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RESEARCH ARTICLE

FORECASTING AND ANALYSIS MORTALITY USING LEE-CARTER MODEL WITH APPLICATION

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ABSTRACT

We demonstrate here a model proposed by Lee and Carter (1992) for fit and to forecast mortality rates. This approach is used widely in demographical applications and academic literature because the structure of the Lee-Carter model allows for the construction of confidence intervals related to mortality and age-specific death protections. To improve the performance of the Lee-Carter model, several extensions to the original version have been proposed. In this paper, we use real data of mortality rates by gender in Kirkuk City in Iraq during the period 2006-2015 to apply a modification of the Lee-Carter model, which accommodates variations in age-specific parameters using a singular values decomposition method to estimate the Lee-Carter parameters model. We also use the autoregressive moving average (ARIMA) with the special case of Random walk with drift (RWD) model to forecast the general index for the time period 2015-2020. The paper further predicts the survival expectancy at birth for each gender. Our results found this survival expectancy to be increasing for age group (0-1) year and to be decreasing for age group (75-80) years. For the long-term forecast it is necessary for the field of Demography to obtain such predications, which depend however on the available data.

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INTRODUCTION

Studying and doing research on demography is a quantitative social science that deals with every aspect of human populations, such as marriage, divorce, giving birth, migration and finally death (Bell and Monsell, 1991). As human beings or population actors our final act on the earth is our death, and thus our death is one of the most important events in our life. It is a certainty that every one of us has been born and every one of us will die. In this research we will consider and analyse (Andreozzi, Blaconá and Arnes 2011) the mortality of individual population actors (Carter and Prskawetz 2001) which is the most important event in our life. Lee and Carter (1992) presented a stochastic model, based on a factor analytic approach, to fit and predict mortality rates. Since then, because of its simplicity and relatively good performance, the Lee-Carter (LC) model has been widely used for demographic and actuarial applications in various countries. In addition, the Lee-Carter model and its extensions have been used by actuaries for multiple purposes. Essentially, the model assumes that the dynamic of mortality trends over time is only ruled by a single parameter called the mortality index. The mortality forecast is based on the index's extrapolation, obtained by selecting an appropriate time series model. The Box-Jenkins models, also known as an autoregressive moving average process (ARIMA) (Box and Jenkins, 1976), are usually used on forecasting. Lee and Carter (1996) used an random walk with drift (RWD) forecasting model. The RWD model does not make any assumption about the structure of the covariance matrix, while the Lee-Carter approach applies the estimation of the systematic non-random structure Lee-Carter model to mortality forecasting, which is based on the Singular Value Decomposition (SVD) of an appropriate data matrix in this approach. Thus, the Lee-Carter estimator is more efficient when data are drawn with the Lee-Carter model. In this research we define a range of demographical concepts in mortality actors. Section (I) describes the properties of the Lee-Carter model, and uses these properties to suggest that the Lee-Carter model is equivalent to a special type of multivariate random walk with drift (RWD) model, in which the covariance matrix depends on the drift vector in section (II). In section (III) we use application data for Kirkuk City for the period 2006-2015. In Section (IV) we present some conclusions, and finally section (V) includes a brief discussion.

Measurements of Mortality

Mortality can be defined as "the frequency with which death occurs in the population".

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The United Nations and the World Health Organization have proposed the following definition of death: “Death is the permanent disappearance of all evidence of life at any time after birth has taken place.” (I): Crude Death Rate (CDR): The simplest and most common measure of mortality is the crude death rate. The crude death rate is defined as:

$$\mathrm{CDR} = \frac{\mathrm{D}}{\mathrm{P}} * 1000 \quad (1\text{-}1)$$

Where (D) represents the total number of deaths in a given year, and (P) is the total mid-year population.

(II): Age-specific Death Rates (ASDR)

The risk of dying differs greatly with age, and this difference is not indicated by the crude death rate. That is why demographers often find it useful to use age-specific death rates. The age-specific death rate is defined as:

$$\mathrm{m}_{\mathrm{x}} = \frac{\mathrm{D}_{\mathrm{x}}}{\mathrm{P}_{\mathrm{x}}} * 1000 \quad (1\text{-}2)$$

Where (m_x) is the age-specific death rate, (D_x) is the number of deaths of people aged (x) years since their last birthday, and (P_x) is the mid-year population of people aged (x) years.

