ISSN: 0975-833X

Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 11, Issue, 03, pp.1901-1905, March, 2019

DOI: https://doi.org/10.24941/ijcr.33847.03.2019

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

RESEARCH ARTICLE

TRUTH TABLE METHOD – ITS RELEVANCE IN DIFFERENT FIELD OF STUDIES: A BRIEF SURVEY

*Trisha Kundu"

Ex-Student, Department of Mathematics, the University of Burdwan and Assistant Teacher in Mathematics at Kazipara Haridas Vidyabhaban, Murshidabad

ARTICLE INFO

Article History:
Received 16th December, 2018
Received in revised form
23rd January, 2019
Accepted 27th February, 2019
Published online 31st March, 2019

Key Words:
Truth table, Proposition calculus, bools, binary operator, substitution instance, genetic ratio, recessive, genotype, phenotype.

ABSTRACT

A truth table is a logical method by which we can test the validity of a deductive argument and so mechanically whether a statement is a tautology, contradictory or contingent. In this paper my objective is not to explain truth table proper but to highlight its efficacy in different field of studies like Mathematics, Physics etc besides its own field (Logic).

***Corresponding author:** Trisha Kundu,

Citation: ***Trisha Kundu*, 2019.** "Truth table method – its relevance in different field of studies: A brief survey", *International Journal of Current Research*, 11, (03), 1901-1905.

INTRODUCTION

A truth table is a mathematical table used in logic specifically in connection with Boolean algebra, Boolean functions, and proposition calculus which sets out the functional values of logical expression on each of their functional arguments, that is, for each combination of values taken by their logical variables. In particular, truth tables can be used to show whether a propositional expression is true for all legitimate input values that is logically valid. In another way, a truth table is a handy little logical device that shows up not only in mathematics, but also in Computer Science, Physics and Philosophy, making it an awesome interdisciplinary tool. The notation may vary depending on what discipline you're working in, but the basic concepts are the same.

Truth Table in different fields of studies:

(I)

Boolean algebra, a branch of algebra that involves bools, or true and false values. They're typically denoted as T or 1 for true and F or 0 for false. Using this simple system we can boil down complex statements into digestible logical formulas.

Unitary Operator: Unitary operators are the simplest operations because they can be applied to a single True or False value.

Identity: The identity is our trivial case. It states that True is True and False is False.

Negation: The negation operator is commonly represented by a tilde (~) or ¬ symbol. It negates, or switches, something's truth value. We can show this relationship in a truth table.

A **truth table** is a way of organizing information to list out all possible scenarios. We title the first column p for proposition. In the second column we apply the operator to p, in this case it's ~p (read: not p). So as you can see if our premise begins as True and we negate it, we obtain False, and vice versa.

Truth Table:

p	~p
T	F
F	T

or

p	~p
1	0
0	1

Binary Operators: Binary operators require two propositions. We'll use p and q as our sample propositions.

AND: The AND operator (symbolically: ∧) also known as logical conjunction requires both p and q to be True for the result to be True. All other cases result in False.

Truth Table

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

OR: The **OR operator** (symbolically: ∨) requires only one premise to be True for the result to be True.

Truth Table

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

NOR: Logical NOR (symbolically: ↓) is the exact opposite of OR. It requires both p and q to be False to result in True.

Truth Table

p	q	p ↓ q
T	T	F
T	F	F
F	T	F
F	F	T

XOR: Exclusive Or, or **XOR** for short, (symbolically: ⊻) requires exactly one True and one False value in order to result in True.

Truth Table

p	q	p ⊻ q
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Operators

Implication: Logical implication (symbolically: p → q), also known as "if-then", results True in all cases except the case T → F. Since this can be a little tricky to remember, it can be helpful to note that this is logically equivalent to ¬p ∨ q (read: not p or q). Let's create a second truth table to demonstrate they're equivalent. To do this, write the p and q columns as usual. Then add a "¬p" column with the opposite truth values of p. Lastly, compute ¬p ∨ q by OR-ing the second and third columns. Remember to result in True for the OR operator, all you need is one True value.

Truth Table

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

p	q	~p	~p ∨ q
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Logical Equality: Also known as the **biconditional or if and only if** (symbolically: ←→), logical equality is the conjunction (p → q) ∧ (q → p). In other words, it's an if-then statement where the converse is also true. The only way we can assert a conditional holds in both directions is if both p and q have the same truth value, meaning they're both True or both False. This is why the biconditional is also known as logical equality.

