Available online at http://www.journalcra.com

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

International Journal of Current Research
Vol. 12, Issue, 08, pp.13359-13361, August, 2020

DOI: https://doi.org/10.24941/ijcr.39533.08.2020

ISSN: 0975-833X

RESEARCH ARTICLE

QUASI M NORMAL OPERATORS LINEAR OPERATORS ON HILBERT SPACE FOR WHICH T + T* AND T*T + T T* COMMUTE, WHERE T* STANDS FOR ADJIONT OF T

[1]Dr. Bakshi Om Prakash Sinha, [2]Dr. Narendra Prasad and [3]Prof. Dudheshwar Mahto

[1]Department of Physics, Ramgarh College, Ramgarh Cantt (Jharkhand)
[2]Department of Mathematics, Ramgarh College, Ramgarh Cantt (Jharkhand)
[3]Department of Mathematics, Maharshi Paramhansh college of education, Ramgarh(Jharkhand)

ARTICLE INFO

Article History:
Received 05th May, 2020
Received in revised form
27th June, 2020
Accepted 14th July, 2020
Published online 30th August, 2020

Key Words:
Self Adjoint Operator, Normal Operator, Quansi Normal Operator, bi-Normal Operator.

ABSTRACT

In this paper we have defined a new class of operators T on a Hilbert space H for which T +T* and T*T + T T*commute where T* stands for adjoint of T. This operator will be called quasi M normal

Citation: ***Dr. Bakshi Om Prakash Sinha, Dr. Narendra Prasad and Prof. Dudheshwar Mahto.*** **2020.** "Quasi M Normal Operators Linear Operators on Hilbert Space for which T + T* and T*T + T T* Commute, where T* stands for adjiont of T", *International Journal of Current Research*, 12, (08), 13359-13361.

INTRODUCTION

Definition and Notation

•If T_1 & T_2 be two operators on Hilbert space H then we define

$[T_1, T_2] = T_1 T_2 - T_2 T_1$

We say that T_1 and T_2 commute if $[T_1, T_2] = 0$
Ie iff $T_1 T_2 - T_2 T_1 = 0$ ie $T_1 T_2 = T_2 T_1$

- Self adjoint operator:- We say that an operator T on a Hilbert space H is self adjoint if T = T*
- Normal operator:- An operator T on a Hilbert space H is called a normal operator if T T* = T* T
- Quasi Normal Operator:- An Operator T on a Hilbert space H is said to be quasi normal operator if

[T, T* T] = 0 ie T T* T = T* T T

- Bi normal Operator:- An operator T on a Hilbert space H is said to be a binormal operator if T T* and T* T commute ie [T T*, T* T] = 0

ie T* T T T* = T T* T* T

- Quasi M normal Operator:- An operator T on a Hilbert space H is said to be a quasi M normal operator if T + T* and T* T + T T* commute. Where T* stands for adjoint of T

Ie [T + T*, T* T + T T*] = 0
Ie [T + T*][T* T + T T*] = [T* T + T T*][T + T*]
Ie T T* T + T* T*T + T* T T* + T T T* = T* T T + T*T T*+T* T T* + T T* T*
Ie T* T* T + T T T* = T* T T + T T* T*

Theorem 1:- If T is a quasi M normal operator and ʎ be any scalar which is real than ʎT is also a quasi M normal operator.

Proof:- Since T is a quasi M normal operator, therefore

$$[T + T^*, T^*T + T T^*] = 0 \qquad (1)$$

Now if ʎ be a real number, then

$$(ʎT)^* = \bar{ʎ} T^* = ʎT^* \qquad (2)$$

$\{ \lambda T +)\lambda T)^*] [ )\lambda T)^*)\lambda T) +)\lambda T))\lambda T)^*\} = \lambda^3 \{T + T^*\} \{ T^*T + T T^*\}$ (3)

$[\{)\lambda T)^*)\lambda T) +)\lambda T)\})\lambda T)^*\} \{\lambda T + (\lambda T)^*\}] = \lambda^3 \{ T^* T + T T^*\}\{T + T^*\}$ (4)

By equation (1), (3) & (4) we see that T is a quasi M normal Operator.