(III): Life Expectancy at Birth: Age-specific life expectancy is an estimation of the average number of the remaining years that a person would be expected to live if current mortality conditions were constant.

Lee-Carter Method: The Lee and Carter model (named LC hereafter) is a demographic and statistical model that is used to project mortality rates (Lee and Carter, 1992). The method can be seen as a special case of a principal components method (Bozik and Bell, 1987; Bell and Monsell, 1991) with a single component. Therefore, this model’s extensions have been used by actuaries for multiple purposes. In addition, the LC model uses an autoregressive moving average process (ARIMA) with a special case of the random walk with drift (RWD) forecasting model.

Model description:

We can write the Lee–Carter model as follows:

$$\mathrm{m}_{\mathrm{x,t}} = \exp^{(\mathrm{a_x} + \mathrm{b_x k_t} + \varepsilon_{\mathrm{x,t}})} \ldots \quad (2\text{-}1)$$

Where the model’s basic premise is that there is a linear relationship among the age-specific death rates () and two explanatory factors: the initial age interval (x), and time (t). This means that the information is distributed in age intervals, so the interval that begins with the (x) age will be called "x age interval.

With$(\mathrm{x} = 0 - 1, 1 - 4, \ldots, \mathrm{A})$ and $(\mathrm{t} = 1, 2, \ldots, \mathrm{T})$

Where:

$m_{x,t}$: Is the age-specific death rate for the (*x*)interval and the year (*t*)
a_x: Is the average age-specific mortality
k_t : Is the mortality index in the year (t).
b_x : Is a deviation in mortality due to changes in the (k_t) index.
$\varepsilon_{x,t}$: Is the random error.
A:Is the beginning of the last age interval.

To solve (2-1) for estimate values of (*ax*, *bx,* and *kt,*), which are the solutions to the system, sowe can write the model in (2-1) as follows:

$$\mathrm{Log}\{ m_{x,t}\} = \{a_x + b_x k_t + \varepsilon_{x,t} \} \quad (2\text{-}2)$$

2-2: (Lee- Carter) model properties:
For properties (LC) model there is not a unique solution for this system. Consequently, it is necessary to add the following two constraints so as to obtain a unique solution:

1- $\sum_{x=0}^{A} b_x = 1$ ... (2-3)
2-$\sum_{t=1}^{T} K_t = 0$ ... (2-4)
3- $b_x \to c b_x \, , k_t \to \frac{1}{c} k_t$ (2-5)
$\forall \mathrm{c} \in \mathrm{R}, \mathrm{c} \neq 0$

$a_x \rightarrow a_x - b_x c\ , k_t \rightarrow k_t +$ (2-6) $\forall c \in R$

Where (c) is constant.

Model Fitting: To a fitting model as in (2-2), we have a constraint $(\sum_{t=1}^{T} K_t = 0)$, which immediately implies that the parameter (a_x) is simply the empirical average over time of the age profile in age group (*x*), then

$a^{\wedge}{}_x = \frac{\sum_{t=1}^{T} Ln\, m_{x,t}}{T}$, withX = 1,2.3, … … . . A and t = 1,2,3 … . . T

Where (A) is the last age of humans and (T) is the last year of data. Therefore $[ln\,(m_{x,t}) - \hat{a}_x] = b_x k_t + \varepsilon_{x,t}$ … (2-7)

Since practical uses of the Lee-Carter model implicitly assume that the disturbances ($\varepsilon_{x,t}$) are normally distributed, and then:{ $ln\,[m_{x,t}] - \hat{a}_x\} \approx N(b_x k_t, \delta^2)$, has a multiplicative fixed effects model for the centered age profile.Now let

$Z_{[x,t]x=1,\dots A,t=1\dots T} = \{ln[m_{x,t}] - \hat{a}_x\}$ (2-8)

$$Z_{x,t} = \begin{array}{c} \\ Ages \\ 0 \\ 1 \\ 2 \\ . \\ . \\ . \\ . \\ . \\ . \\ A \end{array} \left[\begin{array}{ccccccc} & & YEARs & & & & \\ 1 & 2 & & & & & T \\ \ln m_{0,1} - \hat{a_0} & \ln m_{0,2} - \hat{a_0} & . & . & . & . & \ln m_{0,T} - \hat{a_0} \\ \ln m_{1,1} - \hat{a_1} & \ln m_{1,2} - \hat{a_1} & . & . & . & . & \ln m_{1,T} - \hat{a_1} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ \ln m_{A,1} - \hat{a_A} & \ln m_{A,2} - \hat{a_A} & . & . & . & . & \ln m_{A,T} - \hat{a_A} \end{array} \right]_{A \times T}$$

