Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 11, Issue, 04, pp.3006-3009, April, 2019

DOI: https://doi.org/10.24941/ijcr.34485.04.2019

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

MODELING MASS BALANCE OF LIQUID IN STORAGE TANK USING LAPLACE TRANSFORM AND EULER METHOD

***Solomon Kebede, Hailu Geremew and Ayansa Tolesa**

Department of Physics, Wollo University, P.O.box 1145, Dessie, Ethiopia

ARTICLE INFO

Article History:
Received 19th January, 2019
Received in revised form
24th February, 2019
Accepted 27th March, 2019
Published online 30th April, 2019

Key Words:
Modeling; Liquid flow; Laplace transform; Euler Method; storage tank; Mass balance; matlab.

***Corresponding author: Olkeba Assefa**

ABSTRACT

Modeling and simulation are important tools often used for investigating the system's behavior in the industry and also in other fields of living. In this work, the steady state heights of the liquid in the storage tank was determined through plotting the heights of liquid as a function of time and the governing equations of flow of liquid in the tank was derived. The analytic solution and numerical solution obtained from the governing equations of liquid were compared. The steady state height of the liquid in the storage tank was 4.9585m analytically and numerically by using Euler method, the steady state height was 4.969m. At this height the acceleration of liquid is zero and flow rate is uniform. The conservation of volume and mass balance happened at this height.

***Citation: Solomon Kebede, Hailu Geremew and Ayansa Tolesa,* 2019.** "Modeling mass balance of liquid in storage tank using laplace transform and euler method", *International Journal of Current Research*, 11, (04), 3006-3009.

INTRODUCTION

The modeling and simulation are important tools often used nowadays for investigating the system's behavior in the industry and also in other fields of living (Dass and Rajnish Verma, 2010). It is a mathematical representation of a physical, biological or information system. Model allows us to reason about a system and makes predictions about how the system will behave. There are three distinct streams of numerical solution techniques used in fluid flow modeling. These techniques are finite difference, finite element and spectral methods. Those outlined numerical algorithm consists of the following steps. In the first steps the integration of the governing equations of fluid flow over all the finite volumes of the domain is carried out. Next to this, the discretisation process is carried out and this can be done through conversion of the resulting integral equations into a system of algebraic equations and finally the solution of the algebraic equations by an iterative method (Munson *et al.,* 2006).This clear relationship between the numerical algorithm and the underlying physical conservation principle forms one of the main attractions of the finite volume method and makes its concepts much simpler to understand by engineers than the finite element and spectral methods (Versteeg, 2007). In solving fluid flow problems we need to be aware that the underlying physics is complex and the results generated by a matlab codes are at best as good as the physics embedded in it (Victor Udoewa1 and Vinod Kumar2, 2009).

Elaborating on the latter issue first, the user of a code must have skills in a number of areas. Prior to setting up and running a matlab simulation there is a stage of identification and formulation of the flow problem in terms of the physical phenomena that need to be considered. The fluid will be regarded as a continuum. For the analysis of fluid flows at macroscopic length scales, the molecular structure of matter and molecular motions were ignored (World Academy of Science, 2008).

The aims of this works are:

- To establishes the governing equation and solving the established equations using Laplace transform and Euler method.
- To addresses the steady state height of the fluid in the storage tank through plotting the height of liquid as a function of time.
- To compare the analytical solution with the numerical solution obtained from the governing equation.

Reviews of Literatures

Laplace Transform: Laplace transform is an operator takes a function as input and outputs another function. A transform does the same thing with the added twist that the output function has a different independent variable. The Laplace transform takes a function f(t) and produces a function F(s). It can be helpful in solving ordinary and partial differential equations because it can replace an ODE with an algebraic equation or replace a PDE with an ODE. Another reason that the Laplace transform is useful is that it can help deal with the boundary conditions of a PDE on an infinite domain (Mathews, 2004). In practice the Laplace transform has the following benefits. It makes explicit the long-term behavior of f(t), it converts differential equations into algebraic equations and it converts Green's formula, which is a complicated convolution integral in the time view into as simple algebraic statement in the frequency view (Dass, 2010). Most importantly, the frequency view can be summarized in something called the pole diagram. This diagram can show at a glance the stability and frequency response of a system.

