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RESEARCH ARTICLE

SOLVING FUZZY LINEAR PROGRAMMING PROBLEM FOR FUZZY FOURIER MOTZKIN ELIMINATION ALGORITHM BY TRAPEZOIDAL FUZZY NUMBER

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ABSTRACT

In this paper we first change the fuzzy liner programming problem into the fuzzy linear system of equations. Then a Fourier Motzkin elimination method is discussed to solve the above converted fuzzy linear system of equations.

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INTRODUCTION

A method discovered by Fourier (Fourier, 1983) in 1826 for manipulating linear inequalities can be adapted to solve LP models. The theoretical insight given by this method is demonstrated as well as its clear geometrical interpretation. It has been rediscovered a number of times by different authors: Motzkin (Motzkin, 1937) (the name Fourier Motzkin algorithm is often used for this method) Dantzig and Cottle (George Bernard Dantzig and Richard Cottle, 2008) and Kuhn (Kuhn, 1956). The Fourier' method is used for solving a system of linear constraints of the form $a_1x_1 + a_2x_2 + \quad + a_nx_n \quad b$ on the set of real numbers (or more generally on an ordered field) where is >, ≥ or = and $a_1, a_2, a_3, \dots a_{n,} b$ are real numbers. The Fourier Motzkin elimination consists in successive elimination of the unknowns. Each step transforms the constraints system S_n with the unknowns $x_1, x_2, \dots x_n$ to a new system S_{n-1} in which one of the unknowns say $x_n$does not occur anymore: x_n has been eliminated. The concept of fuzzy numbers and arithmetic operations with these numbers was first introduced and investigated by (Zadeh, 1976). One of the major applications of fuzzy number arithmetic is treating linear systems and their parameters that are all partially respected by fuzzy number.

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Friedman *et al.,* 1998 introduced a general modal for solving a fuzzy $n \times n$ linear system whose coefficient matrix is crisp and the right hand side column is an arbitrary fuzzy number vector. They used the parametric form of fuzzy numbers and replaced the original fuzzy $n \times n$ linear system by a crisp $2n \times 2n$ linear system. In this paper, all the variables are considered as fuzzy variables. Applying Fourier Motzkin elimination algorithm in fuzzy linear programming problem and obtaining the values of the fuzzy variables. This paper is organized as follows: section 2 introduces some preliminary definitions. Fourier Motzkin elimination method in fuzzy linear systems is described in section 3 and also Fourier Motzkin elimination algorithm for solving fuzzy linear system is given. Finally in section 4, the effectiveness of the proposed method is illustrated by means of an example.

Preliminaries

Fuzzy set

A fuzzy set $\widetilde{A}$ is defined by $\tilde{A} = \{(x, \mu_a(x)) : x \in A, \mu_A(x) \in [0,1]\}$.In this pair $(x, \mu_A(x))$,the first element x belongs to the classical set A and the second element $\mu_A(x)$ belongs to the interval [0, 1] called membership function.

Fuzzy number

A fuzzy set $\tilde{A}$ on R must posses at least the following three prpperties to qualify as afuzzy number:

- $\tilde{A}$ must be a normal fuzzy set;
- $\alpha_{\tilde{A}}$ must be a closed interval for every $\alpha \in [0,1]$;
- The support of $\tilde{A}$ Must be bounded.

Trapezoidal fuzzy number (Bellman *et al.*, 1970)

A trapezoidal fuzzy number denoted by $\tilde{a}$ and defind as $\tilde{a}$=$(a^l, a^u, \alpha, \beta)$. Where the membership functions of the fuzzy number $\tilde{a}$ as follows

$$\mu_{\tilde{a}}(x) = \begin{cases} 1 - \dfrac{a^l - x}{\alpha}, when\, a^l - \alpha \le x < a^l \\ 1 \qquad whena^l \le x \le a^u \\ 1 - \dfrac{x - a^u}{\beta}, when\, a^u \le x < a^u + \beta \\ 0 \qquad otherwise \end{cases}$$