$\therefore Z_{x,t} = b_x k_t + \varepsilon_{x,t}$ … (2-9)

Where { $Z_{x,t}$} is the centered logged rates matrix as follows:Seen in this matrix, the Lee-Carter model can also be thought of as a special case of a log-linear model for a contingency table. Indeed, this model is the most basic version of a contingency table model, where one assumes independence of rows (age groups) and columns (time periods), and the expected cell value is merely the product of the two parameter values from the respective marginal:

$E(Z_{x,t}) = b_x k_t$

In a contingency table model, this assumption would be appropriate if the variable represented as rows in the table was independent of the variable represented as columns. The same assumption for the log-mortality rate is the absence of (age× time interactions) that (b_x) is fixed over time for all (a_x) and (k_t) is fixed over age groups for all (t).This system provides a unique solution when these constraints are included. All parameters on the right-hand side of equation (2-9) are unobservable, and fitting the model by the ordinary least squares method is impossible. To overcome this situation, we employ Lee and Carter's (1992) two-stage estimation procedure, which gives exact solutions. In the first stage, singular value decomposition (SVD) is applied to the matrix of { $Z_{x,t}$ } to obtain estimates of { b_x and k_t}. This method is used to obtain the exact fitting of least squares (Good, 1969).Therefore, we get the following:

$\{Z_{x,t}\} = uxv'$ (2-10)

where (u, v) are orthogonal matrices and (x) is a nonnegative diagonal matrix.

with $(u'u) = I_T$, and $(v'v) = I_T$ and $u \epsilon R^{A*A}$, $v' \epsilon R^{T*T}$, $x \epsilon R^{A*T}$

Where (u): The Right Eigen Vectors Matrix.

(x): Singular Values matrix.
(v): The Left Eigen Vectors Matrix

Therefore, we have:

$$Z_{x,t} = \sum_{i=1}^{T} \rho_i u_{x,i} v_{t,i} \quad (2\text{-}11)$$

$$Z_{x,t} = \rho_1 * u_{x,1} * v_{t1} + \rho_2 * u_{x2} * v_{t2} + \cdots + \rho_T * u_{x,T} * v_{x,T} \ \ldots \quad (2\text{-}12)$$

Therefore, we can write the above relationship as follows:

$$Z_{x,t} = \rho_1 * u_{x1} * v_{t1} + \sum_{i=2}^{T} \rho_i + u_{x,i} + v_{t,i} \quad (2\text{-}13)$$

That means the first term of (2-13) represents the term, which depends on an estimate of all vectors of (b_x, k_t), but the second term represents the estimate of the standard error as follows:

$$\varepsilon_{x,t} = \sum_{i=2}^{T} \rho_i u_{x,i} v_{t,i} \quad (2\text{-}14)$$

Then, from (2-13) we can estimate all vectors (b_x, k_t) as follows:

$$b_x^{SVD\,\wedge} = u_{x,1} \quad (2\text{-}15)$$

$$b_x^{\wedge} = \frac{b_x^{SVD^\wedge}}{\sum_{x=1}^{A} u_{x,i}} \quad (2\text{-}16)$$

$$k_t^{SVD\,\wedge} = \rho_1 v_{t,1} \quad (2\text{-}17)$$

$$k_t^{\wedge} = k_t^{SVD\,\wedge} \sum_{x=1}^{A} u_{x,i} \quad (2\text{-}18)$$

$$\therefore\ k_t^{\wedge} = \rho_1 v_{t,1} \sum_{x=1}^{A} u_{x,i} \quad (2\text{-}19)$$

Where ($u_{x,1}$): The first column of the matrix (u).

($v_{t,1}$): The first column of the matrix (v).

ρ_1: The first element of the diagonal matrix (x).

