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RESEARCH ARTICLE

SECONDARY UNITARY SIMILARITY OF MATRICES

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ABSTRACT

The concept of s-unitarily similar matrices is introduced. Some theorem on s-unitarily similar matrices are given.

INTRODUCTION

The concept of s-unitary matrix was introduced in our earlier paper [4].Also the concept of s-orthogonal similarity of real matrix is introduced by [5]. In this paper we define s-unitarily similar matrices and established some theorems on s-unitary matrices.

Preliminaries

Let C_{nxn} be the space of nxn complex matrices of order n.For $A \in C_{nxn}$,let A^T $\overline{A}$,, A^*, A^s, $\overline{A}^s$ $(=A^\theta)$ denote transpose, conjugate, conjugate transpose, secondary transpose, conjugate secondary transpose of a matrix A respectively. Anna Lee [1] shown that for a complex matrix A, the usual transpose A^T and secondary transpose A^S are related as $A^S = VA^TV$ Where 'V' is the associated permutation matrix whose elements on the secondary diagonal are 1 and other elements are zero. We denote $A^\theta = \overline{A}^s = (c_{ij})$ where $c_{ij} = \overline{a_{n-j+1,\ n-i+1}}$ and $\overline{A}^s = VA^*V = A^\theta$.More over 'V' satisfies the following properties $V^T = \overline{V} = V^* = V$ and $V^2 = I$

Secondary Unitary Similarity

Definition: 2.1 [4] Let $A \in C_{nxn}$. A matrix A is called s-unitary if $A A^\theta = A^\theta A = I$

Definition: 2.2 [3] The nxn matrices A and B are called similar if there exist a nonsingular matrix S such that $A = S^{-1}BS$.

Definition: 2.3 [6] A and B are unitarily similar if there is a unitary matrix 'U' such that A=U*BU

Now let us define s-unitarily similarity of two matrices A and B.

Definition: 2.4 Let $A,B \in C_{nxn}$. A is said to be s-unitarily similar to B if there is a s-unitary matrix 'U' such that $A=U^\theta BU$

Theorem: 2.5 A is s-unitary iff every matrix s-unitarily similar to A, is s-unitary.

Proof

If A is-unitary then $A A^\theta=A^\theta A= I$.

B is any matrix which is s-unitarily similar to A.
$\therefore$ $B=U^\theta AU$ where U is s-unitary.

$$B^\theta B = (U^\theta AU)^\theta (U^\theta AU) = U^\theta A^\theta U U^\theta AU = U^\theta A^\theta AU = U^\theta U = I$$

$$BB^\theta = (U^\theta AU)(U^\theta AU)^\theta = U^\theta A U U^\theta A^\theta U = U^\theta A A^\theta U = U^\theta U = I$$

$\therefore B^\theta B = BB^\theta = I$ $\therefore$ B is s-unitary.

Coversely, if $B=U^\theta AU$ is s-unitary then $B^\theta B = BB^\theta = I$

$$B^\theta B = (U^\theta AU)^\theta (U^\theta AU) = U^\theta A^\theta U U^\theta AU = U^\theta A^\theta AU$$

$$BB^\theta = (U^\theta AU)(U^\theta AU)^\theta = U^\theta A U U^\theta A^\theta U = U^\theta A A^\theta U$$

$\therefore$ $U^\theta A^\theta AU= U^\theta A A^\theta U=I$

Premultiplying by U and Post multiplying by U^θ we get $A A^\theta = A^\theta A = I$. ∴ A is s-unitary.

Theorem: 2.6 Let $A \in C_{nxn}$. If A is s-unitarily similar to a diagonal matrix which is s-unitary then A is s-unitary.

Proof

A is s-unitarily similar to a diagonal matrix D.Then there exists a s-unitary matrix B such that $B^\theta AB=D$

$\therefore \quad A=BDB^\theta$

$A A^\theta = (BDB^\theta)(BDB^\theta)^\theta$

$=BDB^\theta BD^\theta B^\theta$

$= BDD^\theta B^\theta$

$= BB^\theta = I$ Since Band D are s-unitary.

$A^\theta A = (BDB^\theta)^\theta (BDB^\theta)$

$=BD^\theta B^\theta BDB^\theta$

$= BD^\theta DB^\theta$

$= BB^\theta = I$

$\therefore A A^\theta = A^\theta A = I$. ∴ A is s-unitary.

Corrallory 2.7: Let A and B be s-unitarily similar. Then A is s-unitary iff B is s-unitary.

