)$,where b_1, b_2, b_3 and b_4 are all positive real numbers. If a_1, a_2, a_3, a_4, b_1, b_2, b_3 and b_4 all nonzero positive real numbers, then the division of A and B is

$$A \varnothing B = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1)$$

(5) Let $\alpha \in R$ then

$$\begin{cases} \alpha \ge 0, \alpha \otimes A = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4) \\ \alpha < 0, \alpha \otimes A = (\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1) \end{cases}$$

Recorder Point and Safety Stock under a Unit service Level

Suppose that the inventory manager is willing to accept a 95% of unit service level, S_p =0.95, on any cycle. On the other word, we have the percent of stockout level on a cycle, such as $1 - S_p = 0.05$. In this paper, we suppose fuzzy demand per day $D_p = (dp_1, dp_2, dp_3, dp_4)$ and fuzzy lead time $L = (l_1, l_2, l_3, l_4)$ that be decided by inventory manager as TrFN distribution approached. Therefore, the fuzzy total demanded in fuzzy lead time D_L is equal to fuzzy demand per day multiplied by fuzzy lead time by using Function Principle, such as $D_L = D_p \otimes L = (d_1, d_2, d_3, d_4)$ and is a trapezoidal fuzzy number as shown in Figure 1. In Figure 1, the symbol R is represented as reorder point on a cycle, M_{D_L} is the mean of fuzzy total demand in fuzzy lead time, and the area of triangular TRd_4 is percent of stockout. In addition, the safety stock (S_s) can be determined using $R - M_{D_L}$ on a cycle. What, then, should be the reorder point satisfying the percentage of unit service level on a cycle. Here, we suppose that the inventory manager indicated that he really had meant a

S_p percentage of unit service level. Then the percentage of stockout be calculated as $1-S_p$ we have

$$1-S_p = \text{the area of the triangular } TRd_4 / \text{ the area of } D_L \quad (3)$$

Where the area of triangular $TRd_4 = (d_4 - R)^2 / 2(d_4 - d_3)$
And the area

$$D_L = (d_2 - d_1)/2 + (d_3 - d_2) + (d_4 - d_3)/2$$

From equation (3),we get the recorder point, R, as

$$R = d_4 - \sqrt{(d_4 - d_1 - d_2 + d_3) + (d_4 - d_3)(1 - S_p)} \quad (4)$$

Also, we find the safety stock, S_s as

$$S_s = R - M_{D_L}. \quad (5)$$

Where $M_{D_L} = (d_1 + 2d_2 + 2d_3 + 2d_4)/6$ by equation (2).

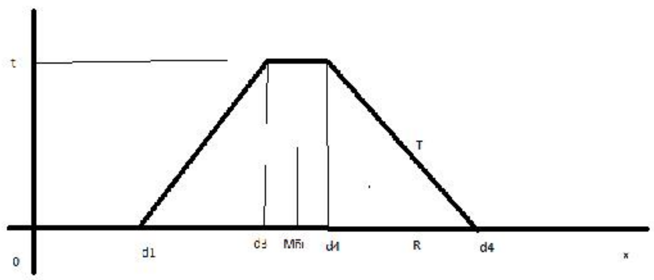

Figure 1. The TrFN Distribution of Fuzzy Total Demand in Fuzzy Lead Time (D_L)

Notations and Assumptions

We need the following notations and assumptions for the treatment of our proposed inventory model. Demand and lead time are fuzzy in nature. Shortages are not allowed. Reorder point and safety stock under unit service level by production manager satisfies trapezoidal distribution. The production manager's preference of fuzzy total annual inventory cost is k which ranges from 0 to 1. Following notations will be used for our proposed inventory model:

T : Fuzzy total cost
C_h : Holding cost per unit
C_0 : Ordering cost per order of spare parts
C_c : Stockout cost per order
C_s : Setup cost per order
D : Fuzzy annual demand
L : Fuzzy lead time on a cycle
Q : Order quantity per order
D_L : Fuzzy total demand in fuzzy lead time
M_{D_L} : Mean of D_L
N : Number of orders per year
S : Fuzzy stockout per order
S_p : Unit service level by production manager
R : Reorder point on a cycle
S_s : Safety stock on a cycle

Model formulation

Crisp model

The classical EPQ model takes into account all the cost components like the ordering cost, holding cost and the set up cost per order and production managers k preference for production quantity Q is either 0 or 1 the annual ordering cost is equal to the product of number of orders per year and the ordering cost per order.Similarly the annual holding cost is the product of average inventory and annual holding cost per unit

Hence the total annual cost(T) now becomes:

$$T = \frac{Q}{2} \times C_h + \frac{D_P}{Q} \times C_0 + C_s \times N \quad (6)$$

