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RESEARCH ARTICLE

SOME PROPERTIES OF CON-SECONDARY K-NORMAL BIMATRICES

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ABSTRACT

Some properties of conjugate secondary k-normal bimatrices are introduced. Some of the equaivalent conditions on con.sk-normal bimatrices are given.

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INTRODUCTION

Matrices provide a very powerful tool for dealing with linear models. Bimatrices are an advanced tool which can handle over one linear model out a time. Bimatrices will be useful when time bound comparisons are needed in the analysis of the model. (Vasanthakandasamy, 2005). Unlikebimatrices can be of several types. We have to combine the properties of bimatrices and con. S-k-normal matrices. The concept of con. S-k-normal bimatrices are introduced (Elumalai and Manikandan). Equivalent conditions on normal matrices are given (David w. Lewis 1991). In this paper our attention is to define con. S-k-normal bimatrices. Also we prove some results on con. S-k-normal matrices.

Preliminaries and Notations

First we wish to mention that when we have a collection of m×nbimatrices say M_B then M_B need not be even closed with respect to addition. Further we make a definition m×nzero bimatrix. Thus we make the following special type of concession in case of zero and unit m×mbimatrices. Appropriate changes are made in case of zero m×nbimatrix. LetC_{nxn} be the space of mxn complex matrices through out, let 'K' be fixed product of disjoint transposition in S_n the set of all permutation on {1,2,3….n} (hence involuntory) and 'K' be the associated permutation matrix and V is the permutation matrix with units in the secondarydiagonal clearly 'k' and 'V' Satisfies the following properties.

$\bar{K}_B= K_B^T=K_B^S=K_B^*=\bar{K}_B^S;K_B^2=I_B$

$\bar{V}_B=V_B^T=V_B^S=V_B^*=\bar{V}_B^S=V_B;V_B^2$

Some of the properties

1.$A_B(B_BC_B)=(A_BB_B)C_B=A_BB_BC_B$

$(ie) if\ \ A_B=A_1 \cup, A_2 B_B=B_1 \cup B_2$ and

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C_B=$C_1 \cup C_2$

Clearly

$A_B(B_B C_B) = A_B(B_1C_1 \cup B_2C_2)$

$=A_1(B_1C_1) \cup A_2(B_2C_2)$

$A_B(B_BC_B)$=$(A_BB_B)C_B$=$A_BB_BC_B$

2. Further bimatrix multiplication satisfies the distributive law for if

A_B=$A_1 \cup A_2$, B_B=$B_1 \cup B_2$ and

$C_B = C_1 \cup C_2$; then

$A_B(B_B + C_B)$=$A_B((B_1$+$C_1)$∪ $(B_2 + C_2))$

$=A_1(B_1 + C_1) \cup A_2(B_2 + C_2)$

$=A_1B_1$+A_1C_1 ∪$(A_2B_2$+$A_2C_2)$

=($A_1B_1 \cup A_2B_2)$+$(A_1A_2C_1 \cup C_2)$

$=A_BB_B + A_BC_B$

bimatrix multiplication is also distributive

Definitions and Theorems

Definition:1

A bimatrix $A_B$is defined as the union of two rectangular array of numbers $A_1$and A_2arranged intorows and columns. It is written as follows A_B=$A_1 \cup A_2$

whereA_1≠$A_2$with

$$A_1=\begin{bmatrix} a_{11}^1 & a_{12}^1 \;\ldots\ldots & a_{1n}^1 \\ a_{21.}^1 & a_{22.}^1 \ldots\ldots & a_{2n.}^1 \\ \vdots & \vdots & \vdots \\ \vdots & . & . \\ a_{m1}^1 & a_{m2}^1 \cdots & a_{mn}^1 \end{bmatrix}$$

$$A_2=\begin{bmatrix} a_{11}^2 & a_{12}^2 \;\ldots\ldots & a_{1n}^2 \\ a_{21.}^2 & a_{22.}^2 \ldots\ldots & a_{2n.}^2 \\ \vdots & \vdots & \vdots \\ \vdots & . & . \\ a_{m1}^2 & a_{m2}^2 \cdots & a_{mn}^2 \end{bmatrix}$$

`∪` the notational convenience (symbol) only.