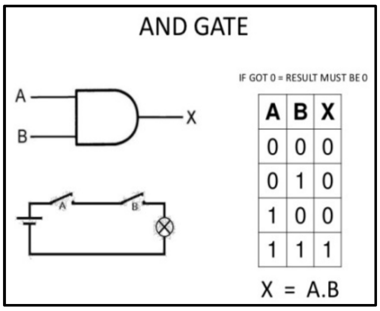
p	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

(II)

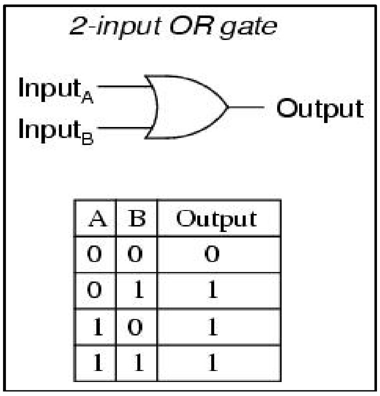
In Digital Electronics, a branch of Physics and Computer Science, a truth table shows how a logic circuit's output responds to various combinations of the inputs, using logic 1 for True and logic 0 for False. All permutations of the inputs are listed on the left, and the output of the circuit is listed on the right. The desired output can be achieved by a combination of logic gates. A truth of two inputs is shown, but it can be extended to any numbers of inputs. The input columns are usually constructed in the order of binary counting with a number of bits equal to the numbers of inputs.

Logic Gates and Truth Table

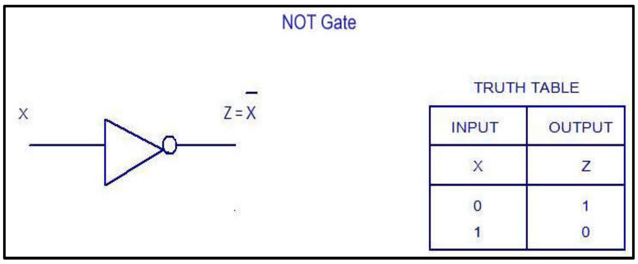
AND GATE



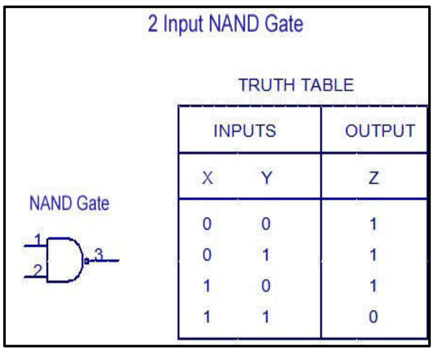
OR GATE



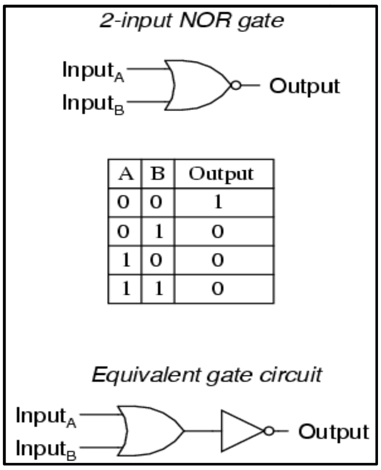
Output = A+B



NAND GATE

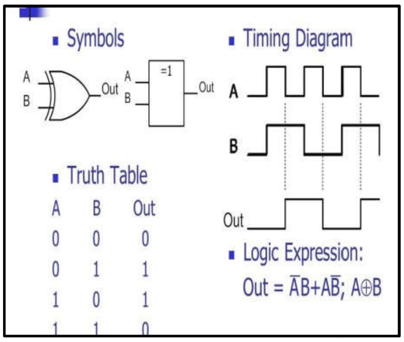


NOR GATE

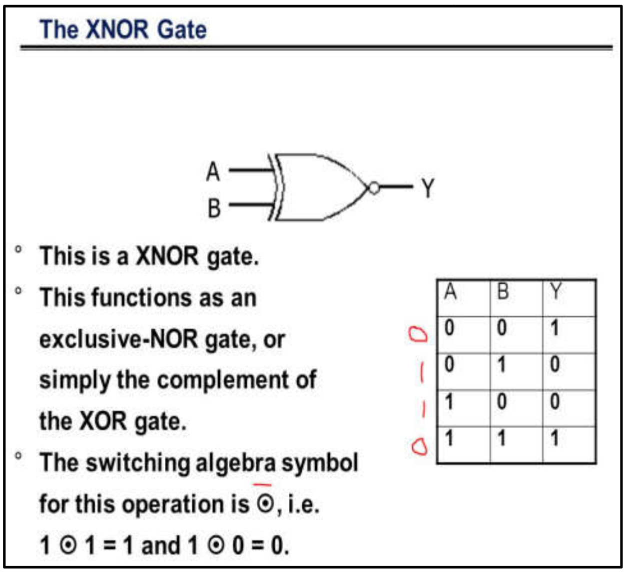


Output = ~ (A +B)

