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RESEARCH ARTICLE

AN ANALYTICAL APPROACH OF THE NONLINEAR IMPLICIT SYSTEM OF PARTIAL DIFFERENTIAL EQUATIONS ON BLOOD FLOW LOOP THROUGH THE ARTERIES IN A HUMAN BODY

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ABSTRACT

The unsteady flow phenomena dominate the blood stream in arteries. The cardiovascular system in the human body can be defined as an inner loop with a complex liquid flowing in multiple branches. Further, the relationship between viscous and unsteady forces is governed primarily, by the Womersley number, a non-dimensional frequency parameter. By the Finite Element method, the researcher is able to resolve the resultant nonlinear implicit system related to partial differential equation. By generating secondary flow in branches and curves, the normal arterial flow remains laminar. The arteries in the human body remain the living organs, capable of changing and adapt to varying hemodynamic conditions, however, in some particular situations, an abnormal biological response is created by unusual hemodynamic conditions. Due to the skewing of the velocity profile, the creation of pockets, takes place in the cardiovascular system, with which the oscillation of the direction of wall shear stress takes place. These sites remain the core of Atherosclerotic disease and that further, results in stenosis or the narrowing of the artery lumen. Due to stenosis, the human body suffers from turbulence and a reduction of flower due to flow choking and viscous head losses. Further, due to extremely high shear stresses near stenosis throat, the platelets are activated, that, further, leads to thrombosis, that, again, retains the capability of blocking the complete blood flow both to the brain and the heart. For surgical intervention, the detection, as well as quantification of stenosis remains the basis. In future studies, the research related to of arterial blood flow would definitely lead to the accurate hemodynamic flow in any particular patient, the development of the accurate diagnostic tools for quantifying the level of the disease, as well as the design various devices required to mimic as well as alter the flow of the blood. When compared to other research, the field of fluid mechanics offers considerable challenges that involve three-dimensional, pulsatile flows at the edge of turbulence.

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INTRODUCTION

According to the report published by the World Health Organization (WHO), various kinds of cardiovascular diseases including strokes, heart attacks, atherosclerosis (stenosis) and aneurisms remains the highest cause of death all over the world. This fact is confirmed by Fuat *et al*. (2001), who states that these diseases claim more than 17.5 million lives per year. Further, according to Fuat *et al*. (2001), in this context, the most arteries affected by these diseases remain the femoral, coronary, the carotid, as well as the abdominal aorta. In this context, the researcher needs to study the non-Newtonian blood flow behavior that includes the yield stress, viscoelasticity, thixotropy, as well as the shear-thinning viscosity. In recent decades, while collaborating during research, different researchers such as the medical researchers, numerical scientists, as well as the bio-engineers have been conducting a deep study on the topic related to the blood flow stimulation in circulation system. The blood itself is defined as a multi-phase, complex mixture of solid corpuscles with different components such as leukocytes on, in common language, WBCs, that is, white blood cells, erythrocytes, or RBCs, that is, red blood cells, and platelets, or thrombocytes. In blood, the solid corpuscles are suspended in the plasma that contains different kinds of proteins, organic molecules, and electrolytes as well as in an aqueous ionic and polymeric solution.

