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RESEARCH ARTICLE

UNIQUENESS OF COMMON FIXED POINTS OF F-CONTRACTION MAPPING WITH GENERALIZED ALTERING DISTANCE FUNCTION IN PARTIALLY ORDERED METRIC SPACES SATISFYING OCCASIONALLY WEAKLY COMPATIBLE MAPPING

*Tesfaye Megerssa Oljira

Department of Mathematics, College of Natural and Computational Sciences, Mizan Tepi University, Ethiopia

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Corresponding author:
Tesfaye Megerssa Oljira

ABSTRACT

In this paper, common fixed point theorems of f-contraction mapping have been established with generalized altering distance function. Existence and uniqueness of common fixed point off-contraction mapping with generalized altering distance function in partially ordered metric spaces satisfying occasionally weakly compatible maps are proved.

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INTRODUCTION

Fixed point theory is among the fundamental tool of nonlinear functional analysis. Banach (1922) showed that every contraction mapping on a complete metric space always possess a unique fixed point. This study focused on proving the existence and uniqueness of common fixed points of f-contraction mapping defined on complete metric spaces endowed with a partial order by using generalized altering distance functions. I tried to answer the question how can we prove the existence and uniqueness of common fixed points of f-contraction mappings defined on complete metric spaces endowed with a partial order by using generalized altering distance functions satisfying occasionally weakly compatible?. Su (2014) proved the following fixed point theorem, which is the generalized type of Yan *et al.* (2012); Let $(X, \preccurlyeq)$ be a partially ordered set and suppose that there exists a metric din X such that $(X, d)$is a complete metric space. Let $T : X \rightarrow X$ be a continuous and non-decreasing mapping such that

$$\eta\big(d(Tx,Ty)\big) \leq \phi\big(d(x,y)\big), \forall y \preccurlyeq x,$$

where $\eta$is a generalized altering distance function and $\phi: (0,\infty) \rightarrow (0,\infty)$is a right uppersemi-continuous function with the condition: $\eta(t) > \phi(t)$for all$t > 0$.If there exists$x_0 \in X$such that $x_0 \preccurlyeq Tx_0$,then T has a fixed point. Many results appeared related to fixed point theorem in complete metric spaces endowed with a partial ordering $\preccurlyeq$ in the literature (Amini-Harandi, 2010; Naidu, 2013; Suzuki, 2008; Yan, 2012). Inspired and motivated by the results mentioned on (Su, 2014), I extend the main theorem of (Su, 2014) to f-contraction mapping in a complete metric space endowed with a partial order by using generalized altering distance functions satisfying occasionally weakly compatible maps and proved the uniqueness of the common fixed point obtained. Examples are given to show that my results are proper extension of the existing one. In (Arvanitakis, 2003; Amini-Harandi, 2010; Beg, 2006; Boyd, 1969; Chidume, 2002; Choudhury, 2000), the researchers proved some types of weak contractions in complete metric spaces. In particular the existence of a fixed point for weak contraction is extended to partial ordered metric spaces under the works of (Amini-Harandi, 2010; Choudhury, 2000; Tesfaye Megerssa Oljira, 2012).

Corresponding author: **Tesfaye Megerssa Oljira,**
Department of Mathematics, College of Natural & Computational Sciences, Mizan Tepi University, Ethiopia.

Definition 1.1 (Tesfaye Megerssa Oljira, 2019) A function $\eta: [0,\infty) \to [0,\infty)$ is called an altering distance function if the following properties are satisfied:

a.$\eta$is continuous and monotonically non-decreasing.

b.$\eta(t) = 0$ if and only if $t = 0$.

Example The following function is an altering distance function:

$\eta(t) = \begin{cases} 0, & t = 0 \\ \beta t, & t \geq 1, \end{cases}$ where $\beta \geq 1$.

Altering distance functions have been used in metric fixed point theory in a number of papers. Some of the works utilizing the concept of altering distance function are noted in (Babu *et al*., 2007; Choudhury, 2005; Khan, 1984; Sastry, 1999) (Babu *et al.*, 2007; Choudhury, 2005 Khan *et al.*, 1984; Sastry *et al.*, 1999).