Theorem 2:- Since T is a self adjoint operator on a Hilbert space H then T is a quasi M normal operator on H.

Proof:- Since T is a self adjoint operator, then T =T* (1)

$\{T + T^*\} \{T^* T + T T^*\} = \{ T + T \} \{ T T + T T \} = 4T^3$ (2)

$\{ T^*T + T T^*\} \{ T + T^*\} = \{ T T + T T \} \{ T + T \} = 4T^3$ (3)

By equation (1), (2) & (3) T is a quasi M normal operator.

Theorem 3:- Since T is a quasi M normal operator, then so is T*

Given that T is a quasi M normal operator.

So, $)T + T^*))T^*T + T T^*) =)T^* T + T T^*))T + T^*)$ (1)

Substituting T* for T in (1),

$L H S = \{ T^* +)T^*)^*\} \{)T^*)^* T^* + T^*)T^*)^*\} = \{T^* + T \} \{ T T^* + T^*T \}$ (2)

Since $)T^*)^* = T$

$R H S = \{)T^*)^* T + T^*)T^*)^*\} \{ T^* +)T^*)^*\} = \{T T^* + T^*T \} \{T^* + T \}$ (3)

By equation (2) & (3), T* is quasi M normal.

Theorem 4:- Let T be any operator on Hilbert space H.
Now consider the following

$N_1 = T + T^*$; $N_2 = T T^*$; $N_3 = T^*T$; $N_4 = T + T^* + T^*T + T T^*$

Then N_1, N_2, N_3 & N_4 are quasi M normal operator on H.

Proof:- Here $N_1 = T + T^*$

$N_1^* = \{ T = T^* \}^* = T^* +)T^*)^* = T^* + T = N_1$
$N_2^* = \{ T T^* \}^* =)T^*)^* T^* = T T^* = N_2$
$N_3^* = \{ T^* T \}^* = T^*)T^*)^* = T^* T = N_3$
$N_4^* = \{ T + T^* + T^* T + T T^*\}^* = T^* +)T^*)^* + T^* (T^*)^* +)T^*)^* T^* = T^* + T + T^* T + T T^* = N_4$

So, N_1, N_2, N_3 & N_4 are self adjoint operator. But every self adjoint operator is a quasi M normal operator, so N_1, N_2, N_3 & N_4 are quasi M normal operator.

Cor(i) :- The zero operator 0 and identity operator I are quasi M normal operator.

Since 0* = 0; I* = I

0 & I are self adjoint operator so they are quasi M normal operator.

Cor (ii):- Let be any operator on Hilbert space H, then I + T* T and I+ T T* are quasi M normal operator

As $\{ I + T^* T\}^* = I^* + T^*)T^*)^* = I + T^* T$
$\{ I + T T^* \} = I^* +)T^*)^* T^* = I + T T^*$

Theorem 5:- If T be an unitary operator on a Hilbert space H, then T is a quasi M normal operator.

Proof:- Since T is an unitary operator on Hilbert space H, therefore T* T = T T* = I

Now $)T + T^*))T^* T + T T^*) =)T + T^*))I + I) = 2)T + T^*) I = 2)T I + T^* I) = 2)T + T^*)$--- (1)

Similarly, $)T^* T + T T^*))T + T^*) =)I + I))T + T^*) = 2)I T + I T^*) = 2)T + T^*)$ ------ (2)

By equation (1) &(2) T is quasi M normal operator.

Theorem 6:- Let T be a self adjoint operator on a Hilbert space H and S be any operator on H, then

S* T S is a quasi M normal operator.

Proof:- Since T is self adjoint operator therefore T* =T

Now, $)S^* T S) = S^* T^*)S^*) = S^* T S$

As, S* T S is self adjoint operator and every self adjoint operator is quasi M normal operator

So, S* T S is a quasi M normal operator.

