ISSN: 0975-833X

Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 14, Issue, 12, pp.22955-22958, December, 2022
DOI: https://doi.org/10.24941/ijcr.44412.12.2022

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

RESEARCH ARTICLE

COMPARISON OF LINEAR AND ANGULAR MEASUREMENTS ACCURACY IN MIXED GEODETIC NETWORKS

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ARTICLE INFO

Article History:
Received 08th September, 2022
Received in revised form
14th October, 2022
Accepted 19th November, 2022
Published online 27th December, 2022

Key words:

Linear-Angular Network, Linear and Angular Precision.

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ABSTRACT

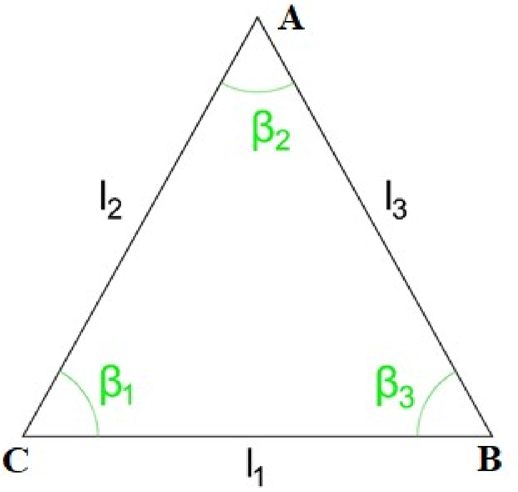
The installation of a linear-angular geodetic network requires the realization of linear and angular measurements contrary to the other methods (triangulation or multilateration) for which it is necessary to measure either the angles or the distances. The question then arises of the relationship between the accuracies of linear measurements and those of angular measurements. It will be a question of comparing the deviations of the measurements carried out according to the arrangement of the points of the network. The experiments carried out on different shapes of figures (disposition of the points of the network) make it possible to formulate the following conclusion: In the linear-angular geodetic networks, in order to obtain linear and angular measurements equivalent in precision, it is necessary that for a given error on the angular measurements, linear measurements give the same relative error.

Citation: ***Dr. YESSOUFOU Moukadamou Joslin, Dr. KOSSOUGBETO Briac Kévin Patrick et al.*** **2022.** "Comparison of linear and angular measurements accuracy in mixed geodetic networks*.". International Journal of Current Research*, 14, (12), 22955-22958.

INTRODUCTION

While keeping its impregnation in various fields of daily life (military, scientific, etc.) geodesy represents an essential tool for the establishment of economic infrastructure through increasingly precise and detailed cartographic documents, obtained thanks to the establishment, by land or space techniques, of a framework of support points. Geodetic markers are an essential basis for any application using geo-referenced data. The evolution of technology has contributed to the creation of various methods for easily and quickly performing a large volume of work. But the main goal sought is to achieve a homogeneous and precise geodesic network that can serve as a basis for various studies of a utilitarian and scientific nature. The classic observation techniques used in the context of the creation of national or regional geodetic networks are terrestrial and astronomical measurements (angles, distances and azimuths) allowing orientation and scaling of the network. These networks presenting certain deformations in orientation and in scale which are highlighted by modern techniques, require certain precautions in the construction of a local geodetic system (planimetric) resulting from terrestrial observations. This means that it is necessary to define and implement a process of adjustment of a classic geodetic network in order to establish an acceptable quality in the geodetic system and to ensure a homogeneous precision on the whole of the canvas (Bachir Gourine, 2003). It has always been preferable to make angular and linear measurements when determining the coordinates of points, buildings or the like. This method is preferred since the precision of the results of the joint adjustment of linear and angular measurements is considered higher than that obtained when adjusting either linear measurements alone or angular measurements alone. However, studies have shown that it is appropriate to carry out the common adjustment of linear and angular measurements only if the ratio of the precisions of these measurements respects given conditions (criteria). This then shows the need to assess the accuracy of these two categories of measurements. For example, in a polygonal traverse, increasing the number of sides decreases the precision required for linear measurements while it increases for angular measurements (Avakyan, 2016). According to the sine theorem, when measurements are made without errors, the ratios between the sines of the angles and the sides opposite these angles are equal in a triangle. It should be noted that the lengths of the three sides make it possible to determine all the other elements of the triangle, namely the perimeter, the area, the angles while differentiating it from the multitude of triangles similar to the one in question. But based on the angles (the sines) we get a multitude of similar triangles. In this case, a scaling operation is therefore necessary in order to conform the result to the specific triangle in question.

