



RESEARCH ARTICLE

NUMERICAL STUDY ON FLOW AND HEAT TRANSFER OF NON NEWTONIAN FLUIDS  
IN COILED MICRO TUBE

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ABSTRACT

In the present study, an attempt was made to explore the flow behavior of non-Newtonian fluids in coiled micro tube. The non-Newtonian fluids were aqueous solutions of carboxymethylcellulose (CMC), a polymer, with concentrations of 0.1 wt% and 0.5 wt%. Initially, the numerical procedure was validated by comparing the present predicted pressure drop of Newtonian fluid flowing in straight micro tube with known experimental data. Numerical computations were then carried out to characterize velocity field and temperature field of the non-Newtonian fluids flowing in coiled and straight micro tube. Friction factor and Nusselt numbers were computed and compared with the existing correlations for conventional tubes for the flow range of  $500 \leq N_{Re} \leq 1000$ . The results showed that the pressure drop in micro coiled tube was higher than the conventional tube. The heat transfer performance of coiled micro tube was more significantly enhanced for Newtonian fluids. Further work was carried out to study the effect of heat transfer for pseudo plastic fluids. It was found that the heat transfer performance of coiled micro tube increased with increase in pseudo plasticity of fluid.

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INTRODUCTION

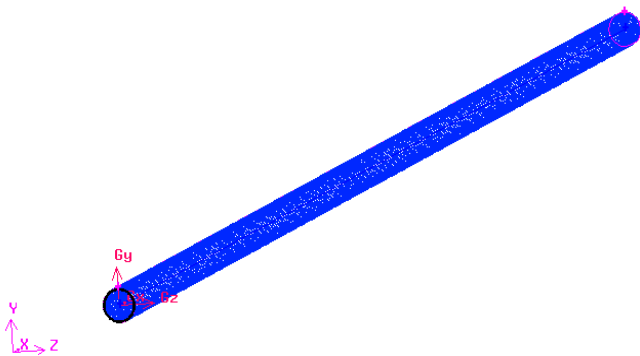
There is increasing trend in development of micro fluidic devices such as micro-pumps, micro-heat exchangers, nozzles, diffusers, heat pipes, sensors, transducers and actuators etc. These devices are portable and expedite time consuming laboratory analysis procedures. The amount of fluid used within these devices is very small hence they are of significant importance while using expensive chemicals. The techniques used to fabricate these devices are comparatively economical. A good understanding of flow characteristics is essential to develop such systems. In the past decade, large number of research works has been carried out in micro channel. One of the earliest research works on flow and heat transfer at micro scale was reported by Tuckermann and Pease (1982). They showed that an electronic chip could be efficiently cooled by means of flow of water through micro channels. Peng and Peterson (1996) experimentally investigated single phase forced convective heat transfer and flow characteristics of water in micro channel structures. They reported that heat transfer was found to be dependent upon the aspect ratio of the micro channels. They observed that resistance was smaller than that predicted by classical relationships for liquid with turbulent flow. The fluid flow transition to fully developed turbulent flow in micro channel was occurring at relatively less

Reynolds number than the ordinary channel flow. Mala and Li (1999) experimentally studied flow of water through straight micro tubes with diameters ranging from 50 to 254  $\mu\text{m}$ . The experimental results indicate significant departure of flow characteristics from the predictions of the conventional theory for micro tubes with smaller diameters. For micro tubes with large diameters, the experimental results are in rough agreement with the conventional theory. They also reported that the pressure drop of liquid flowing at lower flow rates agrees well with the Poiseuille flow theory. Sobhan and Garimella (2001) presented a review on studies on heat transfer and fluid flow in micro- and mini-channels and micro tubes. They observed discrepancies in prediction of flow characteristics and heat transfer in micro channels and the conventional channels by different investigators. They concluded that these discrepancies may be due to several conditions such as entrance and exit effects, surface roughness of micro channels, different channel dimensions, type of the thermal and flow boundary conditions, and uncertainties in instrumentation. Rostami *et al.* (2002) presented a review on flow and heat transfer for gas flowing in micro channels. They observed that further research work on areas such as flow of non-Newtonian fluid in micro channels, local and overall heat transfer characteristics, compressibility and rarefaction effects, etc. is essential to have better understanding to develop more reliable design correlation.