Definition of Laplace transform: Let be a given function defined for all$t \geq 0$, and then the Laplace transformation of F

(t) is defined as $L\{F(t)\} = \int_0^{\infty} e^{-st}F(t)dt = F(s)$ 2.1

where L is called Laplace Transform Operator[8].The function F(t)is known as determining function, depends on the new function which is to be determined F(s) is called generating function depends on s. The Laplace transform, according to this definition, is an operator: It is defined on functions, and it maps functions to another functions. Generally s is a complex variable, but in most of the time we will not bother about the domain of $F(s)$ or about the question on the existence of the Laplace transform, for all the functions we deal with their Laplace transforms are well defined.

In equation 2.1 above, the Laplace transform is defined as an improper integral. Strictly speaking, while evaluating this integral, we need to consider the limit.

$$F(S) = \lim_{c\to\infty} \int_0^{c} F(t)e^{-st}\, d \qquad 2.2$$

It is a good idea to remember about this limit, but in the calculations we will usually use shortcut notations.

Properties of the Laplace Transform: The Laplace transform has several special properties that make it a useful mathematical tool (Makila, 2006).We consider some of these now. Suppose we have two functions along with their respective Laplace transforms:

L {f(t)}=F(s)and L{g(t)}=G(s) 2.3

The property of linearity means that

L{af(t)+bf(t)}=aF(s)+bG(s) 2.4

Where a and b are constants. The second property of the Laplace transform is the first shift theorem. The first shift theorem is defined as

$$L\{e^{-st}F(t)\} = F\{s+t\} \qquad 2.5$$

Transformation of Derivatives: The Laplace transform of F(t) is denoted by F(s). Now consider the Laplace transform of F(t) by $\dot{F(s)}$:

$$L\{\dot{F(t)}\}=\int_0^{\infty} \dot{F(t)}e^{-st}\, dt \qquad 2.6$$

Integrating by parts, the above equation becomes

$$L\{\dot{F(t)}\}=[f(t)e^{-st}]_0^{\infty} -\int_0^{\infty} F(t)(-s)e^{-st}\, dt$$

$$L\{\dot{F(t)}\}=0-F(0)+ s\int_0^{\infty} F(t)e^{-st}\, dt$$

$$L\{\dot{F(t)}\}= -F(0)+s\, F(s)$$

Therefore, $L\{\dot{F(t)}\}$=sF(s)-F(0) 2.7

In a similar fashion, using repeated integration by parts, we can show that

$$L\{\ddot{F(t)}\} = s^2F(s) - sF(0) - F(0) \qquad 2.8$$

This is one of the most important properties of the Laplace transform. The Laplace transform gets rid of derivatives; just the thing for solving differential equations. When we come to solve differential equations using Laplace transforms. we shall use the following alternative notations.

$L\{y\} = \bar{y}$ $L\{\dot{y}\}=s\bar{y}$-y (0),

$$L\{\ddot{y}\}=s\bar{y} - sy(0) - \dot{y} \qquad 2.9$$

It is expressed the Laplace transform of a derivative in terms of the Laplace transform of the undifferentiated function. The Laplace transform has converted the operation of differentiation into the simpler operation of multiplication by *s.*

Laplace Transform Table: From the Laplace transform table only the following formula relations were taken into accounts, because those formulas relation were used in this work. These formula relations are:

$L\{K\}= \frac{K}{s}$,where k is constant 2.10

$L\{\dot{F(t)}\}$=sF(s)-F(0) 2.11

$$L\{1 - e^{-t/\tau}\}=\frac{1}{s(s\tau+1)} \qquad 2.12$$

$$L\{e^{-bt}\}= \frac{1}{s+b} \qquad 2.13$$

Using Laplace Transforms to solve odes: The Laplace transform of the derivative of a function can be expressed in terms of the Laplace transform of the undifferentiated function (10).This property can be used to derive solutions to certain types of differential equations. The process is broken down into the following steps. The first step is transforming both sides of the ode equation into the Laplace transform. The next step is substituting the initial condition and solving for the variable required in ode equation say in this case the variable$\bar{y}$. Finally the manipulated variable $\bar{y}$ can be inverted from table of Laplace transform and the invert gives the solution of ode. The following example illustrates the solution of ode using Laplace transform.

$$\ddot{y} + 6y + 8\dot{y} = 2$$

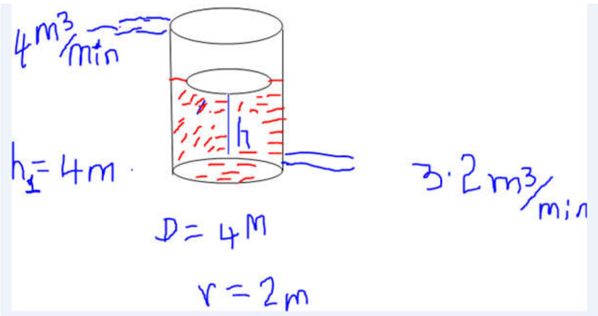
$$\{s^2y(s) - sy(0) - \dot{y}(0)\} + \{s\, y(s) - \dot{y}(0)\} + 8y(s) = \frac{2}{s}$$

Then substituting the initial condition y(0)=1, $\dot{y}(0) = -2$ and taking inverse of the resulting equation ,the following result is obtained.

$$y(t)=\frac{1}{4}(1 + 2e^{-2t} + e^{-4t})u(t) \quad 2.12$$

MATERIALS AND METHODS

The Study Design and experimental set up: The modeling and simulation are important tools often used for investigating the liquid systems behavior in storage tank. This study deals with modeling a storage tank of diameter 4m being filled at the rate of $4\frac{m^3}{min}$. When height of the liquid was 4m in the tank, the bottom of the tank spring was a leak and the rate of the leaking was proportional to the head of liquid so that it was leaking at a rate of $3.2\frac{m^3}{min}$. The modeling can be done by developing a suitable matlab code for the mass balance in liquid by employing mathematical modeling and system analysis. The materials required in this study include: Storage tank, Mat lab software and Personal computer (PC).



Mathematical modeling: The mathematical modeling allows us to reason out about a system and makes predictions about how the system will behave. To establish the governing equation, the mass balance or conservation of mass was used. That is mass in is equal to mass out and using this principle the governing equation of flow liquid was derived. The following equation describes the mass balance of the liquid storage tank.

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} \quad 3.1$$

Since the mass m is the product of volume V and density Þ, the mass is given by m=ÞV and substituting the value of in equation 3.1 the following equation is obtained.

$$\frac{\rho dV}{dt} = \rho\dot{V}_{in} - \rho\,\dot{V}_{out} \quad 3.2$$

Because of the density is the same in both sides of equation 3.2, this equation reduced to

$$\frac{dV}{dt} = \dot{V}_{in} - \dot{V}_{out} \quad 3.3$$

Substituting the values of V=$\pi r^2 h$, $\dot{V}_{in}$ and $\dot{V}_{out}$, the flow rate equation becomes

$$\pi r^2 \frac{dh}{dt} = 4 - 0.8h \quad 3.4$$

By using the method of separation of variable and integrating equation 3.4 above

$$\pi r^2 \int_4^h \frac{dh}{4-0.8h} = \int_0^t dt \quad 3.5$$

From the definition of integral in the form of $\int \frac{dy}{a+by} = \frac{1}{b}\ln(a + by)$, equation 3.5 may be written as:

$$t = \frac{4\pi}{0.8}\ln\left(\frac{0.8}{4-0.8h}\right) \quad 3.6$$

After some rearrangement, equation 3.6 can be written as:

$$h=5- \quad e^{-\frac{0.8t}{4\pi}} \quad 3.7$$

Equation 3.7 is the analytic solution of the steady state height of the liquid.