Definition:2

A bimatrix $A_B \in \mathbf{C_{nxn}}$ is said to be con.s-k-hermitianbimatrix if $K_BV_BA_B^*V_BK_B=\bar{A}_B$

$(K_1V_1A_1^*V_1K_1)\cup(K_2V_2A_2^*V_2K_2)=(\bar{A}_1\cup\bar{A}_2)$

Definition:3

A bimatrix $A_B \in \mathbf{C_{nxn}}$ is said to be con. K-normal (s-k normal)bimatrix if

$A_B(K_BV_BA_B^*V_BK_B)=\overline{(K_BV_BA_B^*V_BK_B)A_B}$ $(A_1K_1V_1A_1^*V_1K_1)\cup(A_2K_2V_2A_2^*V_2K_2)=\overline{(K_1V_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2)}$

Definition:4

A bimatrix $A_B \in \mathbf{C_{nxn}}$ is said to be con.s-k-unitary if $A_B(K_BV_BA_B^*V_BK_B)=\overline{(K_BV_BA_B^*V_BK_B)A_B}=\overline{I_B}$

$(A_1 K_1 V A^*_{1_1} K_1) \cup (A_2 K_2 V_2 A^*_2 V_2 K) = \overline{(K_1 V_1 A^*_1 V_1 K_1 A_1) \cup (K_2 V_2 A^*_2 V_2 K_2 A_2)}$

$= \bar{I}_1 \cup \bar{I}_2$

Definition:5

Sum of con. S-k-normal bimatrices.

$(A_B + C_B)(K_B V_B (A_B + C_B)^* V_B K_B) = \overline{(K_B V_B (A_B + C_B)^* V_B K_B) V_B K_B}$

Where $D_B = A_B + C_B$

$D_B(K_B V_B D_B^* V_B K_B) = \overline{(K_B V_B D^*_B V_B K_B V_B K_B) D_B}$

$(A_1+C_1 \cup A_2+C_2)(K_1 V_1 (A_1^*+C_1^*) V_1 K_1) \cup (K_2 V_2 (A_2^*+C_2^*) V_2 K_2)$

$(K_1 V_1 (A_1^*+C_1^*) V_1 K_1 V_1 K_1) \cup (K_2 V_2 (A_2^*+C_2^*) V_2 K_2 V_2 K_2)(A_1+C_1 \cup A_2+C_2)$

Some of the Theorems

In this section to satisfies the some of the property and also satisfies equaivalent conditions of the con. Sk-normal bimatrices.

Theorem-1

Let A_B bethe con.s-k-normal bimatrices if $\overline{K_B V_B A_B A^*_B V_B K_B}$ $and \overline{(K_B V_B C_B C^*_B V_B K_B)}$ be two con s-k normal mxnbimatrices then $K_B V_B A_B A^*_B V_B K_B + (K_B V_B C_B C_B{}^* V_B K_B) = A_B A_B{}^* + C_B C_B{}^*$

if and only if $A_1 + C_1 \neq A_2 + C_2$

Proof

Suppose that let $A_B = A_1 \cup A_2$ and $C_B = C_1 \cup C_2$ be two m*x*n bimatrices if $\overline{K_B V_B (A_B V_B K_B) A^*_B + (K_B V_B C_B V_B K_B) C^*_B} = \overline{A_B A^*_B} + \overline{C_B C^*_B}$

$K_B V_B (A_B A^*_B + C_B C^*_B) V_B K_B = \overline{A_B A^*_B + C_B C^*_B}$

$(K_1 \cup K_2)(V_1 \cup V_2)((A_1 \cup A_2)(A_1^* \cup A_2^*) + (C_1 \cup C_2)(C_1^* \cup C_2^*)(V_1 \cup V_2)(K_1 \cup K_2))$