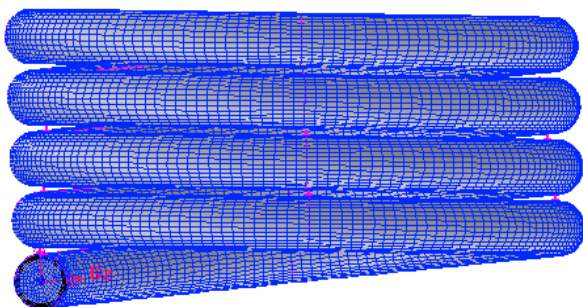
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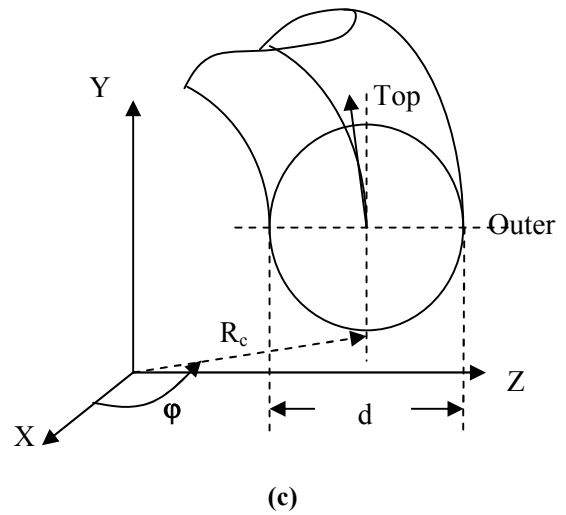
Non-Newtonian fluid plays an important role in various industries such as rayon, plastics, food processing, dye-stuffs, biochemical and pharmaceuticals. An understanding of non-Newtonian fluid flow and its heat transfer behavior is vital. It has been observed that fluids within micro-scale conduits are known to become increasingly viscous and thus difficult to mix (Kim and Lee, 2009). To overcome this problem, various geometries have been designed which show better performance in heat and mass transfer. It is well known that conventional sized helically coiled tube offer several advantages over straight tubes including compactness and enhanced heat transfer coefficients. This is due to secondary flows induced by curvature of the coiled tube (Mashelkar and Devarajan (1976a, b), Kawase and Moo-Young (1987), Kumar and Nigam (2005), Rennie and Raghavan (2007), Vashisth *et al.* (2008)]. In recent years, curved or helically coiled tubes is being studied in the fields of micro-mixers (Schoenfeld and Hardt, 2004), micro-sensors (Svasek *et al.* 1996), and micro-reactors (Mandal *et al.*, 2011, Sasmito *et al.*, 2012). The research area of micro fluidics is a comparatively immature. Numerical modeling of such systems can be tremendously significant and would help in research, design and optimization of micro fluidic devices. The performance of a system can be precisely predicted by including the complexities of conduit geometry, and process conditions into the numerical model. It furthermore helps in having better understanding and visualization of micro fluidic flow phenomena. Due to these reasons, Computational Fluid Dynamics (CFD) simulations were carried out to characterize flow and heat transfer of non Newtonian fluids flowing in straight as well as coiled micro tube for fluid flow range of  $500 \leq N_{Re} \leq 1000$ . Velocity and temperature profiles of the fluids flowing were analyzed. The pressure drop and the heat transfer coefficient were computed for micro tubes and compared with the data available for conventional sized tubes.



(a)



(b)



(c)

Fig. 1 (a) Straight micro tube, (b) Coiled micro tube, (c) Coordinate system

## Numerical Modeling

### Governing equations

The simulations were carried out using a commercial computational fluid dynamic software, FLUENT, ANSYS which uses the finite-volume method to solve the governing equations for a fluid. The geometries of straight micro tube, coiled micro tube and coordinate considered in the present work have been shown in Fig. 1.

The diameter of circular pipe was  $2r$ , and the coiled micro tube had a curvature of  $R_c$ . The distance between the two consecutive turns of coil was  $p$ . The fluid entered the inlet of tube with a velocity of  $U_0$  and temperature  $T_0$ . The wall of tube was heated at constant temperature  $T_w$ . The fluid flow was considered to be steady, and constant thermal properties were assumed. The differential equations governing the three-dimensional laminar flow in the coiled tube could be written as Continuity:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1)$$

Momentum:

$$\frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \rho u_j u_i - \delta_{ij} P \right] + \rho g_i = 0 \quad (2)$$

Energy:

$$\frac{\partial}{\partial x_j} \left[ k \left( \frac{\partial T}{\partial x_j} - \rho u_j C_p T \right) \right] + \mu \phi_v = 0 \quad (3)$$

where  $\mu \phi_v$  is the viscous heating term in energy equation, and  $\phi_v$  is represented by

$$\phi_v = \frac{\partial u_i}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \mu \frac{\partial u_i}{\partial u_j} \delta_{ij} \right) \quad (4)$$

The diffusion flux at the outlet for all variables in the exit direction was set to zero. The dimensionless parameters used to characterize the flow and heat transfer in the study are as follows:

$$N_{Re} = \frac{\rho u_0 d}{\mu} \quad (5)$$

$$\lambda = D/d \quad (6)$$

$$T_b = \left( \frac{1}{uA} \int_0^{2\pi} uT dA \right) \quad (7)$$

$$f_\theta = \frac{\tau_w}{\frac{1}{2} \rho u^2} \quad (8)$$

$$f_m = \frac{1}{2\pi} \int_0^{2\pi} f_\theta d\theta \quad (9)$$

$$N_{Nu,\theta} = \frac{q_w d}{k(T_w - T_b)} \quad (10)$$

$$N_{Nu,m} = (1/2\pi) \left( \int_0^{2\pi} N_{Nu,\theta} d\theta \right) \quad (11)$$

where  $N_{Re}$  is Reynolds Number;  $\lambda$  is the curvature ratio;  $T_b$  is the bulk temperature of fluid;  $f_\theta$  is local friction factor along the circumference of the tube;  $f_m$  is circumference average friction average friction factor;  $N_{Nu,\theta}$  and  $N_{Nu,m}$  are the local and average Nusselt number of the tube respectively.