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When compared to other elements, the RBC or erythrocytes exerts a considerable influence on blood's mechanical properties as its volume in the blood comprises from 40% to 45% in a healthy person (hematocrit). According to the experts, the blood rheology is determined by the surrounding structures, properties of various components, as well as their interaction with each other. According to a study done by Bondar *et al*. (2011), the rheological characteristics of blood are also affected due to outdoor physical environments including temperatures. Nevertheless, this study further states, as the temperature is regulated in living organisms, particularly, in large mammals such as human beings, and while it may be subjected to minor variation, it should not affect the general properties in a general manner. Further, according to Breithupt *et al*. (1997), the properties related to blood as well as blood vessels rheology would be affected by various types of body intakes such as medication, nutrients, as well as fluids. However, according to the conclusion of this study, the effects are not substantial and they do not have lasting consequences. According to a study done by Lee *et al*. (2011), the viscosity of blood is highly affected by various issues for instance the blood cells' mechanical properties, distribution of blood cells, the level of hematocrit, as well as the viscosity of plasma. According to Dintenfass (1962), although the plasma can be described primarily as a Newtonian fluid, the behavior of blood is reminiscent with a non-Newtonian fluid, consistent with all the signs related to non-Newtonian rheology, that containing viscoelasticity, thixotropy, give in stress, and deformation rate dependency. In this context, the origin of most of these effects remains the RBC or red blood cells, mainly because of high density and distinct mechanical possessions like an ability to form three-dimensional structures even at low deformation rate as well as elasticity. In their continuous research, Chien *et al*. (1966, 1967, 1978and 1970), Offer information about three phenomena for explaining the non-Newtonian behavior of blood. One, remains the normal propensity of erythrocytes to form rouleaux or the three-dimensional micro structures at low shear rates, the second phenomena can be defined as their deformability or break up. The third and final phenomena remain the propensity of erythrocytes of alignment with the flow field at high shear rates. The study concludes that while at low shear rates (below $1s^{-1}$), the blood may seem to have high apparent viscosity, and conversely, the blood's viscosity is at a reduction, at the high shear rate. Further, according to an experimental study conducted by Quemada (1978) concludes that the blood displays the effect of stress relaxation, this conclusion has been confirmed by Evans and Hochmuth (1976), that further refines this conclusion by stating that the red blood cell membrane, an important component of blood displays stress relaxation.

However, the experimental study conducted by Thurston (1973) concludes that the time of relaxation in this context relies on the shear rate. The focal point of this analysis is to research the numerical stimulations and investigations of blood flow, various methods such as finite element methods (FEM), as well as the nonlinear system of PDE or partial differential equations related to a combination of elliptic-hyperbolic type, that further, models the that models the non-Newtonian incompressible viscoelastic Oldroyd-B fluids flow in the steady case. In this context, the constitutive equations would characterize the mechanical behavior of fluid related to the Cauchy stress tensor with the kinematics of different quantities. In this case, the constitutive equations for non-Newtonian viscoelastic fluids would include a highly non-linear system of PDE or partial differential equations of a combined parabolic-hyperbolic type or elliptic hyperbolic type. In this context, the Oldroyd-B fluid model is defined primarily a constitutive model of rate type that retains the capability to define the viscoelastic behavior of blood flow during the polymeric process. The iterative Newton-Raphson method would be used for obtaining the numerical solution of the Navier-Stokes problem, discretized with the use of P2 -P1 (Hood-Taylor) finite elements. Using a fixed-point approach application, the iterative method would be implemented in order to resolve the steady tensorial transport equation. This equation would be further discretized by the use of the discontinuous Galerkin finite element approach.

Formulation of the problem: While doing the research, we would consider the coordinates are cylindrical (r, θ, x) that (r = 0) can be defined like the axis of the tube symmetry. Further, the flow of an incompressible hyperbolic tangent fluid of constant viscosity μ_0 and density ρ in a tube having a length L would be considered. In addition, the values$\bar{u}$ and $\bar{U}$would be considered as component of the velocity in the direction $\bar{r}$ and $\bar{x}$- in a respective manner.

h(x) = e(x)[1-η^*(b^{n-1}(x-a)-$(x-a)^n$] $\quad a \le x \le a+b$ (1)

= e(x) , if not

With e(x) = $e_o + \mathcal{E}\bar{x}$ (2)

- The radius of the tapered blood vessel section in the stenotic region is e(x)
- The radius of the non-tapered blood vessel in the non-stenotic area is e_0
- The narrowing factor is Ɛ
- The stenos is length is b

(n≥ 2) is a framework formative the shape of the tightness outline and denoted as the shape parameter (the symmetric stenosis take place for n = 2) and which appears in Figure . The parameter η is defined as follows

$$\eta^* = \frac{\delta^* n^{\frac{n}{n-1}}}{e_0 b^{n(n-1)}} \quad (3)$$

Here, δ refers to the highest altitude stenosis found in

$\bar{x}$ = a + $\frac{b}{\frac{1}{n^{n-1}}}$

These equations which govern the constantly incompressible hyperbolic tangent liquid are specified by