Taking the first order derivative of equation(6) with respect to Q sand equating to zero gives the optimal Q^*

$$Q^* = \sqrt{\frac{2D_P \times C_0}{C_h}} \quad (7)$$

Substituting (7) in (6) gives the optimal T*

$$T^* = \frac{C_h}{2}Q^* + \frac{D_p C_0}{Q^*} + C_s \times N$$

Fuzzy model

We use the arithmetical operations using Function Principle of fuzzy theory to denote the fuzzy total annual inventory cost of the EPQ model with the production manager's k-preference ranging for the values between 0 and 1 as there is fuzziness in the assessment of the total annual demand and cost data as well the fuzzy total annual cost is the summating of total setup cost,total annual holding cost and fuzzy total ordering cost given as bellow:

$$T = C_h \otimes \frac{Q}{2} \oplus C_s \otimes (D \oslash Q) \oplus \frac{D}{Q} \times C_0 1 \quad (8)$$

Where $\oslash, \oplus, \otimes$ are fuzzy Arithmetic operations by Function Principle

We solve equation (8) for optimal order quantity. Let the annual demand be the trapezoidal fuzzy number

$D = (D_1, D_2, D_3, D_4)$ then the fuzzy annual inventory cost is represented as

$$T = \frac{C_h}{2}Q + \frac{C_s}{Q}D_1 + \frac{C_0}{Q}D_1, \frac{C_h}{2}Q + \frac{C_s}{Q}D_2 + \frac{C_0}{Q}D_2,$$
$$\frac{C_h}{2}Q + \frac{C_s}{Q}D_3 + \frac{C_0}{Q}D_3, \frac{C_h}{2}Q + \frac{C_s}{Q}D_4 + \frac{C_0}{Q}D_4,$$

$$T = \frac{C_h}{2}Q + \left(\frac{C_s}{Q} + \frac{C_0}{Q}\right) D_1, \frac{C_h}{2}Q + \left(\frac{C_s}{Q} + \frac{C_0}{Q}\right) D_2, \frac{C_h}{2}Q + \left(\frac{C_s}{Q} + \frac{C_0}{Q}\right) D_3, \frac{C_h}{2}Q + \left(\frac{C_s}{Q} + \frac{C_0}{Q}\right) D_4 \qquad (9)$$

The production manager's k-preference using equation (9) is incorporated into the fuzzy annual inventory cost and we get

$$P_k(\mathrm{T}) = 1/3 \frac{[4\frac{C_h}{2}Q + k(C_0 + C_s)(D_1 + 2D_2) + (1-\mathrm{k})(C_0 + C_s)(2D_3 + D_4)]}{Q} \qquad (10)$$

Now minimize equation (10) by finding its first order derivative and equate to zero to find the optimal order quantity Q^*.

$$\frac{\partial P_k(T)}{\partial Q} = 1/3 \frac{[2C_h - k(C_0 + C_s)(D_1 + 2D_2) + (1-\mathrm{k})(C_0 + C_s)(2D_3 + D_4)]}{Q^2}$$

Let $\frac{\partial^2 P_k(T)}{\partial^2 Q} = 0$ to get optimal Q^* as

$$Q^* = \sqrt{2\frac{C_s}{C_h}\frac{k(D_1 + 2D_2) + (1-\mathrm{k})(2D_3 + D_4)}{3}} \qquad (11)$$

The number of orders on a year N is given by

$$N = D \varnothing Q^* = \left(\frac{D_1}{Q^*}, \frac{D_2}{Q^*}, \frac{D_3}{Q^*}, \frac{D_4}{Q^*}\right)$$

Then any number of orders per year (N) that is the mean of N is given by

$$N = \frac{D_1 + 2D_2 + 2D_3 + D_4}{6Q^*} \qquad (12)$$

Optimization of Fuzzy Model under Safety Stock

We also present a fuzzy inventory model under safety stock when demands per day and lead time are uncertain. This model is given by fuzzy total annual safety stock cost summating of total annual holding cost of safety stock and fuzzy total annual stockout cost. Then, there fuzzy total annual safety stock cost, FTSSC, is obtained by

$$T_S = C_c \times S_s \oplus C_s \otimes (D \varnothing \mathrm{Q}^*) \otimes S \qquad (13)$$

Where $\varnothing$, $\otimes$ and $\oplus$ are the fuzzy arithmetic operations by Function Principle.

In equation (13), the safety stock (S_s) can also be represented by $R - M_{DL}$ with fuzzy total demand in fuzzy lead time, $D_L = (d_1, d_2, d_3, d_4)$ as shown in Figure 1. In addition, $(D \varnothing \mathrm{Q}^*)$ represents the number of orders per year, ($N = D \varnothing \mathrm{Q}^*$), and the result is a trapezoidal fuzzy number after calculating.

Also, the expected stockout per order is combined by the average stockout per orders (S) and the percentage of the average stockout. According to the assumption of a constant demand rate producing a linear decline in the inventory position. For such a linear function, declined between maximum stockout and minimum stockout, the average falls at the midpoint or geometric balance point of (maximum stockout + minimum stockout)/2. The average stockout per orders then is given by $(d_4 - R)/2$ where the maximum stockout $d_4 - R$ which is maximum demand d_4 in fuzzy total demand in fuzzy lead time (D_L) and the reorder point R and the maximum stockout is zero.

