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RESEARCH ARTICLE

A MATHEMATICAL MODEL FOR TWO PHASE HUMAN BLOOD FLOW IN HEPATIC VENUOLES ALONG LIVER ABSCESS

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ABSTRACT

In this paper we describe the mathematical model during Liver Abscess. Clinical evaluation of data collected in a hospital is also presented. The main objective of this paper work is to enhance about Abscess. One of the major problems of Liver Abscess is pollution. Liver abscess (LA) remains a serious and often difficult to diagnose problem. Liver abscess requires a high degree of suspicion for early diagnosis. We apply the Herschel-Bulkley Non - Newtonian model. The stability of this model is one of the most vital aspects in Mathematical field. In this case, the two phase blood flow in which that of first is red blood cells and other is plasma. In this paper we find the graphical presentation between hematocriet and blood pressure drop.

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INTRODUCTION

Structure & function of Liver

The liver constitutes about 2% of the body weight in adults, thus being the largest organ in the body. It receives 25% of the cardiac output via two in flows, the portal vein and the hepatic artery. Total amount of hepatic perfusion is about 1 ml/min per 1 g liver tissue, and oxygen consumption by the liver accounts for 20% of total body oxygen consumption. The liver receives 25% of the cardiac output, although it constitutes only 2.5% of body weight (Kumar Anil, 2009). Function of the liver with respect to bile and plasma proteins, and the process of detoxification, it should be pointed out that the liver has other functions as well. These functions are:

- Carbohydrate storage
- Amino acid metabolism
- Metabolism of steroidal hormones
- Metabolism of fat

Bile is a complex solution secreted by the cells of the liver into the bile duct.

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Approximately 250-1000 ml/day ares secreted. It is golden yellow in color due to the presence of bile pigments (bilirubin and biliverdin). These pigments, it should be noted, are the breakdown products of hemoglobin. Also found in bile are the bile salts which are sodium and potassium salts of bile acids. The Liver considered the production of plasma proteins. The liver plays an intricate role in the synthesis of these plasma proteins and is able to provide for an inter conversion (*i.e.* converting one type of amino acid to another by the process of transamination) of amino acids which, you will recall, are the building blocks of protein structures.

Structure & function of Hepatic circulation

The liver receives 25% of the cardiac output, although it constitutes only 2.5% of body weight. The hepatic parenchymal cells are the most richly perfused of any of the organs, and each parenchymal cell on the average is in contact with perfusate on two sides of the cell. Of the total hepatic blood flow (100–130 ml/min per 100 g of liver, 30 ml/min per kilogram of body weight), one fifth to one third is supplied by the hepatic artery. About two thirds of the hepatic blood supply is portal venous blood. The hepatic parenchymal cells are the most richly perfused of any of the organs, and each parenchymal cell on the average is in contact with perfusate on

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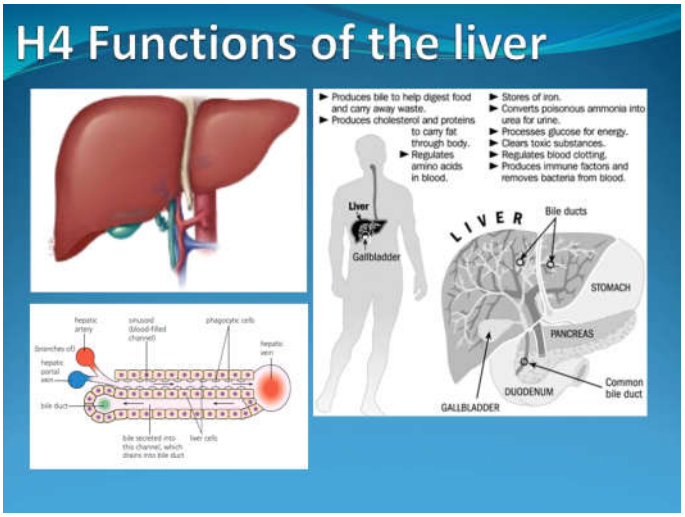


Figure 1. Function of the liver

Structure and function of abscess

Liver Abscesses have been recognized since the age of Hippocrates .In 1938, Ochsnar and Debakey published the largest serious of pyogenic and amebic liver abscesses in the literature. Since the last 20th century, percutaneous drainage has becomes a useful therapeutic option, because of its minimal invasiveness and high cure rate. Hepatic or liver abscesses are infectious, space-occupying lesions in the liver; the two most common abscesses being pyogenic and amoebic (Perrillo, 2010).