Operations on trapezoidal fuzzy number

Let $\tilde{a}$ =$(a^l,\ a^u, \alpha, \beta)and\tilde{b}\ = (b^l, b^u, \gamma, \theta)$be two trapezoidal fuzzy number $\boldsymbol{x} \in \boldsymbol{R}$ the arithmetic trapezoidal fuzzy numbers are shown as follows

- Image of $\tilde{a}$ is $\tilde{a} = (\ \ a^u, \ \ a^l, \beta, \alpha)$
- Addition: $(a^l + b^l, a^u + b^u, \alpha + \gamma, \beta + \theta)$
- Subtraction:$(a^l \quad b^u, a^u \quad b^l, \alpha + \theta, \beta + \gamma)$
- Scalar multiplication: if $x \ge 0, x\tilde{a} = (xa^l, xa^u, x\alpha, x\beta)$

if $x < 0$, $x\tilde{a} = (xa^u, xa^l, \ \ x\beta, \ \ x\alpha\ )$

Fuzzy linear system of equations

Consider the $m \times n$ fuzzy linear system of equations (Motzkin, 1934):

$$a_{11} \times \tilde{x}_{11} + a_{12} \times \tilde{x}_{12} + \quad + a_{1n} \times \tilde{x}_{1n} = \tilde{b}_1$$
$$a_{21} \times \tilde{x}_{21} + a_{22} \times \tilde{x}_{22} + \quad + a_{2n} \times \tilde{x}_{2n} = \tilde{b}_2$$

$$a_{m1} \times \tilde{x}_{m1} + a_{m2} \times \tilde{x}_{m2} + \quad + a_{mn} \times \tilde{x}_{mn} = \tilde{b}_m$$

The matrix form of the above equation is $A \times \tilde{x} = \tilde{b}$, where the coefficient matrix A is (a_{ij}), where $i = 1\ to\ m\ j = 1\ to\ n$, $\tilde{x}$ is a fuzzy variable and $\tilde{b}$ is also fuzzy variable.

Ranking function

The ranking function is approach of ordering fuzzy numbers which is an efficient. The ranking function is denoted by $F(\mathfrak{R})$ and where

$\mathfrak{R}$: $F(\mathfrak{R}) \to \mathfrak{R}$ ***and*** $\boldsymbol{F}(\mathfrak{R})$istheSet of fuzzy numbers defined on a real line, where a natural order exists.

Maleki ranking function (Maleki, 2002)

Maleki ranking function have been introduced which are used for solving linear programing problem with fuzzy variables. The representation of trapezoidal fuzzy number$\tilde{a} = (a^l, a^u, \alpha, \beta)$ and its ranking fuction $\mathfrak{R}(\tilde{a})$ is

$$(\tilde{a})\ = a^l + a^u + \tfrac{1}{2}(\beta \quad \alpha).$$

Where $\alpha = \beta$ or $\alpha \neq \beta$.

FOURIER MOTZKIN ELIMINATION METHOD IN FUZZY LINEAR SYSTEMS

Consider a fuzzy linear system $A\tilde{x} \le \tilde{b}$,$A \in R^{m,n}$,$\tilde{x}, \tilde{b}$ are fuzzy variables and let $I \quad \{1,2,3 \quad ,m\}$. The fuzzy linear system in the following form:

$$a_{11}\tilde{x}_1 + a_{12}\tilde{x}_2 + \quad a_{1n}\tilde{x}_n \le \tilde{b}_1$$
$$a_{21}\tilde{x}_1 + a_{22}\tilde{x}_2 + \quad a_{2n}\tilde{x}_n \le \tilde{b}_2$$

$$a_{m1}\tilde{x}_1 + a_{m2}\tilde{x}_2 + \quad a_{mn}\tilde{x}_n \le \tilde{b}_m \qquad \text{..................(1)}$$