XOR GATE:



XNOR GATE



(III)

Deductive Logic, a branch of Philosophy, uses this method of truth table (in its symbolic form) to test the validity and invalidity of its arguments. An argument form of a deductive logic is valid if and only if it has no substitution instances with true premises and a false conclusion. On the other hand, an argument form is invalid if and only if it has at least one substitution instance with true premises and the false conclusion. So, a valid argument form can have only valid substitution instances, but an invalid argument form can have both valid and invalid substitution instances. Let's have some examples:

p ∨ q
~p

p	q	p ∨ q	~p	q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

Therefore, q
This is a valid argument form as the truth table shows (in the 3rd row).

p →q
q

p	q	p→q	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

therefore, p

This is an invalid argument form as the truth table shows (in the 3rd row).
Again by using truth table we can detect whether a statement form is a Tautology, Contradiction, or Contingent. A statement from that has only true substitution instances is called a Tautology.

Such as,
'p ∨ ~p':

It is a tautologus statement form as the truth table shows (in its result column).

p	~p	p ∨ ~p
T	F	T
F	T	T

Similarly, a statement form that has only false substitution instances is called a contradictory statement form. Such as,
'p . ~p' :

p	~p	p .~p
T	F	F
F	T	F

It is a contradictory statement form as the truth table shows (in its result column).

Moreover, a statement form that has both the true and false substitutions instances is called contingent statement form. Such as,

'p→q' :

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

This is a contingent statement form as the truth table shows (in its result column).

Finally, by the help of truth table, we can show whether the two statement forms have the relation of logical equivalence, so that they can have the same meaning. For example,

p , ~~p (p = I shall eat, ~~p = It is not that I shall not eat.)

p	~p	~~p	p ↔ ~~p
T	F	T	T
F	T	F	T

(IV)

We can use Truth Table method to verify the laws of **Set Theory** in **Mathematics.**

1) **Associative Law:** if p, q and r be three sets then,

$$(p \vee q) \vee r = p \vee (q \vee r)$$
$$(p \wedge q) \wedge r = p \wedge (q \wedge r)$$

Verify by truth table:

p	q	r	p ∨ q	(p∨q)∨r	q ∨ r	p∨(q∨r)
T	T	T	T	**T**	T	**T**
T	T	F	T	**T**	T	**T**
T	F	T	T	**T**	T	**T**
T	F	F	T	**T**	F	**T**
F	T	T	T	**T**	T	**T**
F	T	F	T	**T**	T	**T**
F	F	T	F	**T**	T	**T**
F	F	F	F	**F**	F	**F**

Similarly, we can show, $(p \wedge q) \wedge r = p \wedge (q \wedge r)$ by truth table method.

2) **Distributive law:** if p,q and r be three sets then,

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$$
$$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$$

Verify by Truth Table:

p	q	r	p∨q	p∨r	q∧r	p∨(q∧r)	(p∨q) ∧ (p∨r)
T	T	T	T	T	T	**T**	**T**
T	T	F	T	T	F	**T**	**T**
T	F	T	T	T	F	**T**	**T**
T	F	F	T	T	F	**T**	**T**
F	T	T	T	T	T	**T**	**T**
F	T	F	T	F	F	**F**	**F**
F	F	T	F	T	F	**F**	**F**
F	F	F	F	F	F	**F**	**F**

Similarly, we can prove that, $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$ by truth table method.

3) **Commutative law:** If p and q be two sets then,

$$p \vee q = q \vee p$$
$$p \wedge q = q \wedge p$$

Verify from truth table

p	q	p∨q	q∨p
T	T	**T**	**T**
T	F	**T**	**T**
F	T	**T**	**T**
F	F	**F**	**F**

Similarly, we can verify that, $p \wedge q = q \wedge p$ by truth table method.

4) **De Morgan's law:** If p and q be three sets then,

$$\sim(p \vee q) = \sim p \wedge \sim q$$
$$\sim(p \wedge q) = \sim p \vee \sim q$$

Verify by truth table

p	q	p∨q	~p	~q	~(p∨q)	~p ∧ ~q
T	T	T	F	F	**F**	**F**
T	F	T	F	T	**F**	**F**
F	T	T	T	F	**F**	**F**
F	F	F	T	T	**T**	**T**

Similarly, we can prove that $\sim(p \wedge q) = \sim p \vee \sim q$ by truth table method.

5) **Absorption law:** If p and q be two sets then,

$$p \vee (p \wedge q) = p$$
$$p \wedge (p \vee q) = p$$

Verify by truth table:

p	q	p∧q	p∨ (p∧q)
T	T	T	**T**
T	F	F	**T**
F	T	F	**F**
F	F	F	**F**

Similarly, we can prove that $p \wedge (p \vee q) = p$ by truth table method.

