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RESEARCH ARTICLE

HEAT ENGINE ANALOGOUS OF THE 4-DIMENSIONAL ROTATING-DYONIC BLACK HOLE IN AN EXTENDED PHASE SPACE

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ABSTRACT

In an extended phase-space where the cosmological constant is treated as pressure, the qualitative behavior of the dyonic charged spinning four dimensional Ads black hole is investigated. The system holds a similarity to the Vander Waals liquid-gas. The curves at $T < T_c$ showed the qualitative behavior of the phase transition of small-large black hole in the system. The behavior of the thermodynamic cycle by stating the heat engine's perception of the charged rotating black hole is investigated. It is inferred that the efficiency of the black hole η increases while we increase the angular momentum J and the ratio η/η_C satisfies the second law of thermodynamics for the specific values of J.

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INTRODUCTION

Black Hole Thermodynamics plays an important role in the theoretical physics, particularly it is well known for its study of quantum gravitational effects. The curiosity about the application of the laws of thermodynamics on the event horizon has been initiated preceding the pivotal work of Hawking et.al [1973; Bardeen, 1973]. Stability of the thermodynamic properties of the Anti deSitter (Ads) black holes in a canonical ensemble is quite assured where the exploration of quantum gravity and several super conformal gauge theories can be done in a wider perspective via its critical thermodynamic analysis [Witten, 1998; Chaturvedi, 2017]. Mostly due to the occurrence of a phase transition within the dual thermal field theory and also the confession of the gauge duality of the black holes via Ads/CFT correspondence, black hole thermodynamics had acquired attention in the framework of many researches [Bravetti, 2017; Czinner, 2016; Adamo, 2014; Pourhassan, 2016; Hendi, 2018]. The main advantage in the four dimensional space study is the no hair theorem which characterizes the black hole solutions by mass, electric charge and angular momentum. Kerr-Newman metric, referred as one of the black hole solution of general relativity, is the charged spinning generalization of the Schwarzschild black hole solution [Adamo, 2014].Thermodynamics of Rotating Charged Black holes are in a way similar to the general theory of thermodynamics concerning its area, mass and surface gravity which are related to the entropy, energy and temperature of the black hole respectively [Altamirano, 2014]. Dyonic black holes have shed the new light in the study as they possess electromagnetic duality where magnetic monopole is present along with the electric charge [Sadeghi, 2016; Zingg, 2011] as well as holographic duality, with chemical potential it corresponds to the van der Waals fluid and depending upon the temperature and magnetic field strength, it's Ads/CFT phases are either paramagnetic or diamagnetic in nature [Dutta, 2013]. Apart from the conventional space, with the framework of extended-phase space, the first law of thermodynamics as well as the Smarr relation from where it derived can be satisfied [Kastor, 2009]. In an extended phase space the cosmological constant is established as the thermodynamic variable and the mass is equivalent to the enthalpy of the black hole. Also, it is proved that the analysis of the rotating charged four dimensional Ads black hole in an extended phase space is analogous to the Van der Waals fluid [Kubiznak, 2012]. In this paper, we establish the electric as well as magnetically charged Ads rotating black hole solution from the Kerr-Newman-Ads metric in four space-time dimensions, then the thermodynamic quantities from treating the cosmological constant as independent variable are demonstrated in the segment 2. In segment3, the phase transition of small-large black hole in the system near critical points is analyzed via the P-Vcriticality.

The efficiency of the corresponding black hole is discussed by considering the thermodynamics of the heat engine model in segment 4. To conclude, the summary of the results is deliberated in the segment 5.