In order to limit the effect of errors or weather conditions as much as possible, it is sometimes wise to proceed with the calculation and adjustment based not on the results of measurements but on pure values (without dimension or unit). But these values come from the ratios between the different measurement results (linear, trigonometric functions or others). These values, which represent coefficients, offer advantages compared to the direct use of measurements, such as the automatic elimination of systematic errors (Gubaydullina, 2020; Marcuse, 2016). For a better illustration let us establish the relations on the basis of two triangles in which we have measured all the angles in one and all the distances in the other.



Picture 1. Case 1 : Triangle whose angles have been measured (β_1, β_2 *et* β_3). Case 2: Triangle whose side lengths have been measured (l_1, l_2 *et* l_3)

Let us determine the ratio ω between the sines of the angles β_2 *and* β_3 with respect to β_1 in case 1 and Δ the ratio between the lengths of sides l_2 *and* l_3 with respect to l_1 in case 2.

$$\omega_1 = 1\,;\quad \omega_2 = \frac{\sin\beta_2}{\sin\beta_1}\,;\quad \omega_3 = \frac{\sin\beta_3}{\sin\beta_1} \tag{1}$$

$$\Delta_1 = 1\,;\quad \Delta_2 = \frac{l_2}{l_1}\,;\quad \Delta_3 = \frac{l_3}{l_1}. \tag{2}$$

Since the results of the measurements are marred by errors, we can affirm that the relationships established between the elements of the triangle are also marred by a given margin of error. The expression of the deviation corresponding to each ratio can be established based on the error propagation law. For triangle 1:

$$m_{\omega_2} = \pm\sqrt{\left(\frac{\cos{}_2}{\sin\beta_1}\right)^2 . m_\beta^2 + \left(\frac{\cos\beta_1 . \sin\beta_2}{(\sin\beta_1)^2}\right)^2 . m_\beta^2}\ ;$$

$$m_{\omega_3} = \pm\sqrt{\left(\frac{\cos\beta_2}{\sin\beta_1}\right)^2 . m_\beta^2 + \left(\frac{\cos\beta_1 . \sin\beta_2}{(\sin\beta_1)^2}\right)^2 . m_\beta^2}$$

$$m_{\omega_2} = \pm\sqrt{[(\sin\beta_1)^2 . (\cos\beta_2)^2 + (\cos\beta_1)^2 . (\sin\beta_2)^2] . \frac{m_\beta^2}{(\sin\beta_1)^4}}$$

$$m_{\omega_3} = \pm\sqrt{[(\sin\beta_1)^2 . (\cos\beta_3)^2 + (\cos\beta_1)^2 . (\sin\beta_3)^2] . \frac{m_\beta^2}{(\sin\beta_1)^4}}$$

$$m_{\omega_2} = \pm\sqrt{[(\cos\beta_2)^2 + (\cot\beta_1)^2 . (\sin\beta_2)^2] . \frac{m_\beta^2}{(\sin\beta_1)^4}}$$

$$m_{\omega_3} = \pm\sqrt{[(\cos\beta_3)^2 + (\cot\beta_1)^2 . (\sin\beta_3)^2] . \frac{m_\beta^2}{(\sin\beta_1)^4}}$$

$$m_{\omega_2} = \pm\sqrt{(\cos\beta_2)^2 + (\cot\beta_1)^2 . (\sin\beta_2)^2} . \left[\frac{m_\beta}{(\sin\beta_1)^2}\right]\ ;$$

$$m_{\omega_3} = \pm\sqrt{(\cos\beta_3)^2 + (\cot\beta_1)^2 . (\sin\beta_3)^2} . \left[\frac{m_\beta}{(\sin\beta_1)^2}\right]$$

$$m_{\omega_2} = \pm\sqrt{(\sin(\beta_1 + \beta_2))^2 - \frac{1}{2}\sin 2\beta_2 . \sin 2\beta_1} . \left[\frac{m_\beta}{(\sin\beta_1)^2}\right]\ ;$$

$$m_{\omega_3} = \pm\sqrt{(\sin(\beta_1 + \beta_3))^2 - \frac{1}{2}\sin 2\beta_3 . \sin 2\beta_1} . \left[\frac{m_\beta}{(\sin\beta_1)^2}\right]. \tag{3}$$