### Effect of Non-Newtonian fluids

The stress tensor for an incompressible Newtonian fluid flowing is given as Bird *et al.* (2002)

$$\tau = -\mu \dot{\xi} \quad (12)$$

The stress tensor,  $\tau$  have a non linear relation with the rate-of-strain tensor,  $\dot{\xi}$  for non-Newtonian fluids. The viscosity  $\mu$  is replaced by the apparent viscosity,  $\eta$  in Eq. (12). In this present study water, non-Newtonian power-law was used to model the non Newtonian flow. According to which

$$\eta = K \dot{\xi}^{n-1} e^{T/T_r}$$

where K is the consistency index,  $n$  is the

flow behavior index,  $T_r$  is the reference temperature (ANSYS FLUENT). Fluids with values of  $n < 1$  are pseudo-plastic and those with  $n > 1$  are dilatant. Saxena (1982) have tabulated the properties of various power law fluids which were used for numerical calculations. In this study, properties of 0.1 wt% CMC and 0.5 wt% CMC solutions were used to analyze the behavior of flow and heat transfer in micro tubes. The properties of fluids considered are given in Table 1.

The CMC solutions exhibit pseudo-plastic (shear-thinning) behavior. A generalized Reynolds number,  $N_{Re}^*$ , which takes into account the consistency index and flow behavior index (Rennie and Raghavan (2007)) was computed as

$$N_{Re}^* = \frac{\rho v^{2-n} d^n}{K \left( \frac{a+bn}{n} \right)^n 8^{n-1}} \quad (13)$$

The values of  $a$  and  $b$  are 0.25 and 0.75, respectively for a circular geometry. The flow behavior index of water is one ( $n=1$ ), therefore, the consistency Coefficient,  $K$  is the viscosity of water.

### Grid system

An unstructured non-uniform grid system was used in the geometries to discretize the governing equations. The second-order upwind scheme was used to solve the convection term in the governing equations. The coupling between velocity and pressure was solved using SIMPLE algorithm. The under-relaxation factor for the pressure was 0.3; for temperature was 0.9; for the velocity component was 0.5; and that for body force was 0.8. The numerical computation was assumed to be converged when the residual summed over all the computational nodes at  $n$ th iteration,  $R_\phi^n$  satisfied the criterion:

$$\frac{R_\phi^n}{R_\phi^m} \leq 10^{-8}$$

where  $R_\phi^m$  denotes the maximum residual value of  $\phi$  variable after  $m$  iterations,  $\phi$  applied for p, u, and T. A grid refinement study was carried out to find out the adequate distribution of grids. Table 2 presents the predicted results for different grid distributions on pressure drop of water flowing in a straight micro tube with a diameter of 64  $\mu\text{m}$  and length of 0.055 m. The sectional number represents the total number of elements on one cross-section of the micro tube. It was observed that the 120  $\times$  1000 grid arrangement ensured a satisfactory solution for flow of water in straight micro tube. There was no significant difference in the values of pressure drop after further increase in number of grids.

## RESULTS AND DISCUSSION

The accuracy of the computation technique used in the present study was checked with the experimental data reported by Mala and Li (1999). They have reported the pressure drop data of water flowing in micro channels with diameters ranging from 50 to 254  $\mu\text{m}$ . Figure 2 shows the present CFD predictions of pressure drop as a function of the Reynolds number ( $N_{Re}$ ) for water flowing through straight micro tube of  $d=64 \mu\text{m}$ ,  $L= 0.055 \text{ m}$ . It can be observed from the figure that the CFD predictions of pressure drop were in good agreement with the experimental results of Mala and Li (1999). The maximum deviation between the CFD predictions and the experimental data was less than 2%. To the best of our knowledge, no work has been reported for flow of fluids in coiled micro tube with diameter less than 100  $\mu\text{m}$ .

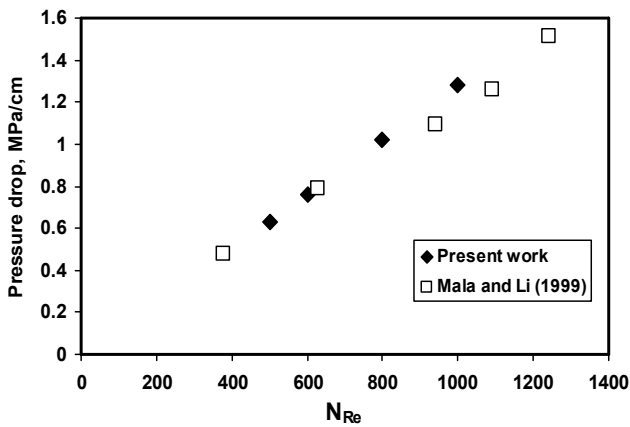


Fig. 2. Comparison of CFD predictions with experimental results for pressure drop of water flowing through straight micro tube of  $d=64 \mu\text{m}$ ,  $L= 0.055 \text{ m}$

Hence, in order to check the reliability of present CFD model for coiled micro tube, the simulations were run on geometry of coiled tube with diameter of  $100 \mu\text{m}$  and  $300 \mu\text{m}$ . The computed data was then validated with experimental results of Kim and Lee (2009). Figures 3 (a) and (b) shows the mean axial velocity profiles within the transverse plane in coiled micro tube diameter of  $100 \mu\text{m}$  and  $300 \mu\text{m}$  respectively. The CFD predictions for coiled tube geometry with nodes distribution of  $195 \times 1120$  agree well with the data reported by Kim and Lee (2009).