Eliminate $\tilde{x}_1$ from the system (1) for each I where $a_{i1} \neq 0$, we multiply the ith inequality by$1/|a_{i1}|$ .This gives an equilant system:

$$\tilde{x}_1 + \ a'_{i2}\tilde{x}_2 + \quad a'_{in}\tilde{x}_n \le b'_i \quad (i \in \ I^+)$$
$$a_{i2}\tilde{x}_2 + \quad + a_{in}\tilde{x}_n \le \tilde{b}_i(i \in I^0\ )$$
$$\tilde{x}_1 + a'_{i2}\tilde{x}_2 + \quad + a'_{in}\tilde{x}_n \le \tilde{b}'_i(i \in I^-\ ) \qquad \text{..............(2)}$$

Where $I^+\ = \{i{:}\, a_{i1} > 0\}$, $I^- = \{\ i{:}\, a_{i1} < 0\ \}$, $a'_{ij}\ = a_{ij}/|a_{i1}|$ and $\tilde{b}'_i = \tilde{b}_i/|a_{i1}|$.

Thus, the row index set $I = \{1,2,3, \quad ,m\}$ is partitioned into subsets $I^+, I^0$and I^-, some of which may be empty. It follows that $\tilde{x}_1, \tilde{x}_2, \quad , \tilde{x}_n$ is a solution of the original system (1) if and only if satisfy $\tilde{x}_2, \tilde{x}_3, \quad , \tilde{x}_n$

$$\sum_{j=2}^{n} a'_{kj}\tilde{x}_j \quad \tilde{b}'_k \le \tilde{b}'_i \quad \sum_{j=2}^{n} a'_{ij}\tilde{x}_j\ (i \in I^+, k \in I^-),$$
$$\textstyle\sum_{j=2}^{n} a_{ij}\tilde{x}_j \le \tilde{b}_i(i \in I^0\ ) \qquad \text{.........................(3)}$$

and $\tilde{x}_1$satisfies

$$\max_{k \in I^-}\left(\textstyle\sum_{j=2}^{n} a'_{kj}\tilde{x}_j \quad \tilde{b}'_k\right) \le \tilde{x}_1 \le \min_{i \in I^+}\left(\tilde{b}'_i \quad \textstyle\sum_{j=2}^{n} a'_{ij}\tilde{x}_j\right) \qquad \text{...............(4)}$$

If $I^- \quad (resp. I^+)$ is empty, then the first set of constraints in (3) vanishes and the maximum (resp.minimum) in (4) is interpreted as ∞ (resp. ∞). if I^0 is empty and either I^-or $I^+$is empty too, then we terminate :the general solution of $A\tilde{x} \le \ \tilde{b}$ is obtained by choosing $\tilde{x}_2, \tilde{x}_3, \quad , \tilde{x}_n$arbitrarily and choosing $\tilde{x}_1$ according to (4). The constraint (4) says that $\tilde{x}_1$ lies in a certain interval which is determined by$\tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \quad , \tilde{x}_n$. The polyhedron defind by (3) is the projection of p along the $\tilde{x}_1$ _axis, i.e, into the space of the variables $\tilde{x}_2, \tilde{x}_3, \quad , \tilde{x}_n$. One may then proceed similarly and eliminate $\tilde{x}_2, \tilde{x}_3,$ etc,.eventually one obtains a systems $l \le \tilde{x}_n \le u.$ Ifl $< u$, then one concludes that $A\tilde{x} \le \tilde{b}$ has no solution , otherwise one may choose $\tilde{x}_n \in [l, u]$, and then choose $\tilde{x}_{n-1}$ in an interval which depends on $\tilde{x}_n$, this back substitution procedure produces a solution $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \quad , \tilde{x}_n)$ to $A\tilde{x} \le \tilde{b}$ is, Moreover ,every solution of $A\tilde{x} \le \tilde{b}$ may be produced in this way(if the system is inconsistent, then ths might possibly be discovered at an early stage and one terminates). The number of constraints may grow exponentially fast as fuzzy variables are eliminated using Fourier Motzkin elimination. Actually, a main problem in practice is that the number of inequalities becomes *" too large"* during the elimination process, even when redundant inequalities are removed. It is therefore of interest to known situations where the projected linear systems are not very large or, at least, have some interesting structure.