The percentage of stockout ($1 - S_p$) is given by

$$\frac{(d_4 - R)^2}{2(d_4 - d_3)(\mathrm{d}_4 + \mathrm{d}_3 - \mathrm{d}_1 - \mathrm{d}_2)}$$

Substituting this in equation (12) we get the fuzzy annual safety stock cost as

$$\tilde{T}_s = C_h \times (R - M_{DL}) \oplus C_c \otimes N \otimes \frac{d_4 - R_x}{2} \times \frac{(d_4 - \mathrm{R})^2}{2(d_4 - d_3)(d_4 + d_3 - d_1 - d_2)} \qquad (14)$$

After simplifying equation (14) and setting

$$t = \frac{C_c N}{4(d_4 - d_3)(d_4 + d_3 - d_1 - d_2)}$$ Where t is a constant

We get

$$T_s = C_h R - M_{DL} C_h + t d_4^3 - 3t d_4^2 R + 3t d_4^2 R^2 - t R^3 \qquad (15)$$

We solve for the optimal reorder R* in equation (15) by taking the first derivative and equating to zero. Hence we get

$$\frac{\partial T_s}{\partial R} = C_h - 3t d_4^2 + 6t d_4 R - 3t R^2$$

Putting $\frac{\partial T_s}{\partial R} = 0$

we get the optimal reorder point R^* as

$$R^* = d_4 - \sqrt{\frac{C_h}{3t}} \qquad (16)$$

And optimal safety stock S_s^* as

$$S_s^* = R^* - M_{DL} \qquad (17)$$

Numerical Example

The Dell Laptop Company's Production Manager receives an approximate annual demand of 1 lakh set of laptops and has 0.5 preference of trapezoidal membership function about his total annual inventory cost. The setup cost is 15000 per order and holding cost is 7.5 per unit and the lead time is about 100 days and stockout cost per unit is 20 and ordering cost per unit 1000. what safety stock should be kept and also obtain the minimum fuzzy total annual safety stock cost.

Solution: Let as assume the annual demand to be trapezoidal fuzzy number

$$D = \left(D_1, D_2, D_3, D_4\right)$$

= [94000,99000,101000,106000]

Let us derive the optimal order quantity with production managers k-preferenceby using equation(11) and we get

$Q*$=20000

By using equation (12) we find the avarage number of orders per year as N=5

The corresponding minimum fuzzy total annual cost T is

$T_{\min}$ = 154700

To calculate the optimal reorder point $R*$ and optimal safety stock S_s* we assume fuzzy lead time as

L = (80,90,110,120) on a cycle of 100 days.

If the company is working for 300 days per year the fuzzy total demand in fuzzy lead time is calculated as

$$D_L = (D \varnothing 300) \otimes L$$

=(25067,29700,37033,42400)

Now we calculateconstant 't' as

t = 0.033

From equation (16) we calculate the optimal reorder point $R*$ as

$$R* = 339$$

From equation (2) we calculate the mean of fuzzy total demand in fuzzy lead time

$$M_{DL} = 109$$

From equation (14) we calculate the minimum fuzzy total annual safety stock cost as

$$Ts_{\min} = 1816$$

Now we calculate the optimal safety stock from equation (17) as

$$S_{s*} = 230$$

The production manager's k-preferences will change according to the changes and trends in the market demand for his commodity. So accordingly the decision maker can denote what optimal production quantity should be produced under different values of k so as to keep the optimal safety stock to meet the requirement of the changing market demand for the different values of k, the results are tabulated as follows:

Table 1. Numerical Inferences

k value	Fuzzy inventory model for order quantity, Equation(6)	
	Minimum fuzzy total annual inventory cost (T^*)	Optimal order quantity Q^*
K=0	155632	20265
K=0.1	155446	20212
K=0.2	155259	20159
K=0.3	155073	20106
K=0.4	154886	20053
K=0.5	154700	20000
K=0.6	154514	19947
K=0.7	154324	19893
K=0.8	154134	19839
K=0.9	153948	19786
K=1	153759	19732

Conclusion

The highlighting factor of this study is that the Function Principle is applied to rationalize and reduce the complexity in the computation of fuzzy total annual inventory cost and fuzzy total demand in a stipulated fuzzy lead time. Also using equation (8) the production quantity Q along with the preference of fuzzy annual inventory cost by the production manager could be easily calculated and when the demand is crisp real number that is D =(D) then the production manager prefers the mean of fuzzy total annual inventory cost and so k=0.5.That is the optimal solution of our formulated model can become the classical inventory model. Hence this fuzzy modeling is highly significant in realistic terms. For further research defuzzification on the inputs and the outputs can be made on a comparative basis and take other parameters other than fuzzy demand and fuzzy lead time as fuzzy numbers and analysis can be done.

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