Spinning-Dyonic Kerr Ads Black Hole: The Ads Kerr metric in the asymptotically Anti-de Sitter space with the magnetic and electric charges [Caldarelli, 2000] is stated as

$$ds^2 = -\frac{\Delta}{\Sigma^2}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)^2 + \frac{\Sigma^2}{\Delta}dr^2 + \frac{\Sigma^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta\sin^2\theta}{\Sigma^2}\left(adt - \frac{r^2+a^2}{\Xi}d\phi\right)^2 \tag{1}$$

were

$$\Delta = \frac{(r^2+a^2)(L^2+r^2)}{L^2} - 2mr + q_e^2 + q_m^2, \Delta_\theta = 1 - \frac{a^2}{L^2}\cos^2\theta,$$
$$\Sigma^2 = r^2 + a^2\cos^2\theta, \Xi = 1 - \frac{a^2}{L^2} \tag{2}$$

The cosmological constant $\left(\Lambda = -\frac{3}{L^2}\right)$is included as a thermodynamic pressure by the equation of state $\Lambda = -8\pi P$considering$G_N = 1$ [Kastor, 2009]. The parameters m,q_e,q_m and a are related to the mass M, electric charge Q_e, magnetic charge Q_m and the angular momentum J as

$$M = \frac{m}{\Xi^2}, \quad Q_e = \frac{q_e}{\Xi},$$
$$Q_m = \frac{q_m}{\Xi}, \quad J = aM. \tag{3}$$

The mass of the black hole is obtained from $\Delta = 0$ as

$$M = \frac{(r^2+a^2)(r^2+L^2) + {q_e}^2L^2 + {q_m}^2L^2}{2rL^2\Xi^2}, \tag{4}$$

The electric potentialϕ_e, the magnetic potential ϕ_m, the angular velocity $\Omega$for the above metric as a function of entropy $\left(S = \frac{\pi(r^2+a^2)}{\Xi}\right)$are written as follows

$$\phi_e = \frac{q_e r}{(r^2+a^2)}, \ \phi_m = \frac{q_m r}{(r^2+a^2)}, \ \Omega = \frac{a(r^2+L^2)}{L^2(r^2+a^2)}. \tag{5}$$

In the same manner the temperature T and Volume V in the case of thermodynamic parameters as

$$T = \frac{1}{2\pi r}\left[\frac{(3r^2+a^2)(r^2+L^2) - {q_e}^2L^2 - {q_m}^2L^2}{2L^2(r^2+a^2)} - 1\right],$$

$$V = \frac{2}{3H\pi}\left[S\left(({Q_e}^2+{Q_m}^2)\pi + \frac{8PS^2}{3} + S\right) + 2J^2\pi^2\right] \tag{6}$$

In an extended phase space the mass is equivalent to the enthalpy of the black hole ($M \equiv H$). Hence the enthalpy of the black hole based on the Smarr relation formulation [Dolan, 2015] is expressed in terms of thermodynamic variables as

$$H = \frac{1}{2}\sqrt{\frac{\left(S + \pi({Q_e}^2+{Q_m}^2) + \frac{8PS^2}{3}\right)^2 + 4\pi^2\left(1+\frac{8PS}{3}\right)J^2}{\pi S}}. \tag{7}$$

The thermodynamic volume of the Ads Kerr Black hole differs from the non-rotating black holes as

$$V = \frac{2}{3H\pi}\left[S\left(({Q_e}^2+{Q_m}^2)\pi + \frac{8PS^2}{3} + S\right) + 2J^2\pi^2\right] \tag{8}$$

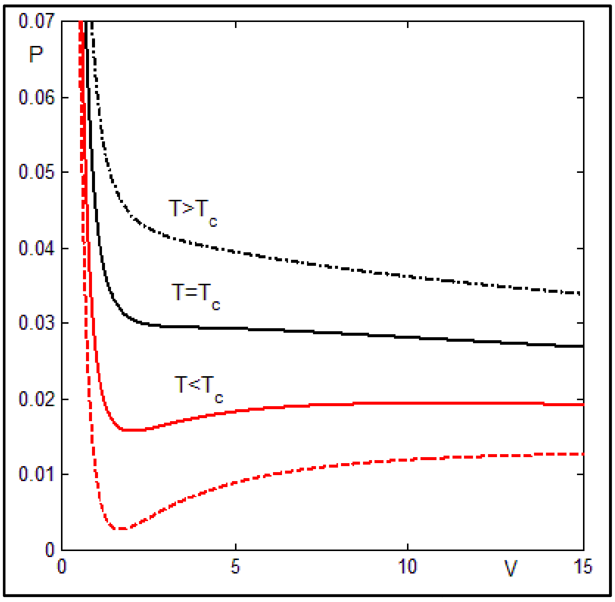

Fig. 1. P- υ diagram at the fixed Q_e, Q_m and J for different temperature of isotherms.