Second stage estimation: We can now use the re-estimate step to re-estimate the parameter ($k_t^{\wedge}$) by an iterative search using the Newton Raph son method to get ($\hat{\hat{k}}_t$). This re estimation step, often called "second stage estimation", is to get a unique solution for the criterion and amore suitable estimation for mortality.

The Time series modeland forecasting: The second distinguishing feature of the Lee-Carter approach is that, having reduced the time dimension of mortality to a single index($\hat{\hat{k}}_t$), they use statistical time series methods to model and forecast this index. We assume that time series (Z_t) follows an auto-regressive integrated moving average, ARIMA (p, d, q) as:

$$Z_t = \emptyset_1 Z_{t-1} + \emptyset_2 Z_{t-2} + \cdots + \emptyset_p Z_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

Where $(Z_t)$is the general time series,

$\emptyset_1, \emptyset_2, \ldots, \emptyset_p$: are unknown parametrics of the autoregressive model;
$\theta_1, \theta_2, \ldots, \theta_q$: are unknown parametrics of the moving average model; and

{e_t} is a sequence of white noise random variables, iid with a mean (0) and constant Variance(σ^2).To produce mortality forecasts, Lee and Carter, testing several ARIMA specifications, found that a random walk with drift (ARIMA(0,1,0))was the most appropriate model for mortality data; therefore, we can write ARIMA(0,1,0)

$$\hat{\hat{k}}_{t+s} = \hat{\hat{k}}_t + \theta + e_t \quad (2\text{-}20)$$

Where: $\left(\hat{\hat{k}}_{t+s}\right)$ (mortality index)as general time series in (t+s).

: (θ) is unknown, and denotes the drift parameter.

To estimate the drift parameter (θ), using (MLE) method as follows:

$$\hat{\theta} = \frac{\hat{\hat{k}}_T - \hat{\hat{k}}_1}{T-1} \qquad (2\text{-}21)$$

Which only depends on the first and last of the ($\hat{\hat{k}}_t$) estimates. Moreover, we can write the variance error as follows:

$$\delta^2_{rw} = \frac{1}{T-1}\sum_{t=1}^{T}(\hat{\hat{k}}_t - \hat{\hat{k}}_{t-1} - \hat{\theta})^2 \qquad (2\text{-}22)$$

The Forecasting of (ASDR): After estimates of all the parameters of the Lee–Carter model

$$m^{\wedge}_{x,T+s} = \exp\left(a^{\wedge}_x + b^{\wedge}_x \hat{\hat{k}}_{t+s}\right) \qquad (2\text{-}23)$$

$$m^{\wedge}_{x,T+s} = m^{\wedge}_{x,T} \exp\left(b^{\wedge}_x\left(\hat{\hat{k}}_{t+s} - \hat{\hat{k}}_T\right)\right) \qquad (2\text{-}24)$$

$s >$, where (s) is the forecasting period with $s = 1,2,3,\ldots$

Application Part: The following analysis describes the application of the Lee-Carter method(1992) to model, estimate, and forecast Age-specific Death Rates (ASDR)in Kirkuk. A general mortality index was created for each age and gender. The indexes were forecast using the ARIMA model and by using (Mat lab) software (Version 7). After forecasts were obtained, it was possible to project age-specific death rates and provide life expectancy at birth. Furthermore, projections of life expectancy at birth from official bodies were included so as to compare the results.

Describe data and estimate model: Available data were composed of the population and death values both by age and gender in Kirkuk from 2006 to2015, and the data were provided by the Ministry of Health and the Ministry of planning. Thus, the age groups and gender mortality were estimated using the Lee-Carter method, and forecasting of the age-specific death rates was as follows:

Table 3-1. The estimate parameter model ($a^{\wedge}_x$) for both female, male and total (2006-2015)