Definition 1.2 (Tesfaye Megerssa Oljira, 2019) We shall say that the mapping S is f-non-decreasing (resp. f-non-increasing) if $fx \preccurlyeq fy \Rightarrow Sx \preccurlyeq Sy$ (respectively $fx \preccurlyeq fy \Rightarrow Sy \preccurlyeq Sx$) holds for each $x, y \in X$.

Definition 1.3 (Akram and Shamailac, 2015) A point $y \in X$is called point of coincidence of two mappings $f, S: X \to X$if there exists a point $x \in X$such that $y = fx = Sx$. In this case x is called the coincidence point of f and S and the set of coincidence points of f and S is denoted by $C(f, S)$. If $x = y$, then y is called common fixed point of f and S.

Definition 1.4 (Pant *et al.*, 2012) Let $f$and Sbe self maps of a metric space (X, d). The pair (f, S) is called occasionally weakly compatible (OWC) if there exists $x \in X$which is a coincidence point for f and S at which f and S commute (i.e. if $f(S(x)) = S(f(x))$ for some $x \in C(f, S)$).

Theorem 1.1(Su, 2014) Let$(\mathrm{X}, \preccurlyeq)$ be a partially ordered set and suppose that there exists a metric $d$on$\mathrm{X}$ such that $(\mathrm{X}, \mathrm{d})$is a complete metric space. Let $\mathrm{T} : \mathrm{X} \to \mathrm{X}$ be a continuous and non-decreasing mapping such that

$$\eta\big(d(Tx, Ty)\big) \leq \phi\big(d(x, y)\big), \forall y \preccurlyeq \mathrm{x},$$

where η is a generalized altering distance function and $\phi: (0,\infty) \to (0,\infty)$is a right upper semi-continuous function with the condition: $\eta(t) > \phi(t)$for all $t > 0$. If there exists $x_0 \in X$such that $x_0 \preccurlyeq Tx_0$, then T has a fixed point.

If (x_n) is a non-decreasing sequence in X such that $x_n \to x$ then $x_n \preccurlyeq x$ for all $n \in \mathbb{N}$. (a)

Theorem 1.2 (Su, 2014) Let $(\mathrm{X}, \preccurlyeq)$ be a partially ordered set and suppose that there exists a metric d in X such that (X, d) is a complete metric space. Assume that X satisfies (a). Let $\mathrm{T} : \mathrm{X} \to \mathrm{X}$ be a continuous and non-decreasing mapping such that

$$\eta(d(Tx, Ty)) \leq \phi(d(x, y)), \forall y \preccurlyeq \mathrm{x},$$

where η is a generalized altering distance function and $\phi: (0,\infty) \to (0,\infty)$ is a right upper semi-continuous function with the condition: $\eta(t) > \phi(t)$ for all$t > 0$. If there exists $x_0 \in X$ such that $x_0 \preccurlyeq Tx_0$, then T has a fixed point.

for $x, y \in X$ *there exists* $z \in X$ *which is comparable to* x *and* y (b)

Theorem 1.3 (Su, 2014) Adding the condition (b) to the hypothesis of Theorem 1.1 (resp. Theorem 1.2) we obtain the uniqueness of the fixed point of T.

Theorem 1.4 (Tesfaye Megerssa Oljira, 2016) Let$(X, \preccurlyeq)$be a partially ordered set and suppose that there exists a metric $d$on X such that $(X, d)$is a complete metric space. Let $f, T: X \to X$be two continues self maps on X satisfying the following conditions:

i) $TX \subset fX$;
ii) fX is closed;
iii) T is f-non-decreasing;
iv) There exists $x_0 \in X$ such that $fx_0 \preccurlyeq Tx_0$;
v) If $z \in C(f, T)$, then $fz \preccurlyeq f(fz)$such that$\eta(d(Tx, Ty)) \leq \phi(d(fx, fy))$ $\forall x, y \in X$ with $fy \preccurlyeq fx$,where η is an altering distance functions and $\phi: (0,\infty) \to (0,\infty)$is a right uppersemi-continuous function with the condition $\eta(t) > \phi(t)$for all $t > 0$ and $\phi(t) = 0 \Leftrightarrow t = 0$. Then f and T have a coincidence point. Furthermore if f and T are occasionally weakly compatible maps, then f and T have common fixed point in X.