Table 1. Properties of fluids used in the simulation

Solvent	Specific heat (J /kg/ K)	Thermal conductivity (W/m/K)	Density (gm/cc)	Flow behavior index $n$	Consistency index $K$ (gm.sec <sup><math>n-2</math></sup> /cm)
water	4184	0.7	1.0	1.0	0.01
0.1 wt% CMC	4100	0.7	1.0	0.915	0.02
0.5 wt% CMC	4100	0.7	1.01	0.725	0.244

Table 2. Grid test for a straight micro tube with  $d= 64 \mu\text{m}$ ,  $L= 0.055 \text{ m}$

S.No.	No. of nodes at cross section X axial length	Pressure drop (Pascal)
1	$97 \times 800$	7125736
2	$110 \times 800$	7058681
3	$120 \times 800$	7039795
4	$120 \times 900$	7038795
5	$120 \times 1000$	7038771
6	$120 \times 1200$	7038770

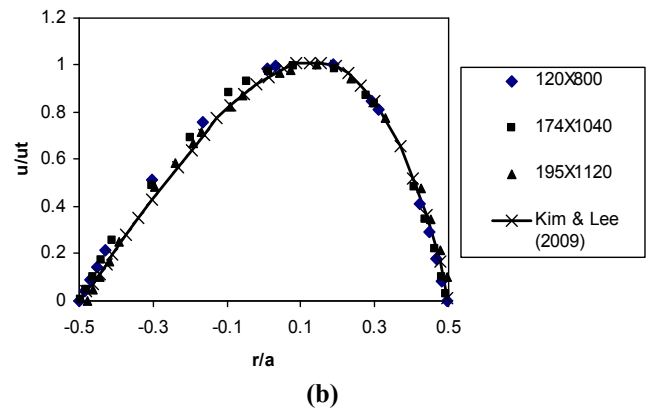
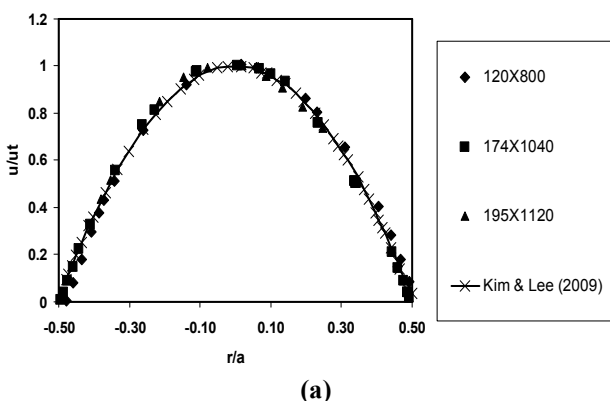


Fig. 3. Comparison of mean axial velocity profiles for different nodes within the transverse plane of coiled micro tube with diameter (a)  $d= 100 \mu\text{m}$ , (b)  $d= 300 \mu\text{m}$

Figure 4(a) and 4(b) illustrates the effect of velocity distribution at outlet of straight micro tube and coiled micro tube, respectively for fluids with  $n=0.7, 0.9, 1$  flowing at  $N_{Re} = 1000$ . Figure 4(a) shows that the velocity of fluids increases uniformly from zero at the wall to the maximum at centre of tube. It was also observed that the velocity gradient was reduced as the value of  $n$  decreased from 1 to 0.7. The maximum velocity at centre of tube was decreased with the decrease in value of  $n$ . The velocity contours became increasingly flattened due to reason that apparent viscosity of the pseudo plastic fluid was minimum at the wall where the shear stress was maximum.

Figure 4(b) depicts that the maximum velocity is shifted towards the outer wall of the tube. This was due to the action of centrifugal force arisen due the curvature of coil. Moreover, the boundary layer present around the wall of coiled micro tube was thinner as compared to the straight micro tube shown in Figure 4(a). The velocity contours were again observed to be blunter for pseudo plastic fluids as compared to Newtonian fluid.

The friction losses can be estimated in straight micro tubes from the magnitude of the Fanning friction factor. Values of friction factor for different fluids flowing through straight micro tube were plotted against  $N_{Re}$ . It can be observed from Figure 5 that the friction factor values decreased with decrease in  $N_{Re}$ .

The effect of  $n$  did not play a significant role on the friction factor values for fluid flowing in laminar flow regime through straight micro tube. However, the friction factor values were 8-14 % higher than the values calculated for laminar flow in round conventional tubes by formula  $f = \frac{16}{N_{Re}}$ . The

dependence of friction factor on the Dean number ( $N_{De}$ ) for pseudo plastic fluids flowing in coiled micro tube was analyzed.  $N_{De}$  is a dimensionless number which characterizes the strength of secondary flow in coiled tubes. Figure 6 demonstrates that the  $n$  value played a more significant role in coiled micro tube. Friction factor values were higher for greater values of  $n$ .