Now we proceed to transform equation 3.4 using table of Laplace transform as:

$$\pi r^2[sH(s)-h(0)]=\frac{4}{s} - 0.8H(s) \quad 3.8$$

Substituting the values of r=2m and h =4m in equation 3.8, we may have:

4π (s H(s)-4]=$\frac{4}{s} - 0.8H(s)$, this equation can be written as;

$$[4\pi s + 0.8]H(s) = \frac{4}{s} + 16 \quad 3.9$$

By rearranging equation 3.9, H(s) is obtained as follows

$$H(s)=\frac{4}{s(4\pi s+0.8)} + \frac{16\pi}{4\pi s+0.8} \quad 3.10$$

By taking the inverse Laplace transform, equation 3.10 takes the form

$$h=5(1- e^{-\frac{0.8t}{4\pi}})+4e^{-\frac{0.8t}{4\pi}}=5- e^{-\frac{0.8t}{4\pi}} \quad 3.11$$

Equation 3.7 and 3.11 gave the same result and then the Euler's method was applied to approximate the value of steady state height. That means it was estimated h(t) with step size $\Delta t = 0.5$,where h(t) is the solution to the initial value problem:

$$\frac{dh}{dt} = (4 - 0.8h)\frac{1}{\pi r^2} = \frac{h_{2-h_1}}{\Delta t} \quad 3.12$$

Where $\Delta t = t_{2-t_1}$ and therefore, equation 3.can be written as

$$h_2 = h_1 + (4 - 0.8 * h_1)\frac{\Delta t}{\pi r^2} \quad 3.13$$

Equation (3.13) is the height of the steady state liquid derived using Euler method.

RESULTS AND DISCUSSION

In this modeling, the differential equations were derived in terms of storage tank heights and solved using Laplace transform and Euler method.

Those derived equations were analyzed by the help of MATLAB software and as listed below.

Laplace Transform: Laplace transform solution graph is plotted for the differential equations derived in terms of liquid storage tank height from equation 3.4 above. The Laplace transform used to solve and give very good information about the liquid in storage tank and the following graph shows the inverse Laplace transform plot:

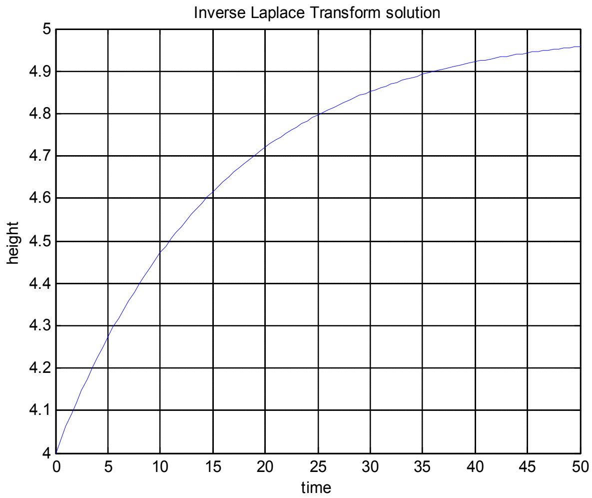


Figure 4.1 shows the inverse Laplace Transform solution curve from equation 3.11.

As it can be seen from the graph of figure 4.1, the liquid reached its steady state height at h=4.9585m. At this level the velocities of liquid into the tank and out of the tank were the same. This is because, as the height of liquid storage tank increases, the pressure of the liquid also increases. This pressure increases the flow rate of liquid out of the storage tank until it become equal to the flow rate into the tank. That means the liquid flow rate is the same; here the concept of mass balance happened at this height.