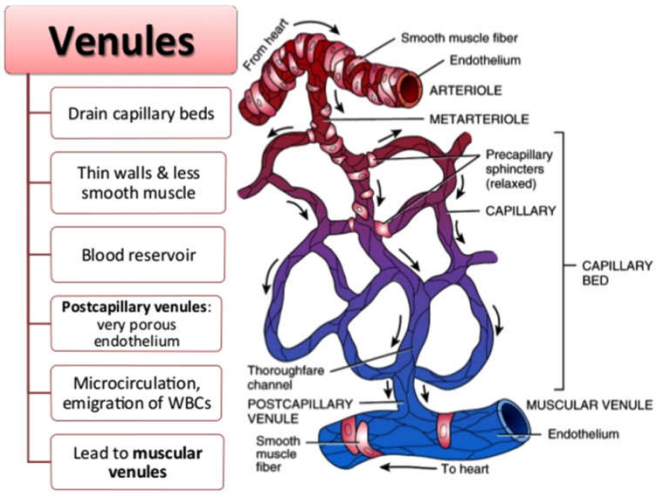


Figure 2. Structure and function of Venules

Pyogenic liver abscess (PLA) is a rare but potentially lethal condition, with a reported incidence of 20 per 1,00,000 hospital admissions in a western population. Its severity depends on the source of the infection and the underlying condition of the patient. Amoebic liver abscesses (ALA) are common in tropical regions mainly where '*Entamoeba histolytica'* is endemic and is more prevalent in individuals (mostly young males) with suppressed cell mediated immunity [4]. In both the types of hepatic abscesses, right lobe of the liver is the most likely site of infection (Khan, 2008). The clinical presentation of both the types may be elusive with combination of fever, right upper quadrant pain and hepatomegaly with or without jaundice.

Structure and function of Venules

Venules are the smallest blood vessel in the microcirculation that allows deoxygenated blood to return from the capillary beds to the larger blood vessels called veins.

Functions of Veins:

- From when capillaries unit
- The smallest venules consist of only a Tunica intima.
- Blood is under low pressure in the venules.

Real Model: Blood is the Non-Newtonian fluids then using the constitutive equation for fluids

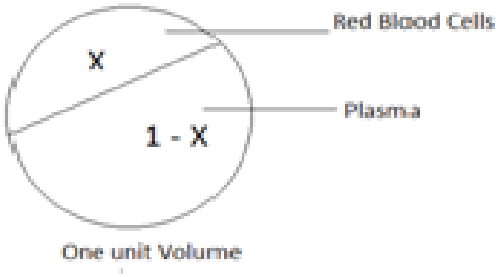
$$\tau = \eta e^{n}$$

If $n = 1$ then the nature of fluid is Newtonian and if $n \neq 1$ then the nature of fluid is Non-Newtonian fluids.Where, τ is denoted by stress,e is denoted by strain rate and n is denoted by the parameter, these equation uses equation of motion. In present study there are five parameter are used but three parameter components of velocity are frequently used namely velocity, pressure P and densityρ. We have using in two phase blood flow through arterioles and whose constitutive equation is as follows-

$$T' = \eta_m e^n + T_p (T' \geq T_p) \text{where, } T_p \text{is the yield stress.}$$

When strain rate $e = 0\,(T' < T_p)$ a core region is formed which flows just like a plug.

They are already conceder two-phases in blood. The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells. Let the volume portion covered by blood cells in unit volume be$X$, X is replaced by H */100*, where H is the Hematocrit the volume percentage of blood cells. Then the volume portion covered by the plasma will be (*1-X*). If mass ratio of cells to plasma is r then clearly:$r = \frac{X\rho_c}{(1-X)\rho_p}$.



Where ρ_c and ρ_p are densities of blood cells and plasma respectivily. Usually this mass ratio is not a constant. Even then this may be supposed to constant in present context.

Mathematical Model / Formulation

The total momentum of any fluid system is conserved in absence of external force.

$\frac{dp}{dt} + P - F_{v(viscosity)} = 0\ (External\ Foorce)$

Equation of Continuity

If mass ratio of cells to plasma is r then clearly:

$$r = \frac{X\rho_c}{(1-X)\rho_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

$$\frac{\partial(X\rho_c)}{\partial t} + (X\rho_c V^i)_{,i} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (2)$$

$$\frac{\partial(1-X)\rho_p}{\partial t} + (1-X)\rho_c V^i)_{,j} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (3)$$

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Then equation (2) and (3) can be combined together as:

$$\frac{\partial \rho_m}{\partial t} + (\rho_m V^i)_{,j} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (5)$$