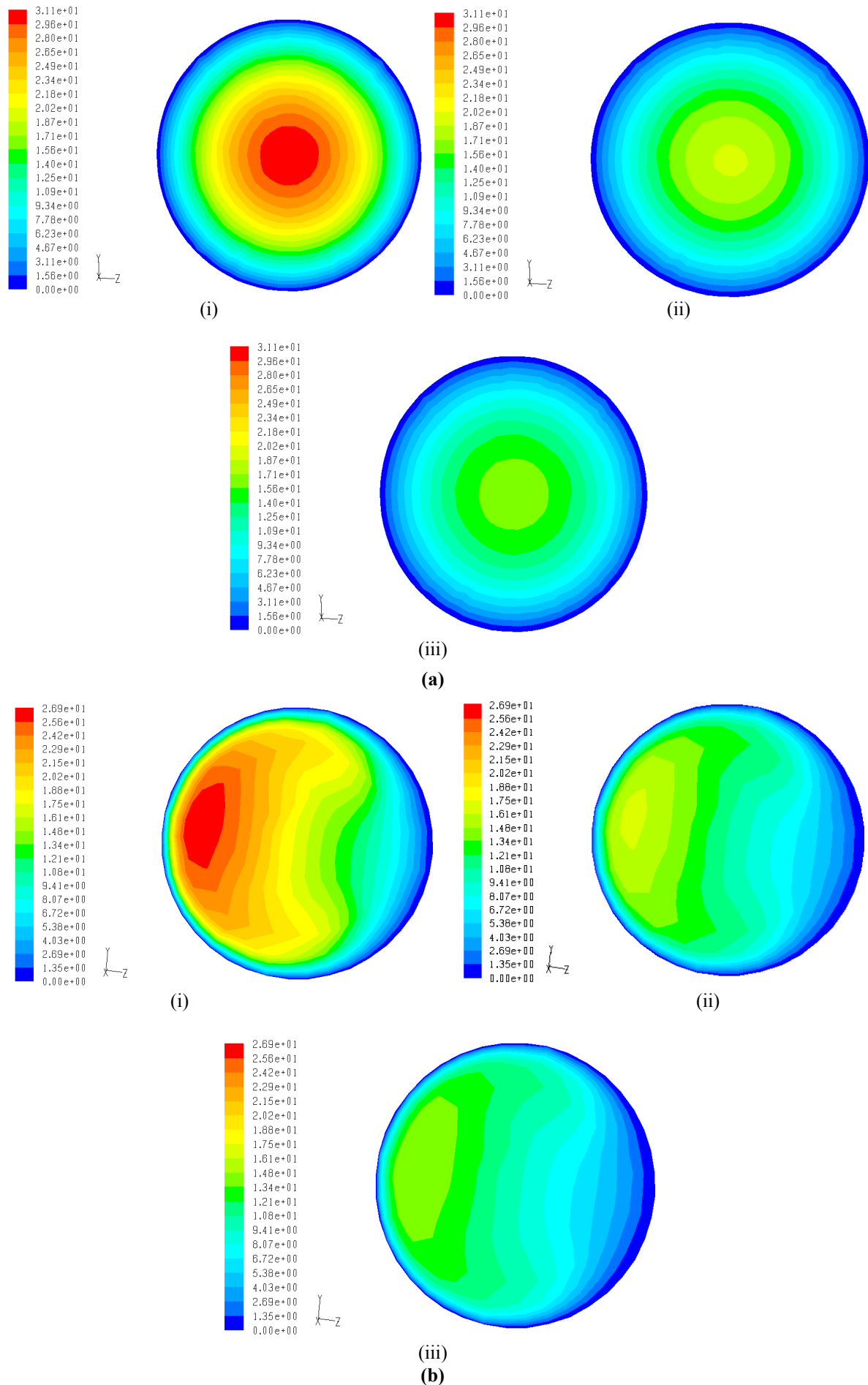


Fig. 4. Velocity profile for fluid with (i)  $n=1$ , (ii)  $n=0.9$ , (iii)  $n=0.7$  in (a) straight (b) coiled micro tube with diameter  $d=64 \mu\text{m}$  at  $N_{Re}=1000$

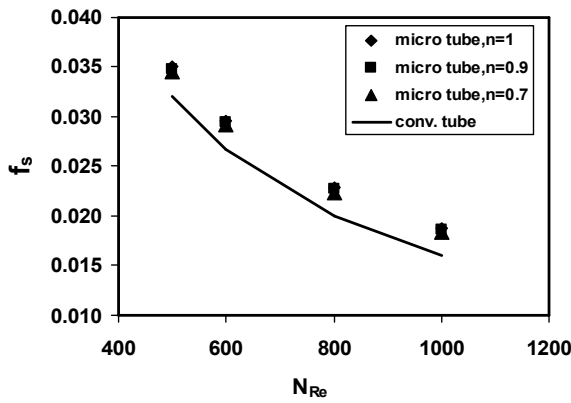


Fig. 5. Comparison of effect of friction factor on  $N_{Re}$  for different fluids flowing in straight micro tube with  $d=64 \mu\text{m}$ ,  $L=0.008 \text{ m}$  with conventional tube

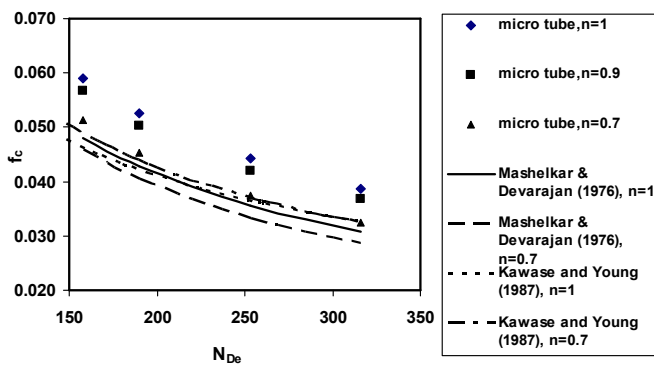


Fig. 6. Comparison of effect of friction factor on  $N_{De}$  for different fluids flowing in coiled micro tube with  $d=64 \mu\text{m}$ ,  $L=0.008 \text{ m}$  with conventional tube

