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RESEARCH ARTICLE

WORKING OF QUEUEING MODEL WITH REMOVABLE SERVER

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ABSTRACT

In this chapter, we have considered a bulk service queueing model with removable serve. We have discussed a Markovian queueing system with a single server and units / customer serve in batches. The server is removed from the system as soon as the system becomes empty for a duration which is exponentially distributed randomly. We derive the time dependent solution of mean number of arrivals in the system or exactly 'i' customer arrive in the system. The study state solution is also discussed.

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INTRODUCTION

In most of the work done in the queuing theory it is assumed that the server is always available. Schall & Kleinrock (1992) Pegden and Rosenshine (2002) discussed on vacation queue and derived some results for transient behavior for a M/M/1 Queue. Levy & Yechiali (1993) considered a queueing system in which server is not available at all time. Some studies such as Balachandran, Heyman and Sobel (1991) considered that the server will resume his work when there is at least N customer accumulate and named this as N policies. In this chapter, we consider that server is removed for an exponentially distributed random time when the queue becomes empty and made a policy in which server removed from service for secondary work. The idle time of the server may be utilized. Many authors including Bailey (1954), Jaiswal(1960), Restrapo(1965) and Gupta and Goyel (1966) studied some queueing problems on the assumption that arrivals occur in batches. Chaudhry and Templton (1972), Murari(1972) and Bagchi and Templton (1973a,b) have studied system with batch arrival and service. In all these studies arrival and service intensities were taken homogenous. First Esien and Tainter(1963) and later Yechiali and Naor(1971), an the assumption of the single arrival and service, have obtained steady state solutions of M/M/1 queue, where arrival and service intensities are subject to Poissonian jumps between two states.

Many practical situations can be represented closely by this system.

(i) During bad weather period, many a times arrival and departure of the plane breakdown at the airport. In such situations the (time) period, during which arrival and departure rates of the passengers at the custom counters are high and low, can be said to follow each other randomly.
(ii) The rush hour problem can be said to correspond to this system, when we consider ordinary periods as one state and rush period as other.

In this chapter we consider queueing models in which server are removed for an exponentially distributed random time when queue is empty. There are many situations in which the server has to perform the secondary work for queue. For example in a bank, the clerk has also to maintain their records in addition to serve the customers.

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This chapter considered a queueing model where the server may be removed from the service facility for an exponential random time 'θ' where there is no customer in the queue. The time dependent solution of the number of arrivals and departures in the system are obtained & finally the steady state solution of the same are also derived.

Notations

$P_{ijk}(t)$: Probability that there are exactly i arrivals and j departures at time t and the server is on removed state.
$P_{ijB}(t)$: Probability that there are exactly i arrivals and j departures at time t and the server is busy.
$P_{ij}(t)$: Probability that there are exactly i arrivals and j departures by time 't' $i \geq j \geq 0$.

Initial condition

$P_{0R}(0) = 1$
$P_{00B}(0) = 0$

The difference-difference-differential equation governing the system is

$$P_{iiR}(t) = -\lambda\, P_{iiR}(t) + \mu\, P_{i,i-1,B}(t) \quad \text{......} (1)$$

$$P_{i,jR}(t) = -(\lambda+\theta)\, P_{ijR}(t) + \lambda\, P_{i-1,jR}(t) \quad \text{......} (2)$$

$$P_{i,jB}(t) = -(\lambda+\mu)\, P_{ijB}(t) + \lambda\, P_{i-1,j,B}(t) + \mu\, P_{ij-1,B}(t) + \theta\, P_{ijR}(t) \;\; i > j \geq 0 \quad \text{......} (3)$$

$$P_{ij}(t) = P_{ijR}(t) + P_{ijB}(t)$$

Taking Laplace transformation of equation 1, 2 and 3

$$S\,\overline{P}_{iiR}(s) + \lambda\,\overline{P}_{iiR}(s) = 1 + \mu\,\overline{P}_{ii-1,B}(s)$$

$$\overline{P}_{iiR}(s) = \frac{1}{S+\lambda}(1 + \mu\,\overline{P}_{i,i-1,B})$$

$$\overline{P}_{00R}(s) = \frac{1}{S+\lambda} \quad \text{......} (4)$$

$$s\,\overline{P}_{i,jR}(s) - \overline{P}_{ijR}(0) = -(\lambda+\theta)\overline{P}_{ijR}(s) + \lambda\,\overline{P}_{i-1,jR}(s)$$

$$(S+\lambda+\theta)\,\overline{P}_{ijR}(s) = \lambda\,\overline{P}_{i-1,j,R}(s) + \overline{P}_{ijR}(0)\;\; (\lambda\,\overline{P}_{i-1,j,R}(s) + \overline{P}_{ijR}(0))$$

$$\overline{P}_{ijR}(s) = \frac{1}{(S+\lambda+\theta)}$$

$$\overline{P}_{1,0,R}(s) = \frac{\lambda}{(S+\lambda)(S+\lambda+\theta)}$$

$$\overline{P}_{2,0,R}(s) = \lambda^2 \frac{1}{(S+\lambda)(S+\lambda+\theta)^2}$$