Theorem: 2.8 If A and B are s-unitarily similar

Then $\sum_{i,j} |b_{ij}|^2 = \sum_{i,j} |a_{ij}|^2$

Proof

We have $\sum_{i,j} |a_{ij}|^2 = tr(A^\theta A)$

$\sum_{i,j} |b_{ij}|^2 = tr(B^\theta B)$

$= tr((U^\theta AU)^\theta (U^\theta AU))$

$= tr(U^\theta A^\theta AU)$

$= tr(A^\theta A)$

$= \sum_{i,j} |a_{ij}|^2$

$\therefore \sum_{i,j} |b_{ij}|^2 = \sum_{i,j} |a_{ij}|^2$

Definition: 2.9 [2] Two families of nxn matrices $\{A_1,A_2,\ldots.A_m\}$ and $\{B_1,B_2,\ldots.B_m\}$are said to be unitarily similar if there exists a unitary matrix U such that $U^* A_i U = B_i$, $i = 1,2\ldots.,m$

Definition: 2.10 Two families of nxn matrices $\{A_1,A_2,\ldots.A_m\}$ and $\{B_1,B_2,\ldots.B_m\}$are said to be s-unitarily similar if there exists a s- unitary matrix U such that $U^\theta A_i U = B_i$, $i = 1,2\ldots.,m$

Theorem: 2.11 Let $A,B \in C_{nxn.}$ If A and B are s-unitarily similar then the families $\{A, A^\theta\}$ and $\{B, B^\theta\}$ are s- unitarily similar.

Proof

A and B are s-unitarily similar ⇒ $B=U^\theta AU$ where U is s-unitary matrix.

We have to show $B=U^\theta AU$ …(i) and $B^\theta=U^\theta A^\theta U$---(ii)

Proof (i) is obvious from definition of s-unitarily similarity.

(i) ⇒ $B=U^\theta AU$

$B^\theta = (U^\theta AU)^\theta = U^\theta A^\theta U$ which is (ii)

Therfore the families $\{A, A^\theta\}$ and $\{B, B^\theta\}$are s- unitarily similar.

Theorem: 2.12 Let $A,B \in C_{nxn.}$ If the families $\{A, A^\theta\}$ and $\{B, B^\theta\}$are s- unitarily similar then the families $\{A, A^\theta\}$ and $\{B, B^\theta\}$are s- similar.

Proof

Given $\{A, A^\theta\}$ and $\{B, B^\theta\}$are s- unitarily similar

$B= U^\theta AU$ and $B^\theta = U^\theta A^\theta U$

We have to show $VB=U^{-1}VAU$ and $(VB)^\theta =U^{-1}(VA)^\theta U$

$B=U^\theta AU$ $VB=VU^\theta AU = U^{-1}VAU$ Since $U^{-1}V= VU^\theta$

Therefore VB is similar to VA

⇒ B is s-similar to A.

Similarly we may prove B^θ is s-similar to A^θ.

Therefore the families $\{A, A^\theta\}$ and $\{B, B^\theta\}$are s- similar

Theorem: 2.13 Let $A,B \in C_{nxn.}$ If the families $\{A, A^\theta\}$ and $\{B, B^\theta\}$are s- similar then A and B are s- unitarily similar .

Proof

Given $\{A, A^\theta\}$ and $\{B, B^\theta\}$are s- similar.

⇒ $P^{-1}VAP = VB$ and $P^{-1}(VA)^\theta P = (VB)^\theta$

$P^{-1}VAP = [P^{-1}(VA)^\theta P]^\theta$

$=P^\theta VA(P^{-1})^\theta$

$P^{-1}VAP= P^\theta VA(P^\theta)^{-1}$

$PP^{-1}VAP= PP^\theta VA(P^\theta)^{-1}$

$VAP= PP^\theta VA(P^\theta)^{-1}$

$(VA)P P^\theta= PP^\theta VA$

Consider polar decomposition i.e $P=SU$, $S=S^\theta$ and $U^*U=I$.

Since the s-hermition S which is a square root PP^θ can be represented as polynomial in PP^θ it follows from the relation above that

$S(VA) = (VA)S$.

$VB= P^{-1}(VA)P= (SU)^{-1}(VA)SU =U^{-1}S^{-1}VASU = U^*VAU$

Therefore VA is unitarily similar to VB.

⇒A is s-unitarily similar to B.

Definition: 2.14: $A \in C_{nxn}$ is called s-unitarily diagonalizable if there exist a s-unitary matrix 'U' for which $U^\theta AU$ is diagonal.

Theorem: 2.15 If $A\in C_{nxn}$ has eigen values $\lambda_1,\lambda_2,\ldots.\lambda n$ counted according to multiplicity then A is s- unitary implies A is s-unitarily diagonalizable.

Proof

A is s-unitary ⇒A is s-normal.

Then $A A^\theta$ is s-hermitian therefore s-unitarily diagonalizable. Thus $U^\theta A^\theta AU = D= U^\theta AA^\theta U$ .Also A, A^θ, $A A^\theta= A^\theta A$ form a commuting family. This implies that eigen vectors of $A^\theta A$ are also eigen vectors of A.Since $A^\theta A$ has a complete orthonormal set .we know that $U^\theta AU$ is also diagonal.

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