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial U}{\partial x} = 0 \quad (4)$$

$$\rho \left(u \frac{\partial}{\partial r} + U \frac{\partial}{\partial x} \right) u = - \frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (r \overline{S_{rr}}) + \frac{\partial}{\partial x} (\overline{S_{rx}}) - \frac{S_{\theta\theta}}{r} \quad (5)$$

$$\rho \left(u \frac{\partial}{\partial r} + U \frac{\partial}{\partial x} \right) U = - \frac{\partial p}{\partial x} + \frac{1}{r} (r \overline{S_{rx}}) + \frac{\partial}{\partial x} (\overline{S_{xx}}) \quad (6)$$

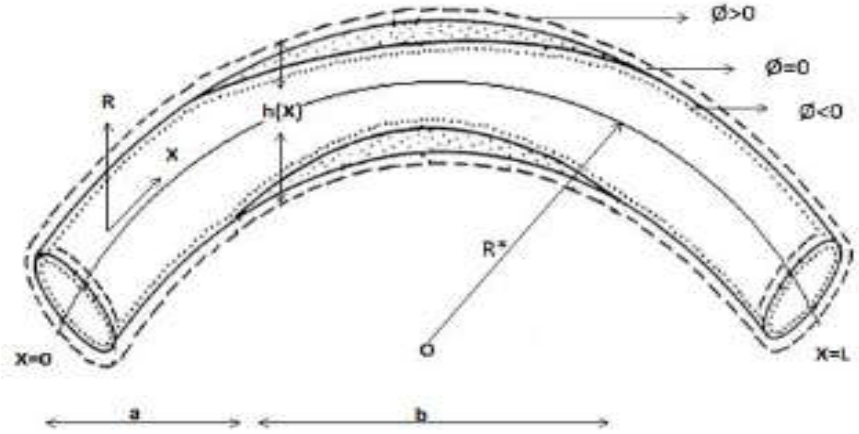


Fig. 1. Geometry of curved artery

The constitutive equation of hyperbolic tangent fluid is defined as

$$\tau = - PI + \bar{s} \quad (7)$$

$$\bar{s} = [\mu_0 + (\mu_\infty + \mu_0) \tanh (\Gamma \overline{\dot{\gamma}})^{n_e}] \overline{\dot{\gamma}} \quad (8)$$

Where, μ_∞ is the viscosity of infinite shear rate, $\bar{s}$ is the tensor of extra stress, Γ is the constant time, μ_0 is the viscosity of zero shear rate, M is the index of power legislation and $\bar{\dot{\gamma}}$ is indicated

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2} \Sigma_i \Sigma_j \dot{\gamma}_{ij} \dot{\gamma}_{ji}} = \sqrt{\frac{1}{2}\Pi} \quad (9)$$

Where Π =$\frac{1}{2}$ trac$(grad\ V + (grad\ v)^T)^2$, the 2nd invariant strain tensor is Π . Now for the case $\Gamma\bar{\dot{\gamma}} << 1$ and μ_∞ = 0. Eq (8) reduced to eq.(10) as,

$$\bar{s} = [\mu_0 (\Gamma\dot{\gamma})^{n_e}] \overline{\dot{\gamma}_0}$$

$$= [\mu_0 (1 - 1 + \Gamma\bar{\dot{\gamma}})^{n_e}] \overline{\dot{\gamma}_0}$$

$$= \mu_0 [1 + n_e (\Gamma\bar{\dot{\gamma}} - 1)] \overline{\dot{\gamma}_0} \quad (10)$$

Where $\overline{\dot{\gamma}_0}$ = L + L^t and extra stress tensor can be mentioned for hyperbolic tangent fluid as

$$\overline{S_{\bar{r}\bar{r}}} = 2\mu_0 [1 + n_e(\Gamma\bar{\dot{\gamma}} - 1)] \frac{\partial \bar{V}}{\partial \bar{r}}$$

$$\overline{S_{\bar{r}\bar{x}}} = \mu_0 [1 + n_e(\Gamma\bar{\dot{\gamma}} - 1)] (\frac{\partial \bar{U}}{\partial \bar{r}} + \frac{R^*}{\bar{r} + R^*} \frac{\partial \bar{V}}{\partial \bar{x}} - \frac{\bar{U}}{\bar{r} + R^*}),$$

$$\overline{S_{\bar{x}\bar{x}}} = 2\mu_0 [1 + n_e(\Gamma\bar{\dot{\gamma}} - 1)] (\frac{R^*}{\bar{r} + R^*} \frac{\partial \bar{U}}{\partial \bar{x}} + \frac{\bar{V}}{r + R^*}) \quad (11)$$