Fourier motzkin elimination algorithm

Step 1. Formulate the fuzzy linear programming problem from the given problem.
Step 2. Then change the objective function as inequality and join it with the constraints. Now we get the fuzzy linear system of equation.(for max problem, use' $\leq$ ' and min problem' $\geq$ 'inequality).
Step 3. Now change all the inequalities of the fuzzy linear system into ' $\geq$ 'for minimization problem' $\leq$ ' for maximization problem.
Step 4. Now we are going to eliminate one by one in the $(\tilde{x}_1, \tilde{x}_2, \quad, \tilde{x}_n.)$

- Divide each equation by its modulus value of $\tilde{x}_1$ coefficient or all the equations.
- Now we have three classes of $\tilde{x}_1$coefficient,i.e , ' 1 ' or ' $+1$ ' or '0 ' linear equations.
- Adding or subtracting any two classes of equations to eliminate $\tilde{x}_1$.

Step 5.Repeat step 4 untilall the 'n'fuzzy variable are eliminated.
Step 6 .After eliminating all the 'n'fuzzy variables ,we get the $\tilde{Z}$ values and substitute the $\tilde{Z}$ in above ,we get the values of fuzzy variables in back to back substitution.

Numerical example

Type 1: $\alpha = \beta$

Example 1: The following example is suggested by the researchers in case $\alpha = \beta$.

Max $\tilde{z} = 5\tilde{x}_1 + 3\tilde{x}_2$

Subject to the constraints

$3\tilde{x}_1 + 5\tilde{x}_2 \leq (5, 10,2,2)$
$5\tilde{x}_1 + 2\tilde{x}_2 \leq (4,6,2,2)$

And the non-negativerestrictions

$\tilde{x}_1 \geq (0,0,0,0)$
$\tilde{x}_2 \geq (0,0,0,0).$

Solution

Formulation of the problem

Max $\tilde{z} = 5\tilde{x}_1 + 3\tilde{x}_2$

Subject to the constraints

$3\tilde{x}_1 + 5\tilde{x}_2 \leq (5, 10,2,2)$
$5\tilde{x}_1 + 2\tilde{x}_2 \leq (4,6,2,2)$

and the non-negative constraints

$\tilde{x}_1 \geq (0,0,0,0)$
$\tilde{x}_2 \geq (0,0,0,0).$

First we have to include the objective function in the constraints to form a fuzzy linear system of equations. For maximization problem, change the equal ' $=$ 'in the objective as' $\leq$ 'and join with it all constraints

$\tilde{z} \leq 5\tilde{x}_1 + 3\tilde{x}_2$
$3\tilde{x}_1 + 5\tilde{x}_2 \leq (5, 10,2,2)$
$5\tilde{x}_1 + 2\tilde{x}_2 \leq (4,6,2,2)$
$\tilde{x}_1 \geq (0,0,0,0)$
$\tilde{x}_2 \geq (0,0,0,0).$ ……………………………..…...……... (5)

Equation(5) is a fuzzy linear system of equation now change all the inequalities in the systems as ' $\leq$ ' for maximization.

$5\tilde{x}_1 \quad 3\tilde{x}_2 + \tilde{z} \leq 0$
$3\tilde{x}_1 + 5\tilde{x}_2 \leq (5, 10,2,2)$
$5\tilde{x}_1 + 2\tilde{x}_2 \leq (4,6,2,2)$
$\tilde{x}_1 \leq (0,0,0,0)$
$\tilde{x}_2 \leq (0,0,0,0).$ …………………...…...……... (6)

Now we are going to eliminate $\tilde{x}_1$ and dividing each co efficient of the system (6) by its co efficient of $\tilde{x}_1$,we have