2. MAIN RESULT

Assuming the following hypothesis in X:

If $\{y_n\}$ *is a non-decreasing sequence in* X *such that* $y_n \to y$ *then* $y_n \preccurlyeq y$ *for all* $n \in \mathbb{N}$.(1)

Theorem 2.1 Let $(X, \preccurlyeq)$be a partially ordered set and suppose that there exists a metric $d$on$X$ such that $(X, d)$is a complete metric space. Assume that X satisfies (1). Let $f, T: X \to X$ be two self maps satisfying the following conditions:

i)$TX \subset fX$;
ii)fX is closed;
iii)T is f-non-decreasing;
iv) there exists $x_0 \in X$ such that $fx_0 \preccurlyeq Tx_0$;
v)if $z \in C(f, T)$, then $fz \preccurlyeq f(fz)$
such that
$\eta(d(Tx, Ty)) \leq \phi(d(fx, fy))\ \forall x, y \in X$ with $fy \preccurlyeq fx$, (2)

where η isan altering distance functions and $\phi: (0, \infty) \to (0, \infty)$is a right upper semi-continuous function with the condition $\eta(t) > \phi(t)$for all $t > 0$. Then f and T have a coincidence point. Furthermore if f and T are occasionally weakly compatible maps, then f and T have common fixed point.

Proof Suppose there exists $x_0 \in X$ such that $fx_0 \preccurlyeq Tx_0$. Since $TX \subset fX$, we can choose $x_1 \in X$ such that $fx_1 = Tx_0$. Again from$TX \subset fX$, we can choose$x_2 \in X$ such that $fx_2 = Tx_1$. Continuing this process, we can choose a sequence $\{y_n\}$ in X such that

$$y_n = fx_{n+1} = Tx_n,\ \forall n \geq 0. \quad (3)$$

By similar procedure we followed in Theorem 1.4, we can show that the sequence $\{y_n\}$ is a Cauchy sequence in X. Since (X, d) is a complete metric space and using (2), $\{y_n\} \subset f(X)$ where $y_n = fx_{n+1}$ for each $n \geq 1$ and $f(X)$ is closed, then there exists $p \in X$ such that$y = fp$. Now we prove that $Tp = y$. Then, by the continuity of η and the upper semi-continuity of ϕ, using the condition (2), we have

$$\eta\big(d(Tp, y)\big) = \eta\left(d\left(Tp, \lim_{n\to\infty} Tx_n\right)\right)$$
$$= \lim_{n\to\infty} \eta\big(d(Tp, Tx_n)\big)$$
$$\leq \lim_{n\to\infty} \phi\big(d(fp, fx_n)\big)$$
$$\leq \lim_{n\to\infty} \eta\big(d(fp, fx_n)\big)$$
$$= \eta\left(\lim_{n\to\infty} d(fp, fx_n)\right)$$
$$= \eta\left(d\left(fp, \lim_{n\to\infty} fx_n\right)\right)$$
$$= \eta\big(d(fp, fp)\big) = \eta(0) = 0.$$

This implies that $\eta\big(d(Tp, y)\big) = 0$ and hence $Tp = y$.

Thus

$$y = Tp = fp.$$

This implies that $\eta\big(d(Tp, y)\big) = 0$ and hence $Tp = y$.

Thus

$$y = Tp = fp$$

Thus p is the coincidence point of f and T, which implies that $C(f, T) \neq \emptyset$. Since f and T are occasionally weakly compatible pair of self maps, f and T commute at some point $z \in C(f, T)$.