Euler Method and Analytic solution: The differential equation is changed to Euler method to approximate the solution. This can be done by plotting the slope of the graph tangent to the solution curve. The plot of this slope is shown below.

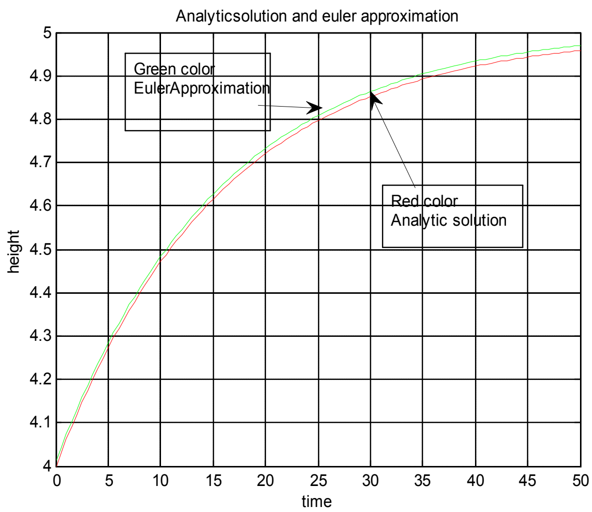


Figure 4.2. Shows the Euler method Approximation and Analytic solution

As it can be seen from the graph, the result obtained from Euler approximation is almost approaches to the exaction solution with the error of 0.011 and therefore the steady state height obtained from Euler method is 4.969m.

Conclusion

In solving fluid flow problems, we need to be aware that the underlying physics is complex and the results generated by a matlab code are best as the physics embedded in it for the predictions about how the system will behave. There are three distinct streams of numerical solution techniques used in this liquid flow modeling. These techniques are Laplace transform, finite element and Euler methods. Those outlined numerical algorithm consists of the following steps. In the first steps the integration of the governing equations of fluid flow over all the finite volumes of the domain is carried out. Next to this, the discretezation process is carried out and this can be done through conversion of the result obtained in integral equations into a time domain or inverse Laplace transform and finally the solution of the Euler method by an iterative method was used.
The Liquid height was determined both analytically and numerically by using Laplace transform equation and Euler approximation method. Those processes were done by plotting the solution curves graph and the results obtained in both techniques were compared. The Liquid's storage tank steady state height solved by analytic solution was 4.9585m and using Euler method the steady state level of liquid was 4.969 which is almost close to the exact solution.

REFERENCES

Dass H.K. and Er. Rajnish Verma, Higher Engineering Mathematics, S. Chand Publication, New Delhi.

Dominique The venin (TEX version) & G´abor Janiga (WWW version), Dominique Thevenin (TEX version) & G´abor Janiga (WWW version) ,2014. Fluid Dynamics for Engineers.

Garnier H. and Wang eds. L. 2008. Continuous-time Model Identification from Sampled Data. Springer-Verlag,

Makila, P. M. 2006. "A note on the Laplace transform method for initial value problems," International Journal of Control, vol. 79, no. 1, pp. 36–41, Jan.

Mathews, J. H., Fink, K. K. 2004. *Numerical Methods Using Matlab*. Prentice-Hall.

Munson, B.R., D.F. Young, and T.H. Okiishi, 2006. Fundamentals of Fluid Mechanics. fifth ed. New York: Wiley.

Peer Mohamed, M. 2016. Department of Mathematics, Marudupandiyar College, Thanjavar, IJSRP,Vol 6,Aug, ISSN 2250-3553.

Versteeg HK. and Malalasekera, W 2007. Introduction to Computational Fluid Dynamics. Second edition,finite volume method.Uk London.

Victor Udoewa1 and Vinod Kumar2, USAID Development Engineer, AAAS Science & Technology Policy Fellow, AAAS, 2009-2011, 2Mechanical Engineering, University of Texas at El Paso,USA.

World Academy of Science, Engineering and Technology International Journal of Mechanical and Mechatronics Engineering, Vol:2, No:7, 2008

****