Equation of motion for blood-flow

We get the equation of motion for the phase of blood cells as follows:

$$X_{\rho_c}\frac{\partial V^i}{\partial t} + \left(X_{\rho_c}V^j\right)V^i,j = -X_p,j\, g^{ij} + X\eta_c\left(g^{jk}V^i,k\right),j \ldots\ldots(6)$$

$$(1-X)_{\rho_c}\frac{\partial V^i}{\partial t} + \{(1-X)_{\rho_c}V^j\}V^i,j = -(1-X)_p,j\, g^{ij} + (1-X)\eta_c\left(g^{jk}V^i,k\right),j \quad \ldots\ldots(7)$$

Now adding equation (6) and (7) and using relation (4), the equation of motion for blood flow with the both phases will be as fallows-

$$\frac{\partial V^i}{\partial t} + (\rho_m V^i)V^i,j = -P,j + \eta_m\left(g^{jk}V^i,k\right) \quad \ldots\ldots\ldots\ldots\ (8)$$

Where $\eta_m = X\eta_c + (1-X)\eta_p$ are the viscosity coefficients of blood as a mixture of two phases.

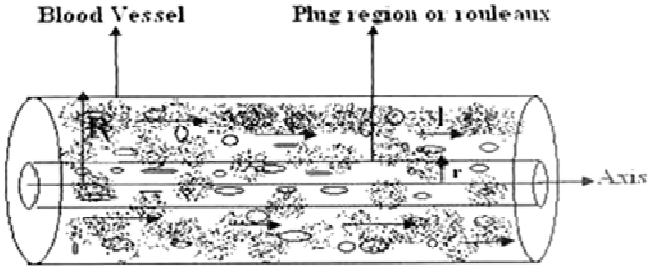


Fig (3): Herschel Bulkley blood flow

$$T^{ij} = -Pg^{ij} + T_e^{ij} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (9)$$

Where $T^{ij} = \eta_m(e^{ij})^n$ while $e^{ij} = (g^{jk}V^i_{,k} + g^{ik}V^j_{,k})$

Equation of Continuity

$$\frac{1}{\sqrt{g\sqrt{(gV^i),i}}} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (10)$$

Equation of Motion

$$\rho_m\frac{\partial V^i}{\partial t} + \rho_m V^j V^i,j = -T^{ij}e,j$$

Analysis

$X^1 = r, X^2 = \theta,\ X^3 = z.$

Matrix of metric tensor in cylindrical co-ordinates is as follows

$$[g^{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Where the Christoffel's symbols of 2^{nd} kind as follows-

$\left\{\begin{matrix}1\\2\end{matrix}\ \begin{matrix}1\\2\end{matrix}\right\} = -r$, $\left\{\begin{matrix}1\\2\end{matrix}\ \begin{matrix}1\\2\end{matrix}\right\} = \left\{\begin{matrix}1\\2\end{matrix}\ \begin{matrix}1\\2\end{matrix}\right\} = \frac{1}{r}$ remaining others are zero.

Relation between contra variant and physical components of velocity of blood flow will be as follows

$\sqrt{g^{11}v^1} = v_r \Rightarrow v_r = v^1$,
$\sqrt{g^{22}v^2} = v_\theta \Rightarrow v_\theta = rv^2$, $\sqrt{g^{33}v^3} = v_z \Rightarrow v_z = v^3$, again the physical components of $p_{,j}g^{ij}$ are $\sqrt{g_{ij}}p_{,j}g^{ij}$

Equation of continuity:

$$\frac{\partial V}{\partial Z} = o$$

The equation of motion

r- component: $-\frac{\partial p}{\partial z} = 0, \emptyset - component: 0 = 0$
z- component: $0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r}\left[r\left(\frac{\partial V_z}{\partial r}\right)^n\right]$

The blood flow the axially symmetric in arteries concerned, i.e. $V_\theta = 0$ and $V_r, V_z and p = p(z) and 0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r}\left[r\left(\frac{\partial V_z}{\partial r}\right)^n\right]$ ..... (11)

Since, pressure gradient $-\frac{dp}{dz} = P$

$r\left(\frac{dV}{dZ}\right)^n = -\frac{Pr^2}{2\eta_m} + A$, we apply boundary condition at $r = 0. V = V_0$ than $A = 0$.