Non-dimensional variables are described

$r = \frac{\bar{r}}{e_0}$, $x = \frac{\bar{x}}{b}$, $U = \frac{\bar{U}}{u_0}$, $V = \frac{b\bar{V}}{u_0\delta}$, $p = \frac{{e_0}^2\bar{p}}{bu_{0\mu}}$,

$w_e = \frac{\Gamma u_0}{e_0}$, $R_{en} = \frac{bu_0\rho}{\mu}$, $s_{rr} = \frac{b\overline{s_{\bar{r}\bar{r}}}}{u_0\mu}$, $s_{rx} = \frac{e_0\overline{s_{\bar{r}\bar{x}}}}{u_0\mu}$,

$s_{xx} = \frac{b\overline{s_{\bar{x}\bar{x}}}}{u_0\mu}$, $\dot{\bar{\gamma}} = \frac{\bar{\gamma}\, e_0}{u_0}$ (12)

Where u_0 is the velocity over the segment of the tube of the breadth e_0.Using Eq.(12) mild stenosis case $\delta^* = \frac{\delta}{e_0} << 1$ and taking the extra conditions

$$\mathcal{E} = \frac{e_0 n^{\frac{1}{n-1}}}{b} \approx 0(1) \quad (13)$$

$$Re_n \frac{\delta n^{\frac{1}{n-1}}}{b} << 1 \quad (14)$$

Can be edited as

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial U}{\partial x} = 0 \quad (15)$$

$$\frac{\partial p}{\partial r} = 0 \quad (16)$$

$$\frac{\partial p}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}[\, r\, (\, (\, n_e\, -1\,)\, (\frac{\partial U}{\partial r}) + w_e n_e (\frac{\partial U}{\partial r})^2\,)] \quad (17)$$

The corresponding boundary condition is

$$\frac{\partial U}{\partial r} = 0 \text{ at } r = 0, \quad (18a)$$

$$U = 0 \text{ at } r = h(x) \quad (18b)$$

The stenosis geometry is described in dimensional-less form

$$h(x) = (\, 1 + \xi x)\, [\, 1 - \eta\, (\, x - \sigma\,) - (\, x - \sigma\,)^n], \sigma \leq x \leq \sigma + 1 \quad (19)$$

where

$$\eta = \frac{\delta n^{\frac{n}{n-1}}}{n-1} , \delta = \frac{\delta^*}{e_0} , \sigma = \frac{a}{b} \quad (20)$$

$$R_c = \frac{R^*}{e_0} , \xi = \frac{\xi' b}{e_0} , \xi = \tan\varphi \quad (21)$$

Where the diverging tapering ($\varphi > 0$), ($\xi = \tan\varphi$), tapered angle is φ and non-tapered blood vessel ($\varphi = 0$) and, for converging tightening($\varphi < 0$), as depicted in the fig

Issue resolution: By considering $w_e$as a small parameter, we can increase speed and the rate of flow according to the system

$U = U_0 + w_e U_1 + {w_e}^2 U_2 + \ldots\ldots$
$F = F_0 + w_e F_1 + {w_e}^2 F_2 + \ldots\ldots\ldots\ldots\ldots\ldots$

The solution of Eq. (17) with respect to boundary condition (18a,18b) takes the following form with the aid of Eq. (21)

$$U = \frac{R_c(r+R_c)}{1-n_e}\frac{dp}{dx}(\, r + R_c\,) \ln (\, r + R_c) = (c_1 + w_e c_3)\, (\, r + R_c\,) + \frac{1}{r+R_c}(\, c_2 + w_e c_4)$$

$$+ \frac{a_2 w_e}{8(r+R_c)^3} + \frac{a_3 w_e}{2}(r + R_c\,) \ln (\, r + R_c\,) \quad (22)$$