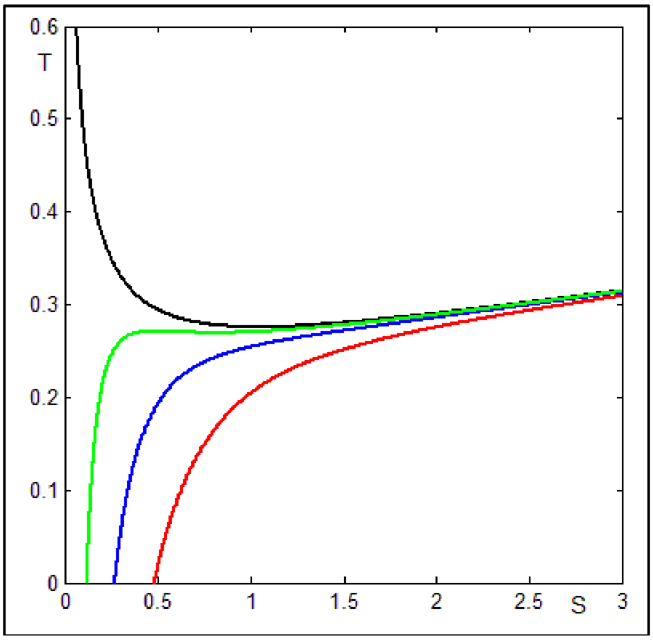

Fig. 2. Equation of state $T = T(S)$, the upper curve is for $Q_e = Q_m = J = 0$ and the lower curves are set at fixed Q_e, Q_m and varying J.

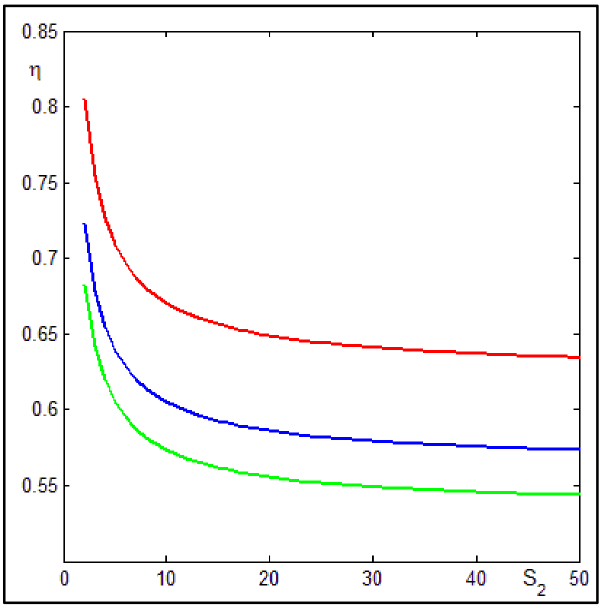

Fig. 3a. η with respect to S_2 by setting $P_4 = 1, P_1 = 4, S_1 = 1, Q_e = Q_m = 0.05$ and $J = 0.01$(green), 0.05(blue) &0.08(red)