For triangle 2:

$$m_{\Delta_2} = \pm\sqrt{\left(\frac{1}{l_1}\right)^2 .m_l^2 + \left(-\frac{l_2}{l_1^2}\right)^2 .m_l^2}\ ;$$

$$m_{\Delta_3} = \pm\sqrt{\left(\frac{1}{l_1}\right)^2 .m_l^2 + \left(-\frac{l_3}{l_1^2}\right)^2 .m_l^2}$$

$$m_{\Delta_2} = \pm\sqrt{\frac{l_1^2+l_2^2}{l_1^4}.m_l^2}\ ;$$

$$m_{\Delta_3} = \pm\sqrt{\frac{l_1^2+l_3^2}{l_1^4}.m_l^2}$$

$$m_{\Delta_2} = \pm\frac{\sqrt{l_1^2+l_2^2}}{l_1^2}.m_l;$$

$$m_{\Delta_3} = \pm\frac{\sqrt{l_1^2+l_3^2}}{l_1^2}.m_l.$$

$$m_{\Delta_2} = \pm\sqrt{l_1^2+l_2^2}.\left[\frac{m_l}{l_1^2}\right];$$

$$m_{\Delta_3} = \pm\sqrt{l_1^2+l_3^2}.\left[\frac{m_l}{l_1^2}\right]. \qquad (4)$$

Based on one of the triangles whose elements have been measured under the same conditions, therefore having equal precision. For an equilateral triangle of expressions (3) and (4) we obtain the following equalities.

$$m_\omega = \pm\sqrt{(\sin(\beta+\beta))^2 - \frac{1}{2}\sin 2\beta\,.\sin 2\beta}\,.\left[\frac{m_\beta}{(\sin\beta)^2}\right]\ ;$$

$$m_\omega = \pm\sqrt{(\sin 2\beta)^2 - \frac{1}{2}(\sin 2\beta)^2}.\left[\frac{m_\beta}{(\sin\beta)^2}\right]$$

$$m_\omega = \pm\frac{\sqrt{(\sin 2\beta)^2.\left(1-\frac{1}{2}\right)}}{(\sin\beta)^2}.m_\beta\ ;$$

With $\beta = 60°$ we have :

$$m_\omega = \pm\frac{2\sqrt{3}}{3\sqrt{2}}.m_\beta\ ; \qquad (5)$$

Where the angular deviation is given in radians

For triangle 2 :

$$m_\Delta = \pm\frac{\sqrt{2l^2}}{l^2}.m_l$$

$$m_\Delta = \pm\frac{m_l}{l}.\sqrt{2} \qquad (6)$$

Expression (6) shows that the deviation of the ratio between the distances does not depend on the absolute error, but rather on the relative error of the measurement.

Since expressions (5) and (6) were obtained based on an equilateral triangle, they can be equated to establish the relationship between the accuracy of linear measurements and angular measurements in an equilateral triangle.

$$\frac{2\sqrt{3}}{3\sqrt{2}}.m_\beta = \frac{m_l}{l}.\sqrt{2} \qquad (7)$$

But in general when fitting a mixed network, the best match between angular and linear deviations is equality between the absolute values of the transverse and longitudinal closure deviations$|m_{longitudinale}| = |m_{transversale}|$. (2)This is explained by:

$$\frac{m_\beta}{\rho} = \frac{m_l}{l} \qquad (8)$$

The analysis (2, 5, 6) of this expression shows that this ratio for $\frac{m_\beta}{\rho} * \frac{l}{m_l} \leq \frac{1}{3}$, the linear measurements do not improve the precision of the elements of the network; and for $\frac{m_\beta}{\rho} * \frac{l}{m_l} \geq 3$, the angular measurements have no significant effect on the accuracy of the array elements. It is therefore recommended to keep it within the following limits:

$$\frac{1}{3} < \frac{m_\beta}{\rho} * \frac{l}{m_l} < 3 \qquad (9)$$

When creating mixed geodetic networks based on linear and angular measurements, it is necessary to have a correspondence between these two categories of measurements. For a better adjustment solution, it is advisable to have an equivalence between the precision of the linear measurements and that of the angular measurements. This equivalence results in the equality between the absolute error of angular measurements and the relative error of linear measurements. Within a given interval, this correspondence is considered acceptable since the various elements involved in the adjustment do not negatively influence the results. However, it should be noted that the inequality (non-correspondence) between the precision of the linear and angular measurements leads to analyzes making it possible to choose the most suitable adjustment method.

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