Now set$w = fz = Tz$. Since f and T are occasionally weakly compatible,

$$fw = Tw.$$

Now we claim that w is a common fixed point of f and T.

Now if $Tw \neq w$, since by (v) of Theorem 2.1, $fz \preccurlyeq f(fz) = fw$, we have

$\eta(d(Tw,w)) = \eta(d(Tw,Tz)) \leq \phi(d(fw,fz)) \leq \phi(d(Tw,w)) < \eta(d(Tw,w))$,
which is absurd. Hence, $Tw = w$.

Therefore $fw = Tw = w$.

Example 2.1.1 Let $X = \{1,2,3,4,5\}$. We define a partial order "$\preccurlyeq$" on X by

$\preccurlyeq = \{(1,1),(2,2),(3,3),(4,4),(5,5),(2,3),(3,4),(2,4)\}$.

Define a metric $d: X \times X \to \mathbb{R}$ by $d(x,y) = |x-y|$ for all $x, y \epsilon X$.

Consider the mappings $f, T: X \to X$ defined by

$f(1) = 1, f(2) = 2, f(3) = 2, f(4) = 3, f(5) = 4$ and
$T(1) = 1, T(2) = 2, T(3) = 2, T(4) = 2, T(5) = 2$.
Then $T(X) = \{1,2\} \subset \{1,2,3,4\} = f(X)$ and $f(X) = \{1,2,3,4\}$ is closed.

Next we show that T is f-non-decreasing.

$2 = f(2) \preccurlyeq f(4) = 3 \Rightarrow 2 = T(2) \preccurlyeq T(4) = 2;$
$2 = f(3) \preccurlyeq f(4) = 3 \Rightarrow 2 = T(3) \preccurlyeq T(4) = 2;$
$3 = f(4) \preccurlyeq f(5) = 4 \Rightarrow 2 = T(4) \preccurlyeq T(5) = 2;$
$2 = f(2) \preccurlyeq f(5) = 4 \Rightarrow 2 = T(2) \preccurlyeq T(5) = 2;$
$2 = f(3) \preccurlyeq f(5) = 4 \Rightarrow 2 = T(3) \preccurlyeq T(5) = 2$

which shows that T is f-non-decreasing. We also observe that $f(1) \preccurlyeq T(1)$ and $z = 2 \in C(f,T) = \{1,2,3\}$ such that $fz \preccurlyeq f(fz)$. Further, f and T satisfy all the contraction conditions of Theorem 2.1 for $\eta(t) = t$ and $\phi(t) = \frac{1}{2}t, t \geq 0$. Since $C(f,T) \neq \emptyset$, f and T are occasionally weakly compatible maps. Moreover, 1 and 2 are common fixed points of f and T. Hence the uniqueness of common fixed point of f and T is not guaranteed by the conditions of Theorem 2.1. Remark 1By choosing a map T to be non-decreasing and f = Identity map in Theorem 2.1 we get Theorem 1.2 as a corollary to Theorem 2.1.

Lemma 2.1. (Pant *et al.*, 2012) Let X be a non-empty set, $f$and $T$are occasionally weakly compatible self maps of X. If $f$and Thave a unique point of coincidence, $w = fx = Tx$then $w$is the unique common fixed point of $f$and T.

Proof: Let z be a unique point of coincidence of $f$and T. Then $z = fx = Tx$ for some $x \in X$ which implies that $C(f,T) \neq \emptyset$. Now since f and$T$ are occasionally weakly compatible maps, $fT(u) = Tf(u)$ for some $u \in C(f,T)$. So, by the uniqueness of z, we have $z = fu = Tu$ and hence $fz = fT(u) = Tf(u) = Tz$, which again follows that $z = fz = Tz$. Thus, z is a common fixed point of f and T. Suppose now that there exists another common fixed point $w \in X$ of f and T.