$\tilde{x}_1 \quad 0.6\tilde{x}_2 + 0.2\tilde{z} \leq 0$
$\tilde{x}_1 + 1.66\tilde{x}_2 \leq (1.66,3.33.0.66,0.66)$
$\tilde{x}_1 + 0.4\tilde{x}_2 \leq (0.8,1.2,0.4,0.4)$
$\tilde{x}_1 \leq (0,0,0,0)$
$\tilde{x}_2 \leq (0,0,0,0).$ …………………...…...……... (7)

Now we have in three classes of equations in fuzzy linear system (4.3) we get the co efficient of $\tilde{x}_{1,}$ in the first class of equations is ' 1 ',in the second class equations ' $+1$ ' and in the third class of equations is'0 '. Now adding first class of equations with the second class of equations to eliminate $\tilde{x}_{1,}$

$1.06\tilde{x}_2 + 0.2\tilde{z} \leq (1.66,3.33,0.66.0.66)$
$0.2\tilde{x}_2 + 0.2\tilde{z} \leq (0.8,1.2,0.4,0.4)$
$1.66\tilde{x}_2 \leq (1.66,3.33,0.66,0.66)$
$0.4\tilde{x}_2 \leq (0.8,1.2,0.4,0.4)$
$\tilde{x}_2 \leq (0,0,0,0)$ …….…………………...…...……... (8)

Now eliminate $\tilde{x}_2$using the same procedure

$\tilde{x}_2 + 0.189\tilde{z} \leq (1.56,3.14,0.62,0.62)$
$\tilde{x}_2 + \tilde{z} \leq (4,6,2,2)$
$\tilde{x}_2 \leq (1,2.01,0.39,0.39)$
$\tilde{x}_2 \leq (2,3,1,1)$
$\tilde{x}_2 \leq (0,0,0,0)$ ……….………………...…...……... (9)

Now adding first class of equation with second class of equation

$1.189\tilde{z} \leq (5.56,9.14,2.62,2.62)$
$\tilde{z} \leq (5,8.01,2.39,2.39)$
$\tilde{z} \leq (6,9,3,3)$
$0.189\tilde{z} \leq (1.56,3.14,0.62,0.62)$
$0 \leq (1,2.01,0.39,0.39)$
$0 \leq (2,3,1,1)$ ……….………………...…...……... (10)

There is no possibility to eliminate $\tilde{z}$in(4.6) so stop the process. We have from (10)

$\tilde{z} \leq (4.67,7.68,2.20,2.20)$
$\tilde{z} \leq (5,8.01,2.39,2.39)$
$\tilde{z} \leq (6,9,3,3)$
$\tilde{z} \leq (8.25,16.61,3.28,3.28)$

Now choosing minimum value for $\tilde{z}$to satisfy all the above conditions. So

$\tilde{z} = (4.67,7.68,2.20,2.20)$

Substitute $\tilde{z}$ in (9)

$\tilde{x}_2 + 0.189(4.67,7.68,2.20,2.20) \leq (1.56,3.14,0.62,0.62)$
$\tilde{x}_2 + (4.67,7.68,2.20,2.20) \leq (4,6,2,2)$
$\tilde{x}_2 \leq (1,2.01,0.39.0.39)$
$\tilde{x}_2 \leq (2,3,1,1)$
$\tilde{x}_2 \leq (0,0,0,0)$

We get

$\tilde{x}_2 \leq (1.56,3.14,0.62,0.62) \quad (0.88,1.45,0.41,0.41)$
$\tilde{x}_2 \leq (4,6,2,2) \quad (4.67,7.68,2.20,2.20)$
$\tilde{x}_2 \leq (1,2.01,0.39,0.39)$
$\tilde{x}_2 \leq (2,3,1,1)$
$\tilde{x}_2 \leq (0,0,0,0)$

$\tilde{x}_2 \leq (0.11,2.26,1.03,1.03)$
$\tilde{x}_2 \geq (\quad 1.33,3.68,4.20,4.20)$
$\tilde{x}_2 \leq (1,2.01,0.39,0.39)$
$\tilde{x}_2 \leq (2,3,1,1)$
$\tilde{x}_2 \geq (0,0,0,0).$

From the above equations

$(\quad 1.33,3.68,4.20,4.20) \leq \tilde{x}_2 \leq (0.11,2.26,1.03,1.03)$

The ranking fuction of $\tilde{x}_2$ on both nearly 2.3.