Age	(a_x) −Female	(a_x) −Male	(a_x) −Combined
0.0	-4.69295810483	-4.78412821815	-4.49789453767
1.0	-6.78717989096	-6.47340256069	-6.81161166898
5.0	-8.14426194384	-7.56322010988	-7.79795991943
10.0	-8.40333158347	-7.53265604305	-7.86438139134
15.0	-7.87186036072	-6.81732489779	-7.19185845834
20.0	-7.65088904638	-6.16648131931	-6.61941882311
25.0	-7.44910360856	-5.91098588289	-6.37564576503
30.0	-7.36008388883	-5.88551954162	-6.35836146181
35.0	-7.08786071024	-5.83507739302	-6.27540667269
40.0	-6.76913845064	-5.71189137645	-6.09594791749
45.0	-6.43290254586	-5.53266641547	-5.88046273399
50.0	-5.62797531094	-5.11879795178	-5.33807513628
55.0	-5.12440023088	-4.51924377243	-4.78043380428
60.0	-4.58906734624	-4.10533654234	-4.32306577501
65.0	-4.10968427361	-3.57009218844	-3.80875059889
70.0	-3.37265189126	-3.12330711039	-3.2382097417
75.0	-2.87429880582	-2.65737640287	-2.76656074833
80.0	-3.3336884197	-3.46843935824	-3.40634205823

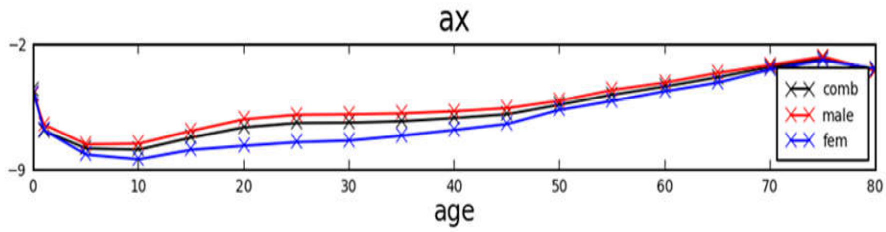

Figure 3-1. Show estimate parameter $(a^{\wedge}_x)$ for female, male and total (2006-2015)

Now, we can find the estimate parameters $(\hat{b}_x, \hat{k}_t)$ as in (2-16) and (2-19) for both male, female and total (2006-2015) as in table (3-2).

To estimate the parameter ($\hat{k}_t$) by (SVD) method for both female, male and total (2006-2015)as in table (3-3).

Now, to re-estimate the parameter ($\hat{k}_t$) by iterative search using the Newton Raphson method to get($\hat{\hat{k}}_t$) as in table (3-4).

Table 3-2. Estimate parameter model $(\hat{b}_x)$ for both female, male and total (2006-2015)

Age	$(\hat{b}_x)$ −Female	$(\hat{b}_x)$ −Male	$(\hat{b}_x)$ −Combined
0.0	0.20635772143	0.111901839596	0.0386444136609
1.0	0.0643690337132	0.044094578335	0.070580538315
5.0	0.0368484433244	0.0293951788937	0.0274653426847
10.0	0.0593828659551	0.039242593742	0.0453922331642
15.0	0.0273442977628	0.0505883895923	0.0565145523605
20.0	-0.0115808197194	0.0648046396509	0.0616459011085
25.0	0.0513503600334	0.0644747394338	0.0708757506129
30.0	0.0484050603783	0.0541154813495	0.0611511617663
35.0	0.0419515624738	0.066399532804	0.0756857678191
40.0	0.0523259913291	0.05863033365	0.0673495787332
45.0	0.0309157897603	0.0430007230073	0.0403804130833
50.0	0.0674796704917	0.0801894303513	0.0831579195289
55.0	0.0781438989681	0.0533291356099	0.0652561564782
60.0	0.0582517262177	0.0507959268876	0.0535057491319
65.0	0.0732531002984	0.0586683423704	0.0702351790099
70.0	0.0294305647245	0.0532208677058	0.0563460870476
75.0	0.0475738716385	0.0476937074206	0.0473090665
80.0	-0.00751762250374	0.0294545596001	0.00850418899482

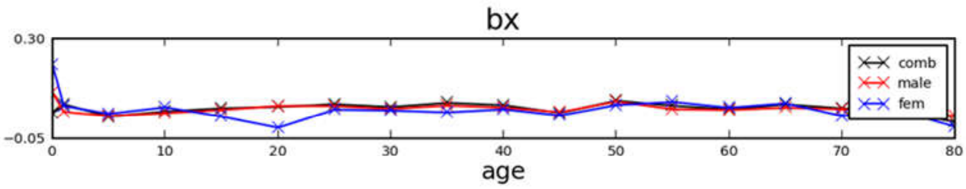

Figure 3-2.Estimates parameter $(\hat{b}_x)$ for female, male and total (2006-2015)