Then w becomes a point of coincidence of f and T. Consequently, by the uniqueness of point of coincidence we obtain, $w = z$. In what follows, we give sufficient condition for the uniqueness of common fixed point of f and T in Theorems 1.4 and Theorem 2.1? Theorem 2.2. In addition to the hypothesis of Theorem 1.4 and Theorem 2.1, suppose that $f: X \to X$ is non-decreasing and for every $x, y \in X$ there exists $z \in X$ which is comparable to $x$and$y$. Then f and T have a unique common fixed point in$X$.

Proof by Theorem 2.1, the set of common fixed points of f and T is non-empty.

Suppose that there exist $y, z \in X$ which are common fixed points of $f$and$T$. We consider two cases.

Case 1.If y is comparable to$z$, then $y = Ty$ is comparable to$z = Tz$. So,

$$\eta(d(y,z)) = \eta(d(Ty,Tz)) \leq \phi(d(fy,fz)) = \phi(d(y,z)).$$

As the condition $\eta(t) > \phi(t)$ for $t > 0$, we obtain $d(y,z) = 0$ which in turn implies $y = z$.

Case 2.If y is not comparable to z, then there exists $x_0 \in X$which is comparable to y and z; i.e., either $x_0 \preccurlyeq y$ and $x_0 \preccurlyeq z$ or $y \preccurlyeq x_0$ and $z \preccurlyeq x_0$.

Without loss of generality let us take $y \preccurlyeq x_0$ and $z \preccurlyeq x_0$.

Now $x_0 \preccurlyeq y \Rightarrow fx_0 \preccurlyeq fy$, since f is non-decreasing on X.

$\Rightarrow Tx_0 \preccurlyeq Ty$, since T is f-non-decreasing on X.

But $TX \subset fX$. Then there exists $x_1 \in X$ such that $Tx_0 = fx_1$. It follows that

$fx_1 \preccurlyeq y = fy.$

Since T is f-non-decreasing on X, this implies

$Tx_1 \preccurlyeq Ty = y.$

Now again since $TX \subset fX$, there exists $x_2 \in X$ such that $Tx_1 = fx_2$. This implies

$fx_2 \preccurlyeq y = fy$

Proceeding this way, inductively we construct a sequence$\{p_n\}$ such that $\forall n \geq 0$,

$p_n \preccurlyeq y,$

where $p_n = Tx_n = fx_{n+1}$ for each $n = 0,1,2,\cdots$.

If there exists $N \in \mathbb{Z}^+$ such that $y = p_N$, then

$$\eta\big(d(y, p_{N+1})\big) = \eta\big(d(Ty, Tx_{N+1})\big) \leq \phi\big(d(fy, fx_{N+1})\big) = \phi\big(d(y, p_N)\big) = 0,$$

which implies that $y = p_n, \forall n \geq N$ and hence the sequence $\{p_n\} \to y$ as $n \to \infty$.

Suppose that$y \neq p_n, \forall n \geq 0$. Then

$$\eta\big(d(y, p_n)\big) = \eta\big(d(Ty, Tx_n)\big) \leq \phi\big(d(fy, fx_n)\big) = \phi\big(d(y, p_{n-1})\big) \tag{4}$$

which implies that

$$\eta\big(d(y, p_n)\big) \leq \phi\big(d(y, p_{n-1})\big) < \eta\big(d(y, p_{n-1})\big), \forall n = 1,2,3,\cdots.$$

From the property of η, we notice that $\{d(y, p_n)\}$ is a non-decreasing sequence and hence there exists $b \geq 0$ such that

$d(y, p_n) \to b$as $n \to \infty$.

We claim that $b = 0$.

Letting $n \to \infty$in (4) and taking into account the properties of η andϕ, we obtain $\eta(b) \leq \phi(b)$. This and the condition $\eta(t) > \phi(t)$for $t > 0$imply$b = 0$. Hence,

$$\lim_{n\to\infty} d(y, p_n) = 0.$$

In similar line, it can be proved that

$$\lim_{n\to\infty} d(z, p_n) = 0.$$

Finally, as

$$\lim_{n\to\infty} d(y, p_n) = \lim_{n\to\infty} d(z, p_n) = 0,$$

by the uniqueness of limit of a convergent sequence in metric spaces we obtain $y = z$. This completes the proof.