$\therefore \tilde{x}_2 = (0.11,2.26,1.03,1.03)$

Substituting $\tilde{x}_2$ and $\tilde{z}$ in (7)

$\tilde{x}_1 \quad 0.6(0.11,2.26,1.03,1.03) + 0.2(4.67,7.68,2.20,2.20) \leq 0$
$\tilde{x}_1 + 1.66(0.11,2.26,1.03,1.03) \leq (1.66,3.33,0.66,0.66)$
$\tilde{x}_1 + 0.4(0.11,2.26,1.03,1.03) \leq (0.8,1.2,0.4,0.4)$
$\tilde{x}_1 \leq (0,0,0,0)$
$(0.11,2.26,1.03,1.03) \leq (0,0,0,0)$

Weget

$\tilde{x}_1 \quad (0.066,1.356,0.618,0.618) + (0.934,1.536.0.44,0.44) \leq 0$
$\tilde{x}_1 + (0.182,3.75,1.70,1.70) \leq (1.66,3.33,0.66,0.66)$
$\tilde{x}_1 + (0.044,0.904,0.412,0.412) \leq (0.8,1.2,0.4,0.4)$
$\tilde{x}_1 \leq (0,0,0,0)$
$(0.11,2.26,1.03,1.03) \leq (0,0,0,0)$.

Then

$\tilde{x}_1 \geq (\quad 0.422,1.47,1.058,1.058)$
$\tilde{x}_1 \leq (\quad 2.09,3.148,2.36,2.36)$
$\tilde{x}_1 \leq (\quad 0.104,1.156,0.812,0.812)$
$\tilde{x}_1 \geq (0,0,0,0)$.

From the above equation we get

$(\quad 0.422,1.47,1.058,1.058) \leq \tilde{x}_1 \leq (\quad 2.09,3.148,2.36,2.36)$

Then the ranking function of$\tilde{x}_1$ on both side nearly 1.05

$\therefore \tilde{x}_1 = (\quad 2.09,3.148,2.36,2.36)$.

We take

$\tilde{x}_1 = (\quad 2.09,3.148,2.36,2.36) = 1.05.$
$\tilde{x}_2 = (0.11,2.26,1.03,1.03) = 2.37$
$\tilde{z} = (4.67,7.68,2.20,2.20)$ =12.35

Type 2: $\boldsymbol{\alpha \neq \beta}$

Example 2: The following example is suggested by the researchers in case $\alpha \neq \beta$

Max $\tilde{z} = 5\tilde{x}_1 + 3\tilde{x}_2$

Subject to the constraints

$3\tilde{x}_1 + 5\tilde{x}_2 \leq (4,9,2,6)$
$5\tilde{x}_1 + 2\tilde{x}_2 \leq (2,7,1,3)$

And the non negative constraints

$\tilde{x}_1 \geq (0,0,0,0)$
$\tilde{x}_2 \geq (0,0,0,0).$

Solution

Formulation of the problem

Max $\tilde{z} = 5\tilde{x}_1 + 3\tilde{x}_2$

Subject to the constraints

$3\tilde{x}_1 + 5\tilde{x}_2 \leq (4,9,2,6)$
$5\tilde{x}_1 + 2\tilde{x}_2 \leq (2,7,1,3)$

And the non _ negative constraints

$\tilde{x}_1 \geq (0,0,0,0)$
$\tilde{x}_2 \geq (0,0,0,0).$

First we have to include the objective function in the constraints to form a fuzzy linear system of equations. For maximization problem, change the equal ‘ = ’in the objective as‘≤ ’and join with it all constraints

$\tilde{z} \leq 5\tilde{x}_1 + 3\tilde{x}_2$
$3\tilde{x}_1 + 5\tilde{x}_2 \leq (4,9,2,6)$
$5\tilde{x}_1 + 2\tilde{x}_2 \leq (2,7,1,3)$
$\tilde{x}_1 \geq (0,0,0,0)$
$\tilde{x}_2 \geq (0,0,0,0).$ ……………………...…...……... (11)

Equation (11) is a fuzzy linear system of equation now change all the inequalities in the systems as ‘ ≤ ’ for maximization.