Theorem 7:- The set L of all quasi M normal operators on a Hilbert space H form a closed subset of B)H) and contains the set of all self adjoint operators and unitary operators on H. B)H) is a class of all operator on H.

Proof:- Let L be the set of all quasi M normal operator on a Hilbert space H. We shall show that L is a closed subset of B)H). Let T be the limit point of L. Then there exists a sequence of quasi normal operator $\{ T_n \}$, such that $T_n \to T$ as $n \to \infty$. We have to show that T belongs to L ie T is a quasi M normal operator.

Now, $\|)T + T^*))T^*T + + T T^*) –)T^* T + T T^*))T + T^*) \|$
$= \|)T + T^*))T^*T + + T T^*) –)T_n + T_n^*))T_n^*T_n + T_n T_n^*) +)T_n + T_n^*))T_n^*T_n + T_n T_n^*)$
$-)T_n^*T_n + T_n T_n^*))T_n + T_n^*) +)T_n^*T_n + T_n T_n^*))T_n + T_n^*)$
$–)T_n^*T_n + T_n T_n^*))T_n + T_n^*) \|$
$\leq \|)T + T^*))T^*T + + T T^*) -)T_n + T_n^*))T_n^*T_n + T_n T_n^*) \| +$
$\|)T_n + T_n^*))T_n^*T_n + T_n T_n^*) –)T_n^*T_n + T_n T_n^*))T_n + T_n^*) \|$
$+$
$\|)T_n^*T_n + T_n T_n^*))T_n + T_n^*) -)T^*T + + T T^*))T + T^*) \|$
$\to 0$ as $n \to \infty$

Which shows that $\|)T + T^*))T^*T + + T T^*) –)T^* T + T T^*))T + T^*) \| = 0$

- T is a quasi M normal operator.
- T belong to L

Thus every limit point of L belong to L. So L is a closed subset of B (H).

Since every self adjoind operator and unitary operator are quasi M normal operator. So, L contain the set of all self adjoint and unitary operators.

Theorem 8:- If T 1 and T2 be two quasi M normal operators such that each is the adjoint of other, then

$T_1 + T_2$ and T_1T_2 are quasi M normal operator.

Here, $T_1^* = T_2$ and $T_2^* = T_1$, since T_1 and T_2 are quasi M normal operator, therefore

$(T_1 + T_1^*)$)$T_1^*T_1 + T_1 T_1^*$) =)$T_1^*T_1 + T_1 T_1^*$))$T_1 + T_1^*$)
)$T_2 + T_2^*$))$T_2^*T_2 + T_2 T_2^*$) =)$T_2^*T_2 + T_2 T_2^*$))$T_2 + T_2^*$)
Now,)$T_1 + T_2$))$T_1 + T_2$)*)$T_1 + T_2$) +)$T_1 + T_2$))$T_1 + T_2$))$T_1 + T_2$)*
=)$T_1^* + T_2^*$))$T_1^* + T_2^*$)*)$T_1 + T_2$) +)$T_1 + T_2$))$T_1 + T_2$))$T_1 + T_2$)*
=)$T_2 + T_1$))$T_2 + T_1$))$T_1 + T_2$) +)$T_1 + T_2$))$T_1 + T_2$))$T_2 + T_1$) ------ ----------- (1)

Similarly,)$T_1 + T_2$)*)$T_1 + T_2$))$T_1 + T_2$) +)$T_1 + T_2$))$T_1 + T_2$)*)$T_1 + T_2$) = 2)$T_1 + T_2$)^3 ------- -------- (2)

By equation (1) & (2) $T_1 + T_2$ is quasi M normal operator.