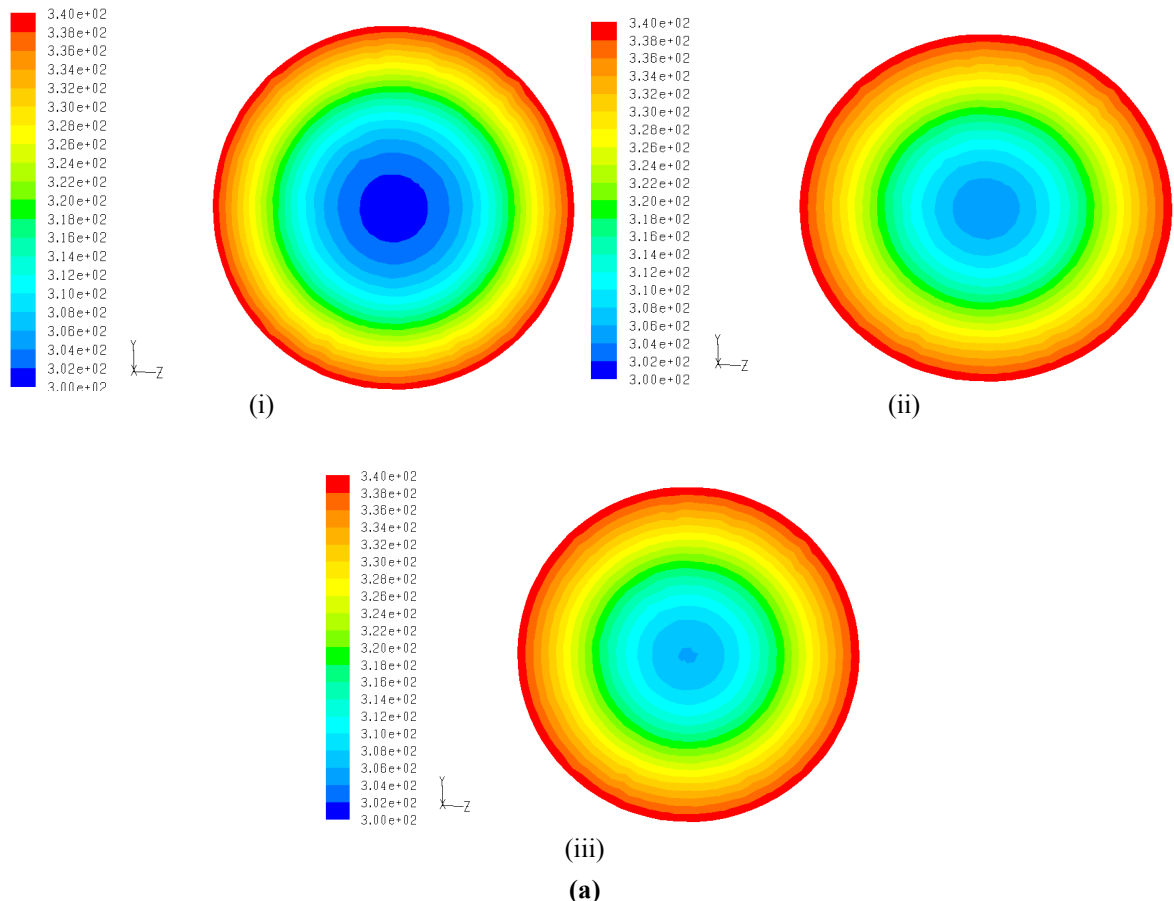
The present computed values of friction factor in coiled micro tube were also compared with the existing correlations proposed for conventional coiled tubes. Mashelkar and Devarajan (1976a) proposed the following correlation for friction for laminar flow of a power law type of non-Newtonian fluid in a coiled tube.

$$f_c = [9.069 - 9.438(n) + 4.37(n)^2] \left(\frac{d}{D_c}\right)^{0.5} (N_{De}^*)^{-0.768+0.122n} \quad (14)$$

This correlation is applicable for  $70 \leq N_{De}^* \leq 400$ ,  $0.01 \leq (d/D_c) \leq 0.135$  and  $0.35 \leq n \leq 1$ , where  $N_{De}^*$  is modified Dean Number. It was observed that the friction factor values for present coiled micro tube was 18-20 % higher than the values calculated by above correlation for fluid with  $n=1$ . The values for friction factor for micro coiled tube were 10-12 % higher than that of conventional tube for fluid with  $n=0.7$ . The difference in friction factor values of Newtonian and pseudo plastic fluids were more significant in micro coiled than the conventional tubes. The present data were also compared with the values calculated by correlation proposed for Newtonian fluids by Kawase and Young (1987). They proposed

$$\frac{f_c}{f_s} = \frac{1}{16} \left\{ C^{-n(2-n)} 2^{2(4-n)} \pi^{-n^2} \left(\frac{3}{2}\right)^{2n(1+n)} A^{-n^2} \left(\frac{1+3n}{4n}\right)^{-2n} N_{De}^{*2n} (d/D)^{n(1-n)} \right\}^{1/[2(1+n)]} \quad (15)$$

Where  $A = \frac{280}{39}(1+n)\left(\frac{3}{2}\right)^n$ . For  $n=1$ , the above equation reduces to  $\frac{f_c}{f_s} = 0.0925C^{-1/4}N_{De}^{1/2}$ . The value of  $C$  was determined as 0.42. The values calculated for present work was nearly 15-22% higher than the values calculated by the correlation proposed by Kawase and Young (1987).