Remark 2 by choosing T to be f-non-decreasing map in Theorem 2.2, we get Theorem 1.3 as a corollary to Theorem 2.1.The following is an example in support of Theorem 2.2.

Example 2.2.1. Let $X = [-3,3]$ and define order relation "$\preccurlyeq$" on X by

$$x \preccurlyeq y \Leftrightarrow \{(x = y)\ or (x \in [-3,0] \& \ y \in [0,3])\}.$$

We observe that $(X, \preccurlyeq)$ is partially ordered set.

Define $d: X \times X \to \mathbb{R}$ by $d(x,y) = |x-y|\ \forall x,y \in X$.

Consider the mapping $f,T: X \to X$ defined by $Tx = \frac{x}{3}$ and $fx = \frac{x}{2}$. Define $\eta, \phi: [0,\infty) \to (0,\infty)$ by $\eta(t) = \begin{cases} \frac{11}{60}t, 0 \le t < 1 \\ \frac{1}{5}t,\ t \ge 1 \end{cases}$ and $\phi(t) = \frac{1}{6}t$. Then η and ϕ satisfy the conditions of the theorem. Here we observe that $T(X) = [-1,1] \subset \left[-\frac{3}{2},\frac{3}{2}\right] = f(X)$ and $f(X) = \left[-\frac{3}{2},\frac{3}{2}\right]$ is closed in X.

Now if $x,y \in X$ such that $fx \preccurlyeq fy$, then either $fx = fy$ or $fx \in [-3,0]$ and $fy \in [0,3]$.

$\Rightarrow x = y$ or $x \in [-3,0]$ and $y \in [0,3]$. $\left(\because fx, fy \in f([-3,3]) = \left[\frac{-3}{2},\frac{3}{2}\right]\right)$
$\Rightarrow Tx = Ty$ or $Tx \in [-3,0]$ and $Ty \in [0,3]$
$\Rightarrow Tx \preccurlyeq Ty$

Clearly there exists $x_0 = 0 \in [-3,3]$ such that $fx_0 = Tx_0$,

i.e., $fx_0 \preccurlyeq Tx_0$.

Also, f is a non-decreasing, since if $x,y \in$ X such that $x \preccurlyeq y$, then either $x = y$ or $x \in [-3,0]$ and $y \in [0,3]$

$\Rightarrow fx = fy$ or $fx = \frac{x}{2} \in [-3,0]$ and $fy = \frac{y}{2} \in [0,3]$
$\Rightarrow fx \preccurlyeq fy$

Now let $x,y \in [-3,3]$ such that $fx \preccurlyeq fy$. Then either $fx = fy$ or $fx \in [-3,0]$ and $fy \in [0,3]$.

Case (i) If $fx = fy$, we have $\frac{x}{2} = \frac{y}{2}$, which implies $Tx = Ty$ and hence obviously the inequality (2) hold.

Case (ii) If $fx \in [-3,0]$ and $fy \in (0,3)$, then $\frac{x}{2} \in (-3,0)$ and $\frac{y}{2} \in (0,3)$.

This implies that

$x \in [-3,0]$ and $y \in (0,3)$ (Since $fx, fy \in f([-3,3]) = (-\frac{3}{2},\frac{3}{2})$).

Now we shall consider two sub-cases

If $0 \le y - x < 1$, then

$$\eta\big(d(Tx,Ty)\big) = \eta\left(\frac{1}{3}(y-x)\right) \le \phi\left(\frac{1}{3}(y-x)\right) = \phi\big(d(fx,fy)\big).$$

If $y - x \ge 1$, then

$$\eta\big(d(Tx,Ty)\big) = \eta\left(\frac{1}{3}(y-x)\right) \le \phi\left(\frac{1}{3}(y-x)\right) = \phi\big(d(fx,fy)\big).$$

Thus

$\eta\big(d(Tx,Ty)\big) \le \phi\big(d(fx,fy)\big)\ \forall x,y \in X$ with $fx \preccurlyeq fy$;

For

$$\eta(t) = \begin{cases} \frac{11}{60}t, 0 \le t < 1 \\ \frac{1}{5}t,\ t \ge 1 \end{cases} \text{ and } \phi(t) = \frac{1}{6}t.$$

Thus, f and T satisfy all the conditions of Theorem 1.4 and Theorem 2.1. Moreover, 0 is a unique common fixed point of f and T.