Thus we can prove more laws of Set Theory like, Identity law, Complement law, Idempotent law etc. by truth table method.

(V)

In case of **Probability** calculation of events this truth table method can also be used. Let us take some example to show this.

Problem 1: What is the probability of getting two heads in tossing two coins?

Solution:

1st coin (p)		2nd coin(q)
H	————————	H
H		T
T		H
T		T

H = head, T= tail. From this table we calculate the probability of the events = ¼ (favourable case/ total case).

Problem 2: In two tosses of a coin what is the probability of getting either two heads or two tails?

Solution:

1st toss (p)		2nd toss (q)
H	————————	H
H		T
T		H
T	————————	T

H = head, T= tail. From this table we calculate the probability of the events = **2/4**= **1/2** (favourable case/ total case).

Problem 3: What is the probability of getting at least on head on two tosses of a coin?

Solution:

1^{st} toss (p)	2^{nd} toss (q)
H	H
H	T
T	H
T	T

H = head, T= tail. From this table we calculate the probability of the events = ¾ (favourable case/ total case).

In this way the calculation of probability is possible through truth table method, though, of course, in more complex cases the use of this method is very difficult.

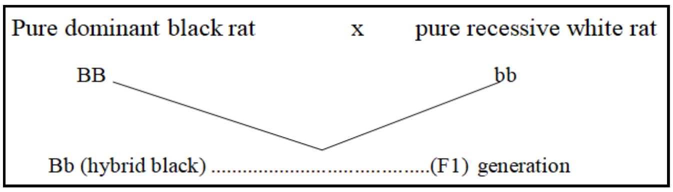
(VI)

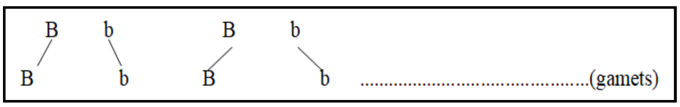
Now before drawing conclusion I can't control my temptation to extend the truth table enquiry to a new field of study like genetics. Here I venture to compare the genetic ratio with truth table ratio in the following way:

In monohybrid cross between a pure dominant black rat and a pure recessive white rat, we get all hybrid black rats in the 1^{st} generation (F1). Now, in the 2^{nd} generation (F2), which arises after crossing two rats from 1^{st} generation (F1), we get, 75% dominant black rats and 25% pure recessive white rats. i.e. the phenotype ratio is = 3:1

Of the 75% dominant black rats, 25% is pure dominant black rats and 50% hybrid black rats.

Therefore, genotype ratio is = 1:2:1. It may be shown in the following figures:





	B	b
B	BB (pure black)	Bb (hybrid black)
b	bB (hybrid black)	Bb (pure white)

.... (F2) generation

Phenotype ratio is = 3:1= dominant black: pure recessive white

Genotype ratio is = (1:2):1= pure black: hybrid black: pure white.

Verify through truth table

Table 1.

p	q
T	T
T	F
F	T
F	F

Table 2.

p	q
T	T
T	F
F	T
F	F

Where T = pure dominant black (B)

F = pure recessive white (b)

From 1^{st} table we get, **(pure black : hybrid black): pure white**

= (1 (TT): 2 (TF/FT)): 1(FF)

From 2^{nd} table this is = **3 (TT/TF/FT): 1(FF)**

=Dominant black: pure recessive white

So, we may conclude that if we advance in this way for further case of studies it might be found that the truth table method help us to calculate or explain or to solve several problems concerning those studies. But it is a matter of further research which is not possible to be done in this limited sphere.

REFERENCES:

Amritava Gupta, 2009. "Groundwork of Mathematically Probability & Statistics", Academic Publishers, Kol-700073.

Ghosh & Chakravorty, 2007. "Advanced Higher Algebra", (9th Edition), U.N. Dhur & Sons Pvt. Ltd., Kol-700073

Irving M. Copi & Carl Cohen, 2001. "Introduction to Logic (9^{th} Edition)", Prentice –Hall of India Pvt Ltd., New Delhi-110001.

Jain, R.P. 2003. "Modern Digital Electronics", Tata McGraw-Hill Education.

Mapa, S.K. 2007. "Higher Algebra (Abstract & Linear)", Sarat Book Distributions, 18- B, Cal-700073.

Nandy, Midya, Santra, 2016. "Life Science & Environment", Santra Publication.

Richard C. Jeffrey, 1980. "Formal Logic: Its Scope and Limits", Tata McGraw- Hill Publishing Company Ltd., New Delhi, TMH edition.

Thomas L. Floyd, "Digital Fundamentals", Eleventh Edition, Pearson Education Publication, India.

****