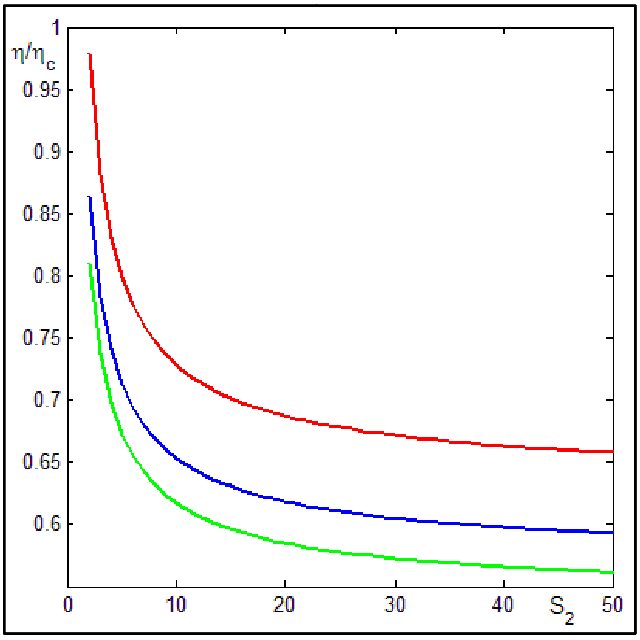

Fig. 3b. η/η_C with respect to S_2 by setting $P_4 = 1, P_1 = 4, S_1 = 1, Q_e = Q_m = 0.05$ and $J = 0.01$ (green), 0.05 (blue) &0.08 (red).

The internal energy is turned into

$$E = \left(\frac{\pi}{S}\right)^3 \left[\left(\frac{3V}{4\pi}\right)\left(\frac{S}{2\pi}\right)\left(\frac{S}{\pi} + {Q_e}^2 + {Q_m}^2\right)\left(\frac{3V}{4\pi}\right)J^2 - J\sqrt{\left(\frac{3V}{4\pi}\right)^2 - \left(\frac{S}{\pi}\right)^3}\sqrt{\left(\frac{S({Q_e}^2 + {Q_m}^2)}{\pi} + J^2\right)}\right] \tag{9}$$

The Pressure in terms of the thermodynamic variables can be yielded as

$$P = -\frac{\partial E}{\partial V} = \left[\frac{3}{8S^3}\left(2J\pi^2\left(\frac{3V\sqrt{J^2\pi + ({Q_e}^2 + {Q_m}^2)S}}{\sqrt{9V^2\pi + 16S^3}} - J\right) - S^2 - ({Q_e}^2 + {Q_m}^2)\pi\right)\right] \tag{10}$$

The Temperature is also written as

$$T = \frac{\partial E}{\partial S} = \frac{1}{8S^4}\left[\frac{9JV^2\pi^2\left(6J^2\pi + 5({Q_e}^2 + {Q_m}^2)S\right)}{\sqrt{J^2\pi + ({Q_e}^2 + {Q_m}^2)S\sqrt{9V^2\pi - 16S^3}}} - \frac{16JS^3\pi\left(3J^2\pi + 2({Q_e}^2 + {Q_m}^2)S\right)}{\sqrt{J^2\pi + ({Q_e}^2 + {Q_m}^2)S\sqrt{9V^2\pi - 16S^3}}} - 3V\left(S^2 + 6J^2\pi^2 + 2({Q_e}^2 + {Q_m}^2)S\right)\right] \tag{11}$$

The Gibbs free energy can be yielded from the equation $G = M - TS$ as

$$G = \frac{(3a^2 - L^2)r^4 + (a^2 + L^2)^2 r^2}{4(L^2 - a^2)^2 r} + \frac{\left(3L^4 - L^2a^2\right)\left({q_e}^2 + {q_m}^2\right) - L^2a^4 + 3L^4a^2}{4(L^2 - a^2)^2 r} \tag{12}$$

The Gibbs free energy elaborates the critical behavior depicted by the thermodynamic potential ,correlated to the Euclidean action, for the fixed values of Q_e, Q_m, P and T.

***P-V* Criticality:** The equation of state related to the P, T, V, Q_e, Q_m and Janalogous to the fluid equation of state defining the properties of black hole [19] is framed as

$$P = \frac{T}{v} - \frac{1}{2\pi v^2} + \frac{2{Q_e}^2}{\pi v^4} + \frac{2{Q_m}^2}{\pi v^4} + \frac{48J^2}{\pi v^6} - \frac{(2304({Q_e}^4 + {Q_m}^4) + 480({Q_e}^2 + {Q_m}^2)v^2)J^2}{\pi v^6\left(v^2 + \pi T v^3 + 8({Q_e}^2 + {Q_m}^2)\right)} + \frac{(576({Q_e}^2 + {Q_m}^2)\pi T v^3)J^2}{\pi v^6\left(v^2 + \pi T v^3 + 8({Q_e}^2 + {Q_m}^2)\right)}. \tag{13}$$

Where v is the specific volume and is equated to the volume by the expression$v = 2\left(\frac{3V}{4\pi}\right)^{1/3}$.
The critical points are then derived from

$$\frac{\partial P}{\partial v} = 0 \;\&\; \frac{\partial^2 P}{\partial v^2} = 0. \tag{14}$$

The critical specific volume is

$$v_c = 2\sqrt{3}\sqrt{\sqrt{{Q_e}^4 + {Q_m}^4 + 10J^2} + {Q_e}^2 + {Q_m}^2} \tag{15}$$

The critical temperature and critical pressure are computed numerically.