$$\overline{P}_{i0,R}(s) = \lambda^i \frac{1}{(S+\lambda)(S+\lambda+\theta)^i}$$

$$\overline{P}_{i0R}(s) = \lambda^i\, \overline{\beta}_{ij}^{\lambda(\lambda+\theta)}(s)$$

$$\overline{P}_{ijR}(s) = \lambda^i\, \mu \frac{1}{(S+\lambda)(S+\lambda+\theta)}\left(\frac{\mu}{S+\lambda}\right)^{S_{ij}} \overline{P}_{i,j-1,B}(s)$$

$$\overline{P}_{ijR}(s) = \lambda^i \mu\, \overline{\beta}_{ij}^{\lambda(\lambda+\theta)}(s)\left(\frac{\mu}{S+\lambda}\right)^{\delta_{ij}} \overline{P}_{i,j-1,B}(s)$$

$$S\ \overline{P}_{ijB}(s) + (\lambda+\mu)\overline{P}_{ijB}(s) = \lambda\,\overline{P}_{i-1,j,B}(s) + \mu\ \overline{P}_{i,j-1,B}(s) + \theta\,\overline{P}_{ijR}(s) + \overline{P}_{i,j,B}(0)$$

$$\overline{P}_{ijB}(s) = \frac{1}{S+\lambda+\mu}[\lambda\,\overline{P}_{i-1,j,B}(s) + \mu\ \overline{P}_{i,j-1,B}(s) + \theta\overline{P}_{ijR}(s) + \overline{P}_{i,j,B}(0)] \qquad \text{.......... (5)}$$

$$\overline{P}_{i,0,B}(s) = \left(\frac{1}{S+\lambda+\mu}\right).\frac{\lambda\theta}{(S+\lambda)(S+\lambda+\theta)}$$

$$\overline{P}_{2,0,B}(s) = \frac{\theta\lambda^2}{(S\_\lambda)(S+\lambda+\mu)(S+\lambda+\theta)^2}$$

$$\overline{P}_{i,0B}(s) = \frac{\theta\,\lambda^i}{(S+\lambda)(S+\lambda+\mu)(S+\lambda+\theta)^i}$$

$$\overline{P}_{ijB}(s) = \left(\frac{\lambda^i}{(S+\lambda+\mu)^i}\right)\{\lambda^i\,\mu\,\theta\,\overline{\beta}_{1,i,j}^{(\lambda+\mu)(\lambda+\theta)}(s)\}\left(\frac{\lambda^i\mu}{(S+\lambda+\mu)^{i+1}}\right)\overline{P}_{i,j-1,B}(s) \qquad \text{.......... (6)}$$

Taking Laplace inverse transformation of equation 4, 5 and 6 we get

$P_{00}(t) = e^{-\lambda t}$

$P_{i0R}(t) = \lambda^i\ \beta_{1,i}^{(\lambda+\theta)}(t)$

$P_{i,jR}(t) = \lambda^i\,\mu\ \overline{\beta}_{ij}^{\lambda(\lambda+\theta)}(t)\,(\mu\ e^{-\lambda t})^{\delta_{ij}}\ P_{ij-1B}(t)$

$$P_{ijB}(t) = \left\{\frac{e^{i(\lambda+\mu)}.t^{i-1}}{\lfloor S-1}\right\}[\lambda^i\mu\theta\overline{\beta}_{1,1,j}^{(\lambda+\mu)(\lambda+\theta)}(t)]\ .\left(\frac{\lambda^i\mu\, e^{-(\lambda+\mu)}t^i}{\lfloor i}\right) P_{i,j-1,B}(t)$$

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}\overline{P}_{ijR}(s) + \overline{P}_{i,j,B}(s) = \frac{1}{S} \text{ and } \sum_{i=0}^{\infty}\sum_{j=0}^{i} P_{ijR}(t) + P_{ijB}(t) = 1$$

Hence the verification.

1) Exactly i units arrive in time‘t’ are

$$\overline{P}_{i0}(s) = \sum_{j=0}^{i}\overline{P}_{iiR}(s) + \overline{P}_{ijB}(s)\ (1-\delta_{ij})$$

$$\overline{P}_{i0}(s) = \frac{1}{(S+\lambda)}\left(\frac{\lambda}{S+\lambda}\right)^i$$

$$\overline{P}_{i0}(s) = \frac{\lambda^i}{(S+\lambda)^{i+1}}$$

$$P_{i0}(t) = \frac{(\lambda\ t)^i}{\lfloor i} e^{-\lambda t}$$

The total number of arrivals is not affected by removal period θ of the server & the arrivals follow a Poisson Distribution. The mean number of arrivals in time 't' is

$$\sum_{i=0}^{\infty} i\ \overline{P}_{i0}(s) = \sum_{i=0}^{\infty} i \left\{ \frac{\lambda^i}{(S+\lambda)^{i+1}} \right\} ;\ i \geq 0$$

$$= \left\{ \frac{\lambda}{S^2} \right\} (6)$$

$$\sum_{i=0}^{\infty} i\ P_{i0}(t) = \{\lambda\ t\}.$$

Conclusion

It is clear from the equation that if the arrival rate is increases than queue length is also increases. Also**,** it is clear that if the service rate is increases than queue length decreases.

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