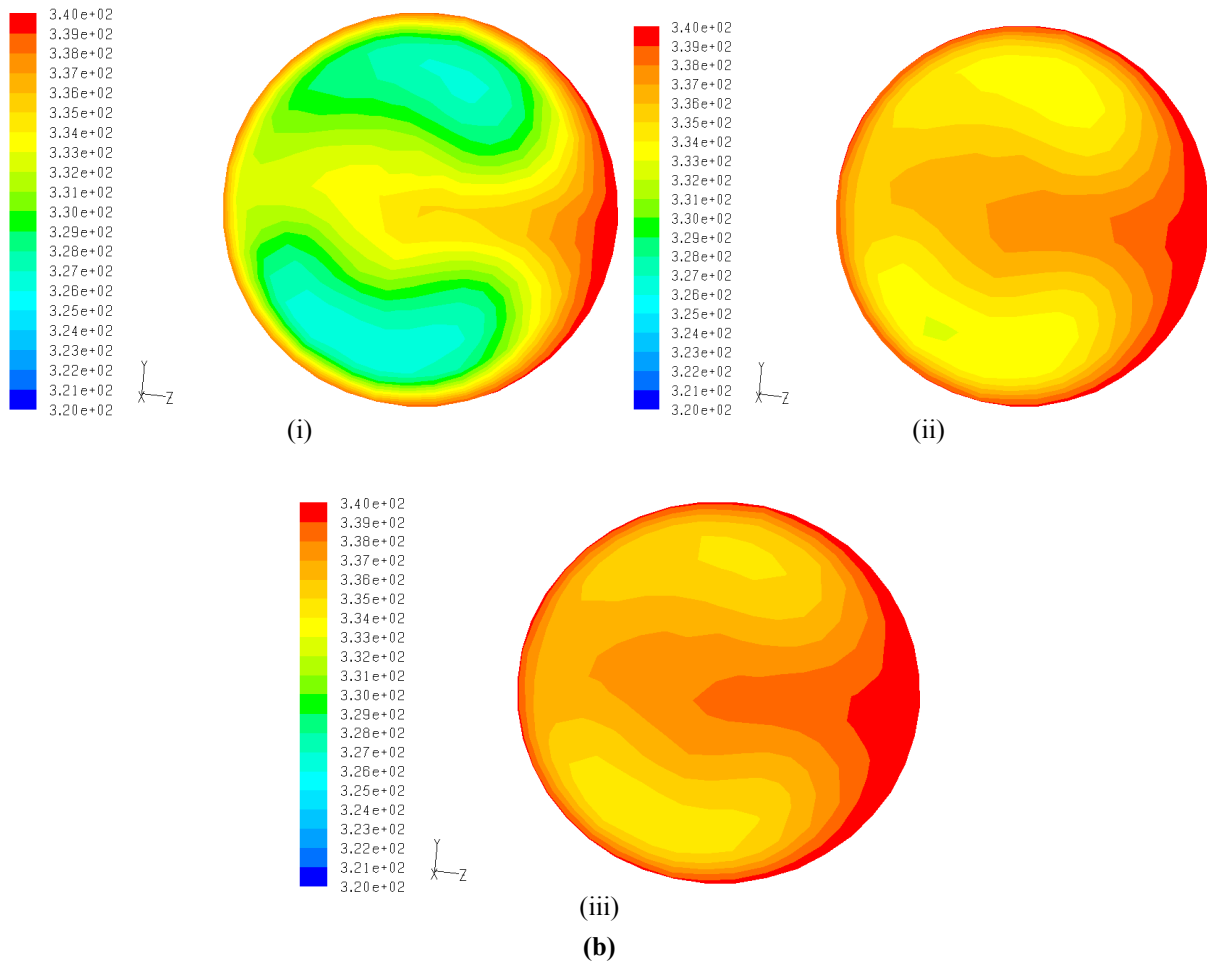


Fig. 7. Temperature profile in (a) straight, (b) coiled micro tube with diameter  $d=64 \mu\text{m}$ ,  $L=0.008 \text{ m}$  at  $N_{Re} = 1000$  (i)  $n=1$ , (ii)  $n=0.9$ , (iii)  $n=0.7$

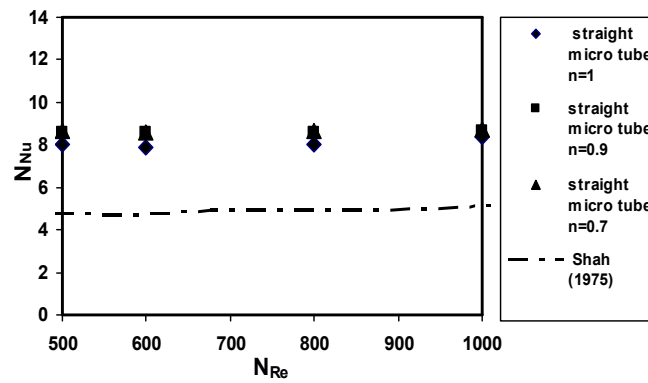


Fig. 8. Comparison of effect of  $N_{Nu}$  with  $N_{Re}$  in straight micro tube with conventional tube