$K_1 V1 (A_1 A^*_1 + C_1 C_2^*) V_1 K_1) \cup (K_2 V_2 (A_2 A_2^* + C_2 C_2^*) V_2 K_2 = \overline{(A_1 A^*_1 + C_1 C^*_1)} \cup$

$\overline{(A_2 A^*_2 + C_2 C^*_2)}$

Hence $A_B + C_B$ be con.s-k- normal bimatrix on the other hand. $A_B + C_B$ be the two con.s-k-normal bimatrix clearly $(K_B V_B A_B A_B^* V_B K_B) + (K_B V_B C_B C_B^* V_B K_B)$ is also a con.s-k normal bimatrix.

Theorem-2

Let $A_B$bethecon.s-k-normal bimatrices if $\overline{K_B V_B A_B A^*_B V_B K_B}$ and $\overline{(K_B V_B C_B C^*_B V_B K_B)}$ be two con s-k normal mxnbimatrices then $K_B V_B A_B A^*_B V_B K_B - (K_B V_B C_B C_B{}^* V_B K_B) = A_B A_B{}^* - C_B C_B{}^*$ if and only if

$A_1 - C_1 \neq A_2 - C_2$

Proof

Suppose that let $A_B = A_1 \cup A_2$ and $C_B = C_1 \cup C_2$ be two m×n bimatrices if $\overline{K_B V_B (A_B V_B K_B) A^*_B - (K_B V_B C_B V_B K_B) C^*_B} = \overline{A_B A^*_B} - \overline{C_B C^*_B}$

$K_B V_B (A_B A^*_B - C_B C^*_B) V_B K_B = \overline{A_B A^*_B - C_B C^*_B}$

$(K_1 \cup K_2)(V_1 \cup V_2)((A_1 \cup A_2)(A_1^* \cup A_2^*) - (C_1 \cup C_2)(C_1^* \cup C_2^*)(V_1 \cup V_2)(K_1 \cup K_2))$

$K_1 V1 (A_1 A_1^* - C_2 C^*_2 V_1 K_1)(K_2 V_2 (A_2 A_2^* - C_2 C_2^*) V_2 K_2 = \overline{(A_1 A^*_1 - C_1 C^*_1)} \cup$

$\overline{(A_2 A^*_2 - C_2 C^*_2)}$

Hence A_B-C_B be con.s-k- normal bimatrix on the other hand. A_B-C_B be the two con.s-k-normal bimatrix clearly ($K_BV_BA_BA_B{}^*V_BK_B$)-($K_BV_B C_BC_B{}^*V_BK_B$) is also a con.s-k normal bimatrix.

Theorem-3

The s-knormal bimatrices are con. S-k normalbimatrices is also a unitarybimatrices.

Proof

We know that.

$A_B(K_BV_BA_B{}^*V_BK_B)=\overline{(K_BV_BA_B^*V_BK_B)A_B}=\overline{I_B}$

$(A_1\cup A_2)(K_1\cup K_2)(V_1\cup V_2)(A_1{}^*\cup A_2{}^*)(V_1\cup V_2)(K_1\cup K_2)=\overline{(K_1\cup K_2)(V_1\cup V_2)(A_1^*\cup A_2^*)(V_1\cup V_2)(K_1\cup K_2)(A_1\cup A_2)}$

$(A_1K_1V_1A_1{}^*V_1K_1)\cup(A_2K_2V_2A_2{}^*V_2K)=\overline{(K_IV_1A_1^*V_1K_1A_1\cup(K_2V_2A_2^*V_2K_2A_2)}=\overline{I_1\cup I_2}$

Hence the k-normal bimatrices are con. S-k- normal Bimatrices it is also unitary bimatrices.

Hence proved.

Theorem-4

The s-k normal bimatrices are con . s-k normalbimatrices.