We have defined flow rate

$$F = \int_0^h U\, dr \tag{23}$$

We get pressure gradient substituting in Eq.(22) into Eq.(23) is described

$$\frac{dp}{dx} = \frac{F - w_e c_6}{c_5} \tag{24}$$

Drop of Pressure (Δp = p at x = 0 and Δp = - p and x = L) can be written through the stenos is between x = 0 and x = L computed from above Eq.(24)

$$\Delta p = \int_0^h (-\frac{dp}{dx})\, dx \tag{25}$$

Confrontation impedance

Impedance of resistance is described in by eq.(25),

$$\bar{\lambda} = \frac{\Delta p}{F} = \{\int_0^a k(x)|h=1\, dx + \int_a^{a+b} k(x)dx + \int_{a+b}^L k(x)|h=1 dx\} \tag{26}$$

Where, $k(x) = -\frac{F - w_e c_6}{c_5 F}$ (27)

Using Eq.(27) into Eq.(26) , takes the form

$$\bar{\lambda} = (-\frac{F - w_e c_6}{c_5 F}|h=1)(L - b) + \int_a^{a+b} k(x)dx \tag{28}$$

Wall-shear stress response

The non-zero dimensional shear stress is generated

$$\overline{s_{rx}} = [(\frac{\partial U}{\partial r})(1 - n_e) + w_e n_e (\frac{\partial U}{\partial r})^2] \tag{29}$$

The representation of shear stress on walls can be measured as

$$\overline{s_{rx}} = [(1 - n_e)(\frac{\partial U}{\partial r} - \frac{U}{r + R_c}) + w_e n_e (\frac{\partial U}{\partial r} - \frac{U}{r + R_c})^2]|r = h \tag{30}$$

The highest shear stress on the stenos is throat is located at x = $\frac{a}{b} + \frac{1}{n^{\frac{1}{n-1}}}$and given as

$$\overline{\tau_{rx}} = \overline{s_{rx}} \,|\, h = 1 - \delta \tag{31}$$

Finally the expression for λ and s_{rx} can be defined as follows:

$$\overline{s_{rx}} = [(1 - n_e)(\frac{\partial U}{\partial r} - \frac{U}{r + R_c}) + w_e n_e (\frac{\partial U}{\partial r} - \frac{U}{r + R_c})^2]|r = h \tag{32}$$

$$\lambda = \{(1 - \frac{b}{L})(-\frac{F - w_e c_6}{c_5 F}|\, h = 1 + \frac{1}{L}\int_a^{a+b} k(x)dx\} \tag{33}$$

in which

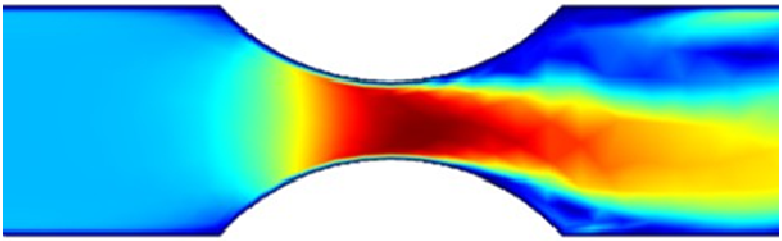
$$\lambda = \frac{\bar{\lambda}}{\lambda_0},\ s_{rx} = \frac{\overline{s_{rx}}}{\tau_0},\ \tau_{rx} = \frac{\overline{\tau_{rx}}}{\tau_0},\ \lambda_0 = L,\ \tau_0 = F \tag{34}$$

Velocity Flow Field and Streamlines

In this horizontal flow channel, blood would flow from left and would enter with a parabolic velocity profile resulted by a pressure gradient at the outlet as well as the inlet. As the fluid velocity retains much variety in the stenosis region, it becomes highest at the stenosis throat (high-velocity gradient). In this study, the flow velocity field at time t = 1s, and the resultant streamlines have been

displayed in the below Fig. In this research, the researchers are interested in the region where the flow separates after stenosis as in the recirculation zone, there is a high particle density.