REFERENCES

partial metricspaces, *J. Nonlinear Sci. Appl.,* 8), 184–192.

2. Amini-Harandi A. Emami H. 2010. A fixed point theorem for contraction type maps in partially ordered metric spaces and application to ordinary differential equations. *Nonlinear Anal.* 72, 2238-2242.
3. Arvanitakis A. D. 2003. A proof of the generalized Banach contraction conjecture, *Proceedings of the American Mathematical Society,* 131(12), 3647–3656.
4. Babu G. V. R., Lalitha B. and Sandhya M. L. 2007. Common fixed point theorems involving two generalized Altering distance functions in four variables, Proceedings *of the Jangjeon Mathematical Society*, 10(1), 83–93.
5. Banach S. 1922. Sur le operations dans les ensembles abstraitetleur application aux equations, integrals. *Fundam. Math., 3*, 133-181.
6. Beg I. and Abbas M. Coincidence point and invariant approximation for mappings satisfying generalized weak contractive condition, Fixed *Point Theory and Applications*, 2006 (74503), (2006) 7-18.
7. Boyd D. W. and Wong J. S. W. 1969. On nonlinear contractions, *Proceedings of the American Mathematical Society*, 20(2) 458–464.
8. Chidume C. E., Zegeye H. and Aneke S. J. 2002. Approximation of fixed points of weakly contractive non-self-maps in Banach spaces, *Journal of Mathematical Analysis and Applications*, 270(1) 189–199.
9. Choudhury B. S. 2005. A common unique fixed point result in metric spaces involving generalised altering distances, *Mathematical Communications*, 10(2) 2005105–110.
10. Choudhury B. S. and Dutta P. N. 2000. A unified fixed point result in metric spaces involving a two variable function, *Fixed Point Theory and Applications.Filomat*, (14) 43–48.
11. Ciri´ L., Caki´ N., Rajovi´ M. and Ume J. S. 2008. "Monotone generalized nonlinear contractions in partially ordered metric spaces," *Fixed Point Theory and Applications*, (131294) (2008)11.
12. Khan M.S., Swaleh M. and Sessa S. 1984. Fixed point theorems by altering distances between the points. Bull. Aust. Math. Soc. 30(1) 1-9.
13. Naidu S. V. R. 2003. "Some fixed point theorems in metric spaces by altering distances," *zechoslovak Mathematical Journal*, 53(1) 205–212.
14. Pant B. D. And Sunny C. 2012. Fixed Point Theorems for Occasionally Weakly Compatible Mappings InMenger Spaces. *Matematiqki Vesnik* 64 (4) 267–274.
15. Sastry K. P. R. and Babu G. V. R. 1999. Some fixed point theorems by altering distances between the points, *Indian Journal of Pure and Applied Mathematics*, 30(6) 641–647.
16. Su Y. 2014. Contraction mapping principle with generalized altering distance function in ordered metric spaces and applications to ordinary differential equations. *Fixed Point Theory and Applications* 227.
17. Suzuki T. 2008. A generalized Banach contraction principle that characterizes metric completeness, *Proceedings of the American Mathematical Society*, 136 (5) 1861–1869.
18. Tesfaye Megerssa Oljira, 2019. "Common fixed points of f-contraction mapping with generalized altering distance function in partially ordered metric spaces", International Journal of Current Research, 11, (08), 6050-6054.
19. Yan F., Su Y. and Feng Q. 2012. A new contraction mapping principle in partially ordered metric spaces and Applications to ordinary differential equations. *Fixed Point Theory Appl*. (152) .

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