Proof

We know that,

Hence

$A_B(K_BV_BA_B{}^*V_BK_B)=\overline{(K_BV_BA_B^*V_BK_B)A_B}$

$(A_1\cup A_2)(K_1\cup K_2)(V_1\cup V_2)(A_1{}^*\cup A_2{}^*)(V_1\cup V_2)(K_1\cup K_2)=\overline{(K_1\cup K_2)(V_1\cup V_2)(A_1^*\cup A_2^*)(V_1\cup V_2)(K_1\cup K_2)(A_1\cup A_2)}$

$(A_1K_1V_1A_1{}^*V_1K_1)\cup(A_2K_2V_2A_2{}^*V_2K)=\overline{(K_IV_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2)}$

Hence the k- normal bimatrices are con. S-k normal bimatrices.

Hence proved.

Example-2

The con.s-k-normal bimatrix.

$A_B(K_BV_BA_B{}^*V_BK_B)=\overline{(K_BV_BA_B^*V_BK_B)A_B}$

$(A_1\cup A_2)(K_1\cup K_2)(V_1\cup V_2)(A_1{}^*\cup A_2{}^*)(V_1\cup V_2)(K_1\cup K_2)=\overline{(K_1\cup K_2)(V_1\cup V_2)(A_1^*\cup A_2^*)(V_1\cup V_2)(K_1\cup K_2)(A_1\cup A_2)}$

$(A_1K_1V_1A_1{}^*V_1K_1)\cup(A_2K_2V_2A_2{}^*V_2K)=\overline{(K_IV_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2)}$

$(A_1K_1V_1A_1{}^*V_1K_1)\cup(A_2K_2V_2A_2{}^*V_2K)=(\overline{K_IV_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2)}$

$(A_1K_1V_1A_1{}^*V_1K_1)=\begin{pmatrix}0 & i\\ i & 0\end{pmatrix}\begin{pmatrix}i & o\\ 0 & i\end{pmatrix}\begin{pmatrix}0 & i\\ i & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix}\begin{pmatrix}0 & i\\ i & 0\end{pmatrix}\begin{pmatrix}i & o\\ 0 & i\end{pmatrix}$

$=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$

$=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$

$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

Similarly

$(A_2K_2V_2A_2{*}V_2K)=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$(A_1K_1V_1A_1{*}V_1K_1)\cup(A_2K_2V_2A_2{*}V_2K)=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\cup\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$\overline{(K_IV_1A_1^*V_1K_1A_1)}=\begin{pmatrix}i & o\\0 & i\end{pmatrix}\begin{pmatrix}0 & i\\i & 0\end{pmatrix}\begin{pmatrix}0 & -i\\-i & 0\end{pmatrix}\begin{pmatrix}0 & i\\i & 0\end{pmatrix}\begin{pmatrix}i & o\\0 & i\end{pmatrix}\begin{pmatrix}0 & i\\i & 0\end{pmatrix}=\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$

$=\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$

$=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

Similarly

$(\overline{(K_2V_2A_2^*V_2K_2A_2)}=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$\overline{(K_IV_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*K_2A_2)}=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\cup\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$(A_1K_1V_1A_1{*}V_1K_1)\cup(A_2K_2V_2A_2{*}V_2K)=\overline{(K_IV_1A_1^*V_1K_1A_1)\cup(K_2V_2A_2^*V_2K_2A_2)}$

$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\cup\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\cup\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

Equivalent Properties on con. secondary k- normal bimatrices

Theorem-5

Let $A_B\in C_{nxn.}$ if A_B is secondary k- unitarily equivalent to a diagonal matrix then A_B is secondary k- normal.