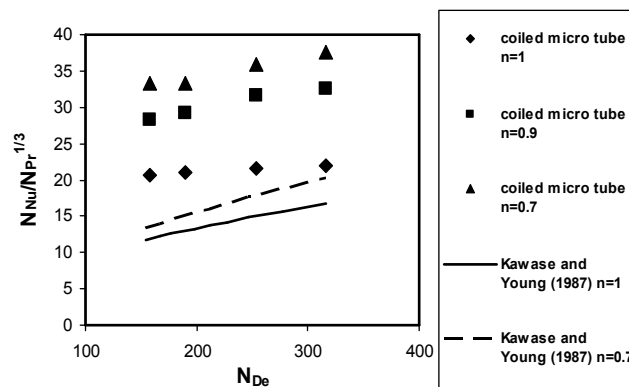


Fig. 9. Comparison of effect of  $N_{Nu}$  with  $N_{De}$  in coiled micro tube with conventional tube

Heat transfer simulations under uniform wall temperature were carried out for the two geometries. The nature of temperature distribution for fluids with  $N_{Re}=1000$  flowing at the outlet of straight micro tube and coiled micro tube is shown in Figure 7(a) and 7(b) respectively. The Figure 7(a) shows that the maximum temperature (red colour) was at the wall of straight micro tube. The temperature gradually decreases till the minimum at the centre of tube. The temperature profiles were more uniform as the flow behavior index decreased from  $n=1$  to  $n=0.7$ . This could be due to the reason that for pseudo plastic fluids, the non Newtonian behavior made the temperature profile more like plug flow and increased the heat transfer coefficient. Similar trend could be observed for coiled micro tube in Figure 7(b). However it is quite clear that the temperature profile is more significantly uniform for coiled micro tube than straight micro tube. This shows that heat transfer of pseudo plastic fluids is increased significantly in coiled micro tubes.

The effect of  $N_{Nu}$  with  $N_{Re}$  in straight micro tube was studied and is shown in Figure 8. It was found that there was  $N_{Nu}$  increment of 3-8 % for non Newtonian fluid ( $n=0.7$ ) as compared to Newtonian fluid. A comparison was made with the heat transfer data proposed by Shah (1975) for conventional sized tubes. It was found that the heat transfer was significantly increased in straight micro tube as compared to conventional sized tubes. Figure 9 shows the effect of heat transfer with  $N_{De}$  in coiled micro tube. The figure shows that the heat transfer increased with increase in  $N_{De}$  as well as the value of  $n$ . The data computed for coiled micro tube was also compared to conventional macro tube [Kawase and Young (1987)]. They proposed heat transfer correlation as follows.

$$N_{Nu} = 0.698 \left\{ 4.76 \left( 2^{7(n-1)} \left( \frac{\pi}{2} \right)^{-(2n-1)/3} \left( \frac{3n+1}{4n} \right)^{2n} \left( \frac{3+3n}{1+2n} \right)^{[2(n+1)]/3} A^{1/3} N_{De}^{*2} \right)^{1/[2(1+n)]} \right\} N_{Pr}^{1/3} \quad (16)$$

For  $n=1$ , the above equation reduces to  $N_{Nu} = 0.944 N_{De}^{1/2} N_{Pr}^{1/3}$ . It was observed that the heat transfer performance was increased by 23-40 % in coiled micro tube as compared to conventional tube.

## Conclusion

In the present work, numerical investigations have been carried out to investigate flow behavior and heat transfer of non-Newtonian fluids in straight as well as coiled micro tube. The data obtained from computations were validated with the data already existing in literature. The velocity field and temperature field of the both Newtonian as well as non-Newtonian fluids flowing in straight and coiled micro tube were observed. It was found that as the profiles were more uniform with increase in pseudoplasticity of fluid. The friction factor and  $N_{Nu}$  values were computed for the present micro tubes and were compared with that of conventional tubes. The values were found to be higher than conventional tubes. It was also observed that heat transfer performance of coiled micro tube was significantly enhanced for non Newtonian fluids as compared to conventional tubes. The study shows that coiled micro tubes can be used as efficient device to enhance mixing and heat transfer of fluids.

## Notations

A	cross-sectional area (m <sup>2</sup> )
C <sub>p</sub>	specific heat, kJ/(kg K)
d	internal diameter of tube (m)
D	coil diameter (m)
g	gravity (m <sup>2</sup> /s)
H	dimensionless pitch, H = p/d
k	thermal conductivity, W/(m K)
K	consistency coefficient
L	length (m)
n	flow behavior index
$N_{De}$	Dean number = $\frac{N_{Re}}{\lambda}$
$N_{De}^*$	generalized Dean number
$N_{Nu}$	Nusselt number
$N_{Pr}$	Prandtl number
$N_{Re}$	Reynolds number
$N_{Re}^*$	generalized Reynolds number
p	pitch (m)
P	pressure (N/m <sup>2</sup> )
q <sub>w</sub>	wall heat flux (W/m <sup>2</sup> )
r	radius of pipe (m)
R <sub>c</sub>	coil radius (m)
u	velocity, m/s
u <sub>i</sub>	velocity component in i-direction (i = 1, 2, 3), m/s
U <sub>0</sub>	inlet velocity
T	Temperature, K
T <sub>0</sub>	inlet temperature
T <sub>w</sub>	wall temperature
x <sub>i</sub>	master Cartesian coordinate in i-direction (i = 1, 2, 3), m

## Greek symbols

$\delta_{ij}$	Dirac delta function
$\lambda$	curvature ratio (D/d)
$\sigma$	surface tension (N/m)
$\mu$	viscosity (kg/(m.s))
$\rho$	density of fluid (kg/m <sup>3</sup> )
$\tau_w$	shear stress at wall ((N/m <sup>2</sup> ))

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