$5\tilde{x}_1 \quad 3\tilde{x}_2 + \tilde{z} \leq 0$
$3\tilde{x}_1 + 5\tilde{x}_2 \leq (4,9,2,6)$
$5\tilde{x}_1 + 2\tilde{x}_2 \leq (2,7,1,3)$
$\tilde{x}_1 \leq (0,0,0,0)$
$\tilde{x}_2 \leq (0,0,0,0).$ ……………...…...…...……... (12)

Now we are going to eliminate $\tilde{x}_1$ and dividing each co efficient of the system (12) by its co efficient of $\tilde{x}_1$, we have

$\tilde{x}_1 \quad 0.6\tilde{x}_2 + 0.2\tilde{z} \leq 0$
$\tilde{x}_1 + 1.66\tilde{x}_2 \leq (1.33,3,0.66,2)$
$\tilde{x}_1 + 0.4\tilde{x}_2 \leq (0.4,1.4,0.2,0.6)$
$\tilde{x}_1 \leq (0,0,0,0)$
$\tilde{x}_2 \leq (0,0,0,0).$ …………………...…...……... (13)

Now we have in three classes of equations in fuzzy linear system (13) we get the co efficient of $\tilde{x}_{1,}$ in the first class of equations is ' 1',in the second class equations '$+1$'and in the third class of equations is'0' Now adding first class of equations with the second class of equations to eliminate $\tilde{x}_{1,}$

$1.06\tilde{x}_2 + 0.2\tilde{z} \leq (1.33,3,0.66,2)$
$0.2\tilde{x}_2 + 0.2\tilde{z} \leq (0.4,1.4,0.2,0.6)$
$1.66\tilde{x}_2 \leq (1.33,3,0.66,2)$
$0.4\tilde{x}_2 \leq (0.4,1.4,0.2,,0.6)$
$\tilde{x}_2 \leq (0,0,0,0)$ ……………………………………… (14)

Now eliminate $\tilde{x}_2$using the same procedure

$\tilde{x}_2 + 0.189\tilde{z} \leq (1.25,2.83,0.62,1.89)$
$\tilde{x}_2 + \tilde{z} \leq (2,7,1,3)$
$\tilde{x}_2 \leq (0.80,1.81,0.40,1.20)$
$\tilde{x}_2 \leq (1,3.5,0.5,1.5)$
$\tilde{x}_2 \leq (0,0,0,0)$ ……………………………………… (15)

Now adding first class of equation with second class of equation

$1.189\tilde{z} \leq (3.25,9.83,1.62,4.89)$
$\tilde{z} \leq (2.80,8.81,1.40,4.20)$
$\tilde{z} \leq (3,10.5,1.5,4.5)$
$0.189\tilde{z} \leq (1.25,2.83,0.62,1.89)$
$0 \leq (0.80,1.81,0.40,1.20)$
$0 \leq (1,3.5,0.5,1.5)$ ………………………………… (16)

There is no possibility to eliminate $\tilde{z}$in (16) so stop the process.