Proof-

Let $A_B\in C_{nxn}$ if A is secondary k- unitarily equivalent to a diagonal bimatrixD_B.then there exist an secondary k-unitary matrix C_B such that

$(K_BV_BC_B{*}V_BK_B)A_BC_B=D_B$ say

$(K_BV_BC_B{*}V_BK_B)=C_B^{\Theta}$Similarly$A_B^{\Theta}$and D_B^{Θ}

There fore

$A_B=C_BD_BC_B^{\Theta}$ Since C_B is s-k unitary

Now

$A_BA_B^{\Theta}=C_BD_BC_B^{\Theta}\ (C_BD_BC_B^{\Theta})^{\Theta}$

$=\ C_BD_BC_B^{\Theta}\ C_BD_B^{\Theta}\ C_B^{\Theta}$

$=C_B^{\Theta}D_BD_B^{\Theta}\ C_B^{\Theta}$

Since $D_B$and$D_B^{\Theta}$are each diagonal $D_BD_B^{\Theta}=D_B^{\Theta}\ D_B$ therefore

$A_BA_B^{\Theta}=\ C_BD_B^{\Theta}D_BC_B^{\Theta}$

$=C_BD_B^{\Theta}C_B^{\Theta}\ (C_BD_BC_B^{\Theta})$

$A_BA_B^{\Theta}=A_B^{\Theta}A_B$

$A_1\cup A_2(A_1^{\Theta}\cup A_2^{\Theta})=(A_1^{\Theta}\cup A_2^{\Theta})(A_1\cup A_2)$

$A_1A_1^{\Theta}\cup A_2A_2^{\Theta}=A_1^{\Theta}A_1\cup A_2^{\Theta}A_1$

Hence A_B is con. S-k normal.

Theorem-6

Let G_B, $N_B \in C_{nxn}$ be invertible if $B_B= G_B N_B G_B$ where G_B is s-k hermitian and then N_B is s-k normal then $B_B^{-1}(K_BV_BB^*_BV_BK_B)$.

It is similar to an s-k unitary bimatrix.

Proof

Let G_B,$N_B \in C_{nxn}$ be invertible if $B_B= G_B N_B G_B$ then

$B_B^{-1}(K_BV_BB^*_BV_BK_B)=G_B^{-1}N_B^{-1}G_B^{-1}K_BV_B(G_BN_BG_B)^*V_BK_B$

$G_B^{-1}N_B^{-1}G_B^{-1}(K_BV_BG_B^*V_BK_B)\ (K_BV_BN_B^*V_BK_B)\ (K_BV_BG_B^*V_BK_B)$

$G_B^{-1}N_B^{-1}G_B^{-1}G_B(K_BV_BN_B^*V_BK_B)G_B$

Since N_B is s-k normal$N_B^{-1}(K_BV_BN_B^*V_BK_B)$ is s-k unitary

$(N_1^{-1} \cup N_2^{-1})((K_1 \cup K2)(V_1 \cup V_2)(N_1^* \cup N_2^*)(V_1 \cup V_2)(K_1 \cup K_2))$

$N_1^{-1}K_1V_1N_1^*V_1K_1 \cup N_2^{-2}K_2V_2N_1^*V_2K_2$ is s-k unitary.

Theorem-7

Let $A_B \in C_{nxn}$ assume that $A_B=U_BC_B$ where U_B is s-k unitary and C_B is non- singular and s-k hermitian such that if C_B^2 commutes with U_B then thefollowing conditions are equivalent.

(i) A_B is s-k normal.
(ii) $U_BC_B=C_BU_B$
(iii)$A_BU_B=U_BA_B$
(iv)$A_BC_B=C_BA_B$