$\Rightarrow -\frac{dv}{dr} = \left(\frac{Pr}{2\eta_m}\right)^{\frac{1}{n}}$ Replace from $r - r_p$

$$-\frac{dv}{dr} = \left(\frac{\frac{1}{2}pr - \frac{1}{2}p_r}{\eta_m}\right)^{\frac{1}{n}} \Rightarrow \frac{dv}{dr} = -\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (r - r_p)^{\frac{1}{n}} \ldots\ldots\ldots\ldots (12)$$

Integrating above equation (12) under the no slip boundary condition $V = 0$ at $r = R$ so we get:

$$v = \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \frac{n}{n+1}\left[(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1}\right] \ldots\ldots\ldots\ldots (13)$$

This is the formula for velocity of blood flow in arterioles.
Putting $r = r_p$ to get the velocity v_p of plug flow as follows:

$$v_P = \frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} \ldots\ldots\ldots\ldots (14)$$

RESULT AND DISCUSSION

Clinical data for diagnosis of Abscess

The value of flow flux Q = 900ml/min = o.o15m^3/sec

Where $z_f = 100, z_i = 8, \quad z = z_f - z_i = 100 - 8 = 92\ \mu m \Rightarrow 9.2 \times 10^{-5} meter$

Apply the formula we find the value of n.

$$\frac{27Q}{2\pi} = \left(\frac{p_f - p_i}{z_f - z_i\ 3\eta_m}\right)^{1/n} \frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1}$$

$$0.636171975 = \left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]\left[\frac{34168616.3}{9.2\times10^{-5}}\right]^{1/n}$$

Solve this formula find the value of n = -4.8

Again we apply the formula we find pressure drop

$$p_f - p_i = 3\eta_m(z_f - z_i)\left(\frac{27Q}{2\pi A}\right)^n$$

Where A= $\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}$]

$$p_f - p_i = 3[\eta_c X + \eta_p(1 - X)](z_f - z_i)\left(\frac{27Q}{2\pi A}\right)^n$$

$$3[87.1844741\times \frac{H}{100} + 0.0015(1 - 0.03849)\ (0.6361719)^n \left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]^{1/-4.8}$$

Solve this equation we find the different values of Hematocrit and find different values of pressure drop.

Table 1. Clinical data for diagnosis of Abscess

S.N.	B.P.(mm hg)	B.P. drop (2/3(S+D/2+D)/3) – (1/3 (S+D/2+D)/3)	B.P (Pascal)	HB	H
1	120/60	16.667	2218.911	11.9	0.0336792
2	110/70	17.778	2366.8207	11.2	0.0316981
3	110/80	19.443	2588.4854	12.7	0.0359434
4	140/95	23.612	3143.5127	13.6	0.0384905
5	120/70	18.333	2440.70896	13.2	0.0373584

Table 2. Different value of Hematocrit and pressure drop

Hematocrit(H)	Pressure- dropΔp
0.033667	2.9015428×10^{-4}
0.031698	2.7381382×10^{-4}
0.0359434	$3.088191828 \times 10^{-4}$
0.03849057	$3.297654987 \times 10^{-4}$
0.03735849	$3.204037962 \times 10^{-4}$

If

We take the value of H = 0.0384905 then pressure drop P = 3143.5127

Where η_p = o.oo15
$\eta_m = 0.035$

Apply the formula and find the value of η_c.

Let $\eta_m = \eta_c X + \eta_p(1 - X)$

Where X = H/100 where 0.0384905/100 = 0.000384905
By solve we find the value of $\eta_c = 87.1844741\ pascle/sec$
If pressure drop P = $-\int \frac{dp}{dz}$ = 3143.5127 / 9.2×10^{-5} = 34168616.3

Where $z = z_f - z_i$= $8 to 100\ \mu m\ (micro\ meter)$ Length of Venules in hepatic.

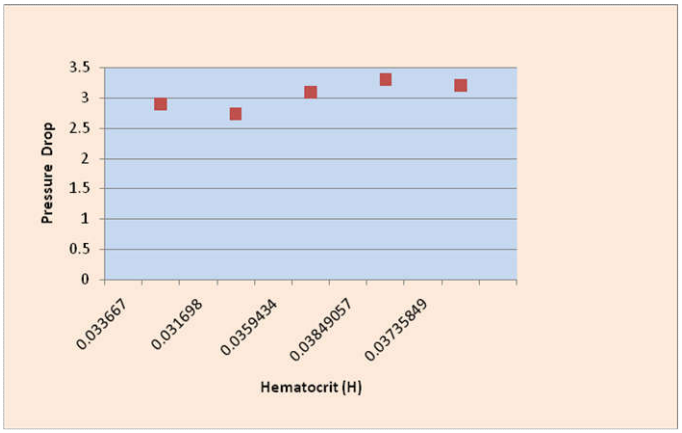


Figure 4. Graphical presentation of Clinical Data

Conclusion

A simple investigation of the graph is upper convex on 0.03367 and lower convex on 0.031698 and again upper

convex on 0.0359434 and 0.03849057 and again lower convex on 0.03735849.The blood pressure drop and hematocrit in suffering Abscess patient and confirmations that when hematocrit increase then blood pressure drop also increased and shows a non-linier graph.

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