Table 3-3.The parameter model ($\hat{k}_t$) from (SVD) for both female, male and total (2006-2015)

Year	($\hat{k}_t$) For female	$(\hat{k}_t)$For male	$(\hat{k}_t)$ For total
2006	2.73141162755	8.48308687132	6.93754912411
2007	3.57760720792	9.5452010685	8.26571765494
2008	3.51416453906	6.17769151894	5.43529255946
2009	0.771191471143	-0.585948080638	-0.811737822471
2010	-0.512447723847	-4.38143602432	-3.77953059094
2011	-0.182066914348	-3.02272079387	-2.22502594882
2012	-0.155436387383	-4.6481250256	-3.39861284322
2013	-9.11611711314	-8.02437510295	-4.30655949623
2014	0.823101202278	-0.139910773273	-0.853348414314
2015	-1.45140790923	-3.40346365811	-5.2637442225

Table 3-4. Re-estimates the parameter ($\hat{\hat{k}}_t$) for female, male and total (2006- 2015)

Year	($\hat{\hat{k}}_t$) for Female	($\hat{\hat{k}}_t$) for male	($\hat{\hat{k}}_t$) for total
2006	3.76563271749	8.13027990038	6.24325485777
2007	4.65158525909	9.76428368969	7.98793956096
2008	6.08900207344	7.58023040235	7.17162301903
2009	0.20034052635	-1.0711107867	-1.17673255379
2010	-0.37059858611	-4.2401759385	-3.45113293463
2011	-0.48091197009	-3.0866339134	-2.62252725021
2012	-1.78761679328	-4.5730915541	-2.93459547062
2013	-6.2585628774	-5.7500560097	-4.94122976693
2014	0.02320713688	-0.9912842657	0.044127987258
2015	-3.94452798814	-1.9660901156	-4.15677197313

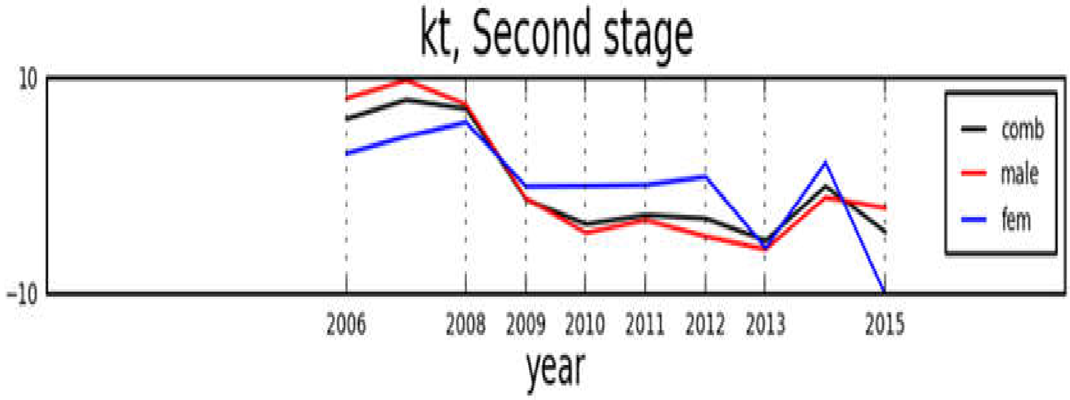

Figure 3-3.Decreasing re-estimate parameter ($\ddot{k}_t$)for female, male and total (2006- 2015)

Forecasting mortality: To produce mortality forecasts, Lee and Carter(1992) assumed that the most appropriate model wasa random walk with drift (RWD) or (ARIMA)(0, 1,0) model as in (2-20).Thus, according to our mortality data,we found forecasting values($\hat{\theta}$) and ($\hat{k}_{t+s}$) as in (3-5) and (3-6) tables.