Proof

$\widehat{U}_B=\widehat{U}_1 \cup \widehat{U}_2$ (ii)-

If A_B is s-k normal then

$A_B(K_BV_BA_B^*V_BK_B)=(K_BV_BA_B^*V_BK_B)A_B$

Since $A_B=U_BC_B$

$(U_BC_B)(K_BV_B(U_BC_B)^*V_BK_B)=(K_BV_B(U_BC_B)^*V_BK_B)(U_BC_B)$

$U_BC_BK_BV_BC_B^*U_B^*V_BK_B=K_BV_BC_B^*U_B^*V_BK_BU_BC_B$

$U_BC_BK_BV_BC_B^*V_BK_BK_BV_BU_B^*V_BK_B=K_BV_BC_B^*V_BK_BK_BV_BU_B^*V_BK_BU_BK_B^*$

$U_BC_BC_BU_B^{-1}=C_BU_B^{-1}U_BC_B$

$U_BC_B=C_BU_B$

Conversely if

$U_BC_B=C_BU_B$

Then

$K_BV_B(U_BC_B)^*V_BK_B=K_BV_B(C_BU_B)^*V_BK_B$

Now

$$
\begin{aligned}
A_B(K_BV_BA_B^*V_BK_B) &=(U_BC_B)(K_BV_B(U_BC_B)^*V_BK_B)\\
&=(U_BC_B)(K_BV_B(C_BU_B)^*V_BK_B)\\
&=U_BK_BV_BC_B^*V_BK_BK_BV_BU_B^*V_BK_BC_B
\end{aligned}
$$

Since C_B is s-k hermitian

$=U_B(K_BV_BU_B^*V_BK_B)K_BV_BC_B^*V_BK_BC_B$

$=(K_BV_BU_B^*V_BK_B)U_BC_BC_B$

Since $C_B$is s-k hermitian and s-k unitary.

$=(K_BV_BU_B^*V_BK_B)C_BU_BC_B$

Since $U_BC_B=C_BU_B$

$=(K_BV_BU_B^*V_BK_B)(K_BV_BC_B^*V_BK_B)U_BC_B$

$=(K_BV_B(C_BU_B)^*V_BK_B)(U_BC_B)$

$A_B(K_BV_BA_B^*V_BK_B)=(K_BV_BA_B^*V_BK_B)A_B$

$(A_1\cup A_2)(K_1\cup K_2)(V_1\cup V_2)(A_1^*\cup A_2^*)(V_1\cup V_2)(K_1\cup K_2)= (K_1\cup K_2)(V_1\cup V_2)(A_1^*\cup A_2^*)(V_1\cup V_2)(K_1\cup K_2)(A_1\cup A_2)$

$(A_1K_1V_1A_1^*V_1K_1) = (A_2K_2V_2A_2^*V_2K_2)$

$=A_1K_1A_1^*V_1K_1A_1\cup K_2V_2A_2^*V_2K_2A_2$

Hence A_B is s-k normal

(i) $\hat{U}_B=\hat{U}_1\cup \hat{U}_2$(iii)
If $A_B$is s-k normal then
$A_BU_B=(U_BC_B)U_B=U_B(C_BU_B)=U_B(U_BC_B)$ by
(ii) Conversely if
$A_BU_B=(U_BC_B)$ then
$(U_BC_B)U_B=U_B(C_BU_B)$
$(K_BV_BU_B^*V_BK_B)(U_BC_B)U_B=((K_BV_BU_B^*V_BK_B)U_B)(U_BC_B)$
$U_BC_B=U_BC_B$
$(C_1U_1)\cup(C_2U_2)=(U_1C_1)\cup(U_2C_2)$
Therefore A_B is s-k normal
(i) $\hat{U}_B=\hat{U}_1\cup \hat{U}_2$(iv)-
If A_B is s-k normal
$A_BC_B=(U_BC_B)C_B=C_B(U_BC_B)=C_BA_B$
Conversely if
$A_BC_B=C_BA_B$ then $(U_BC_B)C_B=C_B(U_BC_B)$
Post multiply by P^{-1},we have
$U_BC_B=C_BU_B$
$(U_1\cup U_2)(C_1\cup C_2)=(C_1\cup C_2)(U_1\cup U_2)$
$(U_1C_1)\cup(U_2C_2)=(C_1U_1)\cup(C_2U_2)$ and so
$A_B$is s-k normal.

Conclusion

Some of the characterization and properties of con. Secondary k-normal bimatrices can be verified.

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