Again,)T_1T_2)*)T_1T_2)*)T_1T_2) +)T_1T_2))T_1T_2))T_1T_2)*
=)T_2T_1)*)$T_2^*T_1^*$))T_1T_2) +)T_1T_2))T_1T_2))T2*T_1^*)
=)T_1T_2))T_1T_2))T_1T_2) +)T_1T_2))T_1T_2))T_1T_2) = 2)$T_1 T_2$)^3 ------- ---------- (3)
Similarly,)T_1T_2)*)T_1T_2))T_1T_2) +)T_1T_2))T_1T_2)*)T_1T_2)* = 2)$T_1 T_2$)^3 ------- -------- ----- (4)

By equations (3) & (4) we see that $T_1 T_2$ is a quasi M normal operator.

Theorem 9:- If T = UP be a polar decomposition of an operator, where the null space of P and U is a unitary operator. Then

[T + T*, T* T + T T*] =0 [U P + P U*, P^2 + UP^2U] =0
Here T = U P
T* =)U P)* = P* U* = P U*
Hence N)U) = N)P)
Now, [T + T*, T*T + T T*] = 0
[U P + P U*, P U*U P + U P P U*] = 0
[U P + P U*, P.P + U P* U*] = 0
[U P + P U*, P^2 + U P*U*] = 0
[U U* = U U* = T]T

Theorem 10:- Let S be self adjoint operator on a Hilbert space H and T be quasi M normal operator on H Such that S T = T S, then S T is a quasi M normal operator.

Proof:- Since S is a selfadjoint operator, therefore S* = S
Now, S T = T S

•)S T)* =)T S)*
•T* S* = S* T*
•T* S = S T* (1)

Since, T is a quasi M normal operator, so

)T + T*))T* T + T T*) =)T* T + T T*))T + T*)
ie T T* T + T T T* + T* T* T + T* T T* = T* T T + T T* T + T* T T* + T T*T*
ie T T T* + T* T* T = T* T T + T T*T* ------- ----- (2)
Now,)S T))S T))S T)* +)S T)*)S T)*)S T)
=)S T))S T))T* S*) +)T* S*))T* S*))S T)
=)S T))S T))T* S) +)T* S))T* S))S T)
=)S T))S T))S T*) +)S T*))S T*))S T)
= S^3)T T T* + T* T* T) (3)
Similarly,)S T)*)S T))S T) +)S T))S T)*)S T)*
= S^3)T* T T + T T* T*) (4)

By equation (2), (3) & (4), we find that S T is a quasi M normal operator.

Theorem 11:- Let T = R + i S be any operator on a Hilbert space H, where R S = S R

Then T is quasi M normal operator if R S S = S S R
Proof:- Here T = R + i S
Therefore T* = R – i S
T T* =)R + i S))R – i S) = R R + S S + i)S R – R S)
ie T T* = R R + S S
Similarly, T*T =)R – i S))R + i S)
Now,)T + T*))T* T + T T*) = {)R + i S) +)R – i S) } {2 R R + 2 S S }
= 2 R)2 R R + 2 S S) = 4)R R R + R S S) (1)

Similarly,)T* T + T T*))T + T*)= 4)R R R + S S R) (2)
We find that)T + T*))T* T + T T*) =)T* T + T T*))T + T*)
Hence, if S S R = R S S
Therefore T is quasi M normal if S S R = R S S
----------- * --------- * ---------

REFERENCES

Berberian, S.K. 1974. Lectures in Functional Analysis and operator theory : Springer verlag NEW YORK.

Bhattacharya D. & Prasad N. 2012. Quasi P Normal operators ultra scientist of Physical science Vol24(2), A pp 269 – 272, Bhopal.

Halmus, P.R. 1967. A Hilbert space problem book, Princeton, N.J.: Van Nostrand

Maddox I. J. 1970. Elements of functional analysis, Cambridge University Press, Cambridge.

Rudin W. 1973. Functional Analysis NEW YORK Mc Graw Hill.

Simmons, G.F. 1963. Introduction to topology and modern analysis, NEW YORK; Mc Graw – Hill.

Taylor A. E. 1958. Introduction to Functional Analysis, new york Walley.

****