Table 3-5. Forecasting values ($\hat{\theta}$) with standard error (See) for female, male and total

Item	$\hat{\theta}$	See	Sec
Female	-0.85668	3.466	1.155
Male	-1.12181	3.488	1.163
Total	-1.15555	3.557	1.185

Table 3-6. Forecasting values ($\hat{k}_{t+s}$) for female, male and total

years	$\hat{k}_{t+s}$ -female	S.E.	$\hat{k}_{t+s}$-male	S.E.	$\hat{k}_{t+s}$-total	S.E.
2016	-0.8566	3.6542	-1.121819	3.677158	-1.155559	3.749441
2017	-1.7133	5.4200	-2.243638	5.454106	-2.311117	5.561320
2018	-2.5700	6.9333	-3.365457	6.976916	-3.466676	7.114064
2019	-3.4267	8.3328	-4.487276	8.385209	-4.622234	8.550041
2020	-4.2834	9.6681	-5.609094	9.728845	-5.777793	9.920089

Now, substituting ($\hat{\theta}$) and $(\hat{b}_x, \hat{k}_t)$in (Lee-Carter) as in (2-2) model to forecast the values of log-mortality as in the following figures:

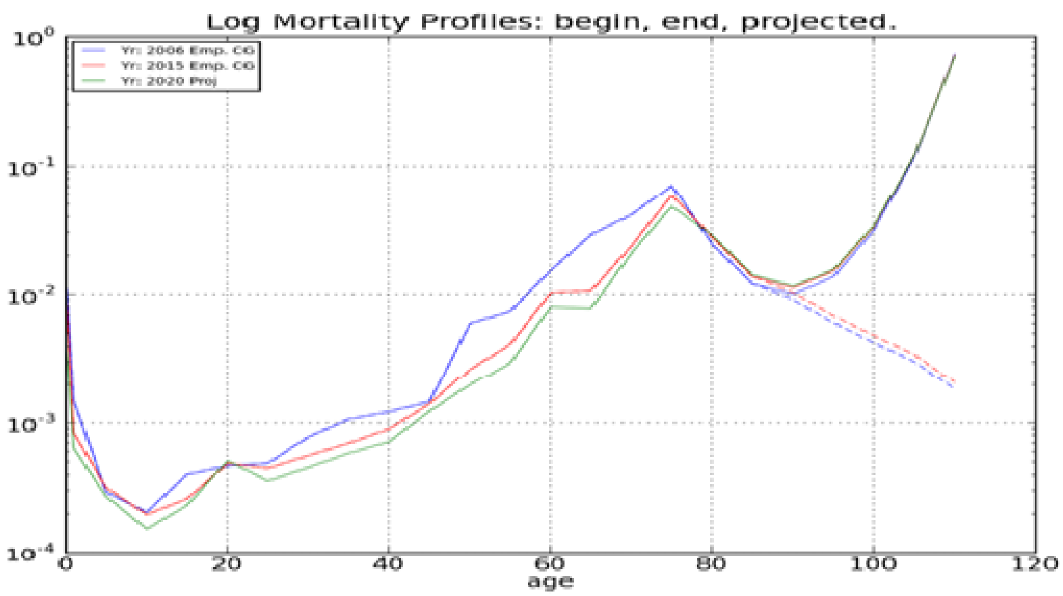

Figure 3-4. Forecast the values of log-mortality for Female with actual data (2006- 2015) and forecasting values in (2020)