P- $v$diagram in Fig. (1)exhibits similarity to the van der Waals fluid with the temperature declining from the top. The magnetic monopole parameter $Q_m$significantly increased the magnitudes of critical temperature T_c and pressure as compared to the kerr-Newman Ads black holes whereas in the $T - S$diagram the upper curve showed an expected similarity to the Schwarzschild Ads black hole explicitly and the lower curves correspond to $Q_e = Q_m = 0.05$ and $J = 0.02$ (green), 0.05 (blue) and 0.1 (red) suggested the existence of small and large black hole phases in the canonical ensemble and also the incidence of a probable second-order phase transition is indicated by an inflection point of critical value $J_c = 0.02$.

Thermodynamic Cycle corresponding to Heat Engine Paradigm: We know that the heat capacity at constant volume $C_v = T\frac{\partial S}{\partial T}\Big|_v$ is non-zero from the equ. 11.Hence, for a heat engine analysis, efficiency is achieved by

$$\eta = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}, \quad (16)$$

where W, Q_H, Q_C are the work done, total heat inflow and total heat outflow, respectively. They are expressed as follows:

$$W = (P_1 - P_4)(V_2 - V_1),$$

$$Q_c = T_4(S_1,P_4)\,(S_2 - S_1), Q_H = T_2(S_2,P_1)(S_2 - S_1). \quad (17)$$

Thus the thermodynamic analysis of heat engine can be examined lucidly with the correlation of maximum efficiency thermodynamic cycle called the Carnot cycle and the cycle's efficiency is related to the isotherms of temperature as

$$\eta_c = 1 - \frac{T_4(S_1,P_4)}{T_2(S_2,P_1)}. \quad (18)$$

The plots generated between the ratio $\eta/\eta_c$with the entropy of large black hole, S_2in the Fig. 3b shows that the second law of thermodynamics holds good for the dyonic-rotating black holes for all values of $Q_m$and Q_e comparatively and it may be violated for above certain values of J, e.g. see Ref. 20 and the η/η_cincreases with increasing J. As for the curves calculated between η and S_2 are concerned, η increases as J increases and decreases to achieve a specific value when the ΔV (thermodynamic volume difference of small and large black hole) increases. between η and S_2 are concerned, η increases as J increases and decreases to achieve a specific value when the ΔV (thermodynamic volume difference of small and large black hole) increases.

Conclusion

We have probed the dyonic charged spinning black hole thermodynamics in the four space-time dimension in the framework of extended phase-space. The thermodynamic properties such as Pressure and temperature are recognized as a function of thermodynamic variables as a result of the extended phase where the enthalpy equals the mass instead of the internal energy by $M = H = U + PV$. The system exhibited a correspondence to the Vander Waals liquid-gasin Fig.2 and the curves at $T < T_c$, showed the qualitative behavior of the phase transition of small-large black hole in the system and it can be observed in higher dimensions further [19]. The behavior of the thermodynamic cycle by stating the heat engine analogous of the charged rotating black hole is investigated. It is inferred that the efficiency of the black hole ηincreases while we increase the angular momentum J and it may increase further by reducing the Q_e and Q_m simultaneously. By the association of S_2 with theV_2, we can say that when the volume difference of small and large black hole grows, the efficiency will probably reduce. The pressure difference $(P_4 - P_1)$ is kept small during the process since it obeys the second law of thermodynamics, there may be a possible violation of the second law if it gets larger. The ratio η/η_C satisfies the second law of thermodynamics for the specific values of J. Hence, it is safe to inspect the system within the particular range of angular momentum.

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