Time=1s Surface Velocity Magnitude (m/s)



Time=1s Streamline Velocity Field (Spatial)

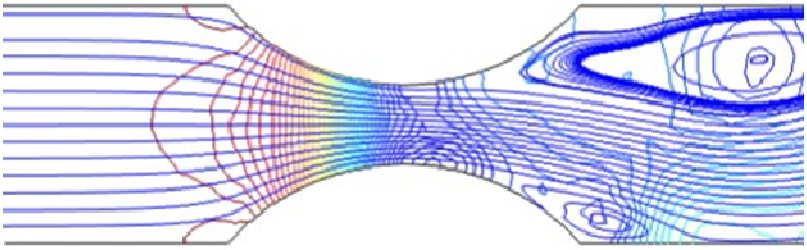


Fig 2. Blood flow field (first one) and the streamlines (latter one) at last time frame

Pressure Contours: Following figure shows the pressure contours for the fluid domain. The study concludes that as it crosses the stenotic region with a pressure profile at parabolic form, the increase of blood pressure is gradual in a normal artery. Simultaneously, there is a reduction in pressure at the next section of artery, while there is a continuous change in pressure profile.

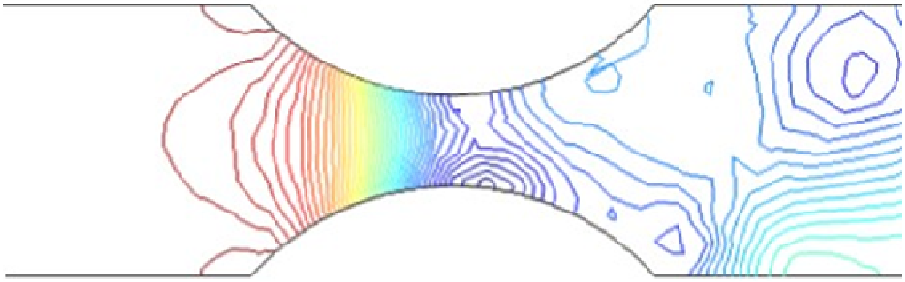


Fig 3. Pressure contour

Velocity Plot Group: The following figures (Fig) illustrate the plot groups for blood velocity profile with respect to the diameter of the blood vessel.

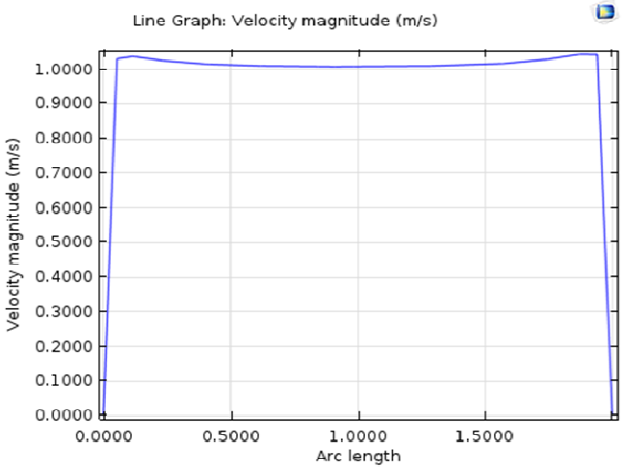


Fig 4. Velocity plot Group

Pressure Contours: Following figure shows the pressure contours for the fluid domain. The study concludes that as it crosses the stenotic region with a pressure profile at parabolic form, the increase of blood pressure is gradual in a normal artery. Simultaneously, there is a reduction in pressure at the next section of artery, while there is a continuous change in pressure profile.

Velocity Plot Group: The following figures (Fig) illustrate the plot groups for blood velocity profile with respect to the diameter of the blood vessel.

Conclusion

Thus, after analyzing the results, the study concludes that in the most occulted region; the peak velocity of the blood is at its supreme. Further, with an increase in severity in stenosis also leads to an increase in pressure profile at a delicate occluded region by stenotic region. As the wall shear stress is very high in this region, the flow pattern changes with the flow of the blood. With the core study of the atherosclerosis, ICAs, as well as an improvement in clinical solutions, such as medical advice, advanced surgical solutions, and treatment methods, it is possible to bring a reduction in mortality and morbidity linked to diseases.

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