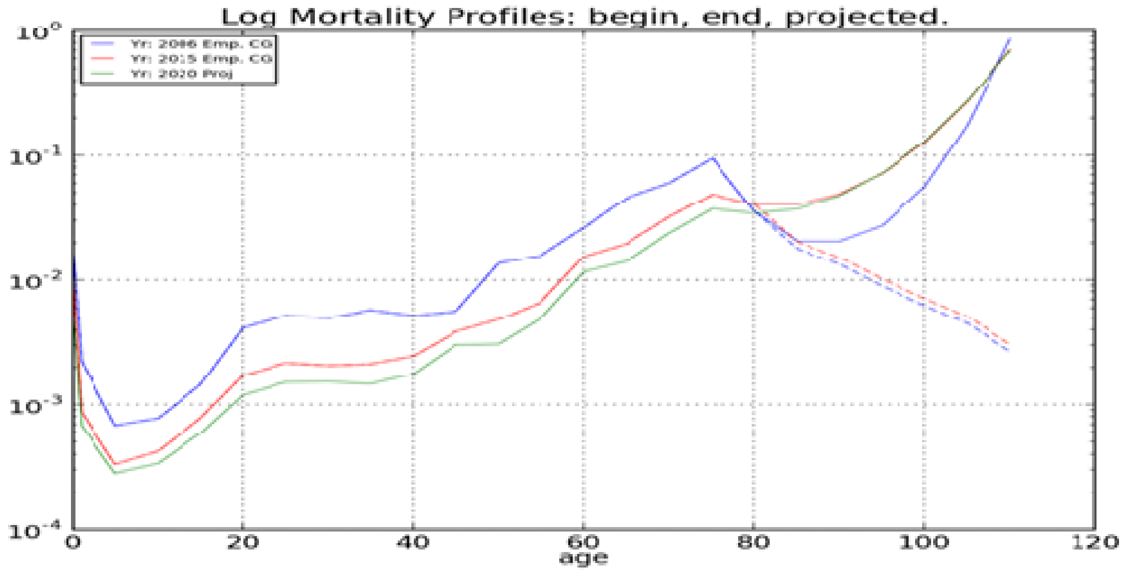

Figure 3-5. Forecast the values of log-mortality for Male with actual data (2006- 2015) and forecasting values in (2020).

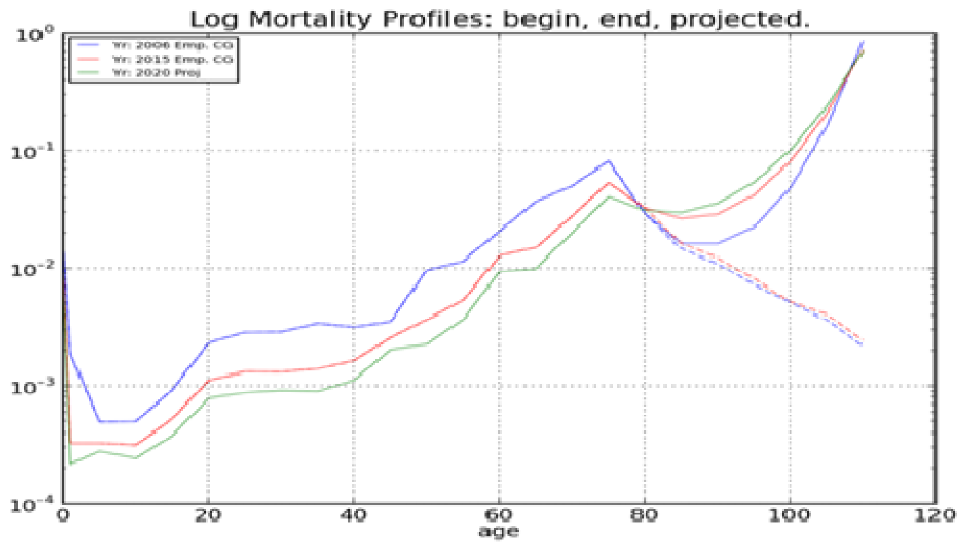

Figure 3-6.Forecast the values of log-mortality for Total with actual data (2006- 2015) and forecasting values in (2020).

DISCUSSION AND CONCLUSION

In this paper the application of the Lee-Carter model to specific ages has been described in relation to mortality rates by gender in Kirkuk, Iraq. The conclusions of this research are as follows:

- The estimates of the average age-specific mortality parameter ($\hat{a}_x$) are available for the period 2006-2015, which is increasing for the age group (0-1) year and decreasing for the age group (75-80) years.
- We have shown from an analysis of average age-specific mortality and an estimate of (b_x) and (k_x) parameters, that using the (SVD) method plays an important rule for identifying trends in mortality for the period of estimation and forecasting.
- We computed (18) age groups for each gender index; levels for mortality and coefficients were obtained through the Lee-Carter method.
- The general index of mortality and forecasting for the period 2016-2020 used the important method (Box-Jenkins) such that (ARIMA) (0,1,0) model.
- Forecast models such as ARIMA(0, 1, 0), with a constant, was used to project the ($\hat{k}_t$) index and presented an adequate fit model.
- During the second stage, age-specific death rates were predicted using forecasts of the ($\hat{k}_{t+s}$) index obtained for three groups (female, male and total).

- According to the forecast method used, such rates showed increasing and decreasing mortality depending on different times and ages.
- The long-term forecast was necessary for the field of Demography and obtaining them depends on the available data.

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