We have from (16).
$\tilde{z} \leq (2.73,8.27,1.36,4.11)$
$\tilde{z} \leq (2.80,8.81,1.40,4.20)$
$\tilde{z} \leq (3,10.5,1.5,4.5)$
$\tilde{z} \leq (6.61,14.9,3.28,10)$

Now choosing minimum value for $\tilde{z}$to satisfy all the above conditions.So Substitute$\tilde{z}$ in (15)

$\tilde{z} = (2.73,8.27,1.36,4.11)$

Substitute $\tilde{z}$in (15)

$\tilde{x}_2 + (0.189)(2.73,8.27,1.36,4.11) \leq (1.25,2.83,0.62,1.89)$
$\tilde{x}_2 + (2.73,8.27,1.36,4.11) \leq (2,7,1,3)$
$\tilde{x}_2 \leq (0.80,1.81,0.40,1.20)$
$\tilde{x}_2 \leq (1,3.5,0.5,1.5)$
$\tilde{x}_2 \leq (0,0,0,0)$

We get

$\tilde{x}_2 \leq (1.25,2.83,0.62,1.89) \quad (0.52,1.56,0.26,0.77)$
$\tilde{x}_2 \leq (2,7,1,3) \quad (2.73,8.27,1.36,4.11)$
$\tilde{x}_2 \leq (0.80,1.81,0.40,1.20)$
$\tilde{x}_2 \leq (1,3.5,0.5,1.5)$
$\tilde{x}_2 \leq (0,0,0,0)$

Then

$\tilde{x}_2 \leq (\ 0.31,2.31,1.39,2.15)$
$\tilde{x}_2 \geq (\ 4.27,6.27,4.36,5.11)$
$\tilde{x}_2 \leq (0.80,1.81,0.40,1.20)$
$\tilde{x}_2 \leq (1,3.5,0.5,1.5)$
$\tilde{x}_2 \geq (0,0,0,0)$

From the above equations

$(\ 4.27,6.27,4.36,5.11) \leq \tilde{x}_2 \leq (\ 0.31,2.31,1.39,2.15)$

The ranking fuction of $\tilde{x}_2$ on both nearly 2.3.

$\therefore \tilde{x}_2 = (\ 0.31,2.31,1.39,2.15)$

Substituting $\tilde{x}_2$ and $\tilde{z}$ in (13)

$\tilde{x}_1 \quad 0.6(\ 0.31,2.31,1.39,2.15) + 0.2(2.73,8.27,1.36,4.11) \leq 0$
$\tilde{x}_1 + 1.66(\ 0.31,2.31,1.39,2.15) \leq (1.33,3,0.66,2)$
$\tilde{x}_1 + 0.4(\ 0.31,2.31,1.39,2.15) \leq (0.4,1.4,0.2,.6)$
$\tilde{x}_1 \leq (0,0,0,0)$
$(\ 0.31,2.31,1.39,2.15) \leq (0,0,0,0)$

Weget

$\tilde{x}_1 \quad (\ 0.186,1.386,0.834,1.29)) + (0.546,1.654,0.272,0.822) \leq 0$
$\tilde{x}_1 + (\ 0.51,3.83,2.31,3.57) \leq (1.33,3,0.66,2)$
$\tilde{x}_1 + (\ 0.124,0.924,0.56,0.86) \leq (0.4,1.4,0.2,0.6)$
$\tilde{x}_1 \leq (0,0,0,0)$
$(\ 0.31,2.31,1.39,2.15) \leq (0,0,0,0)$.

Then

$\tilde{x}_1 \geq (\ 0.84,1.84,1.562,1.656)$
$\tilde{x}_1 \leq (\ 2.5,3.51,4.23,4.31)$
$\tilde{x}_1 \leq (\ 0.524,1.524,1.06,1.16)$
$\tilde{x}_1 \geq (0,0,0,0)$.

From the above equation we get

$(\ 0.84,1.84,1.562,1.656) \leq \tilde{x}_1 \leq (\ 0.524,1.524,1.06,1.16)$

Then the ranking function of$\tilde{x}_1$ on both side nearly 1.05

$\therefore \tilde{x}_1 = (\ 0.524,1.524,1.06,1.16)$

We take

$\tilde{x}_1 = (\ 0.524,1.524,1.06,1.16) = 1.05$
$\tilde{x}_2 = (\ 0.31,2.31,1.39,2.15)$ =2.38
$\tilde{z} = (2.73,8.27,1.36,4.11)$ =12.37.

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