



RESEARCH ARTICLE

TWENTY-FOUR POINTS CALCULUS OPTIMUM VALUE ROTATABLE DESIGN OF SECOND ORDER USING RESPONSE SURFACE METHODOLOGY

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ABSTRACT

Response Surface Methodology (RSM) is a collection of mathematical and statistical techniques designed for modeling and analyzing problems where a response of interest is influenced by several variables, with the ultimate goal of optimizing the response. This study applied RSM to model the yield of *rose coco* beans (*Phaseolus vulgaris*) using an existing A-optimum and D-efficient second-order rotatable design of twenty-four points in three dimensions under greenhouse conditions. The experimental factors were three inorganic fertilizers: nitrogen, phosphorus, and potassium. The design was implemented using the calculus optimum value of the free/letter parameter $f=1.1072569$, with model parameters estimated via least squares techniques. This provided, for the first time, yield response estimates for *rose coco* beans at the calculus optimum value design. Results indicated that nitrogen, phosphorus, and potassium significantly influenced bean yield ($p<0.05$). In the GP2G design, the second-order model was statistically adequate at the 1% significance level ($p=0.0065$). The analysis of variance (ANOVA) confirmed the adequacy of the model, supported by a coefficient of determination ($R^2=0.6704$) and a coefficient of variation (CV) of 14.47. The findings demonstrated the utility of statistical methods for optimal and efficient *rose coco* bean production. It is recommended that a randomized screening of fertilizer components be undertaken to establish the most effective initial levels of each fertilizer for achieving maximum yield.

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INTRODUCTION

This study investigated the impact of varying levels of inorganic fertilizers treatments on the yield of *Rose coco* beans (*Phaseolus vulgaris*) using a second-order rotatable design. The experiment was conducted in a 15 m × 10 m greenhouse between February and July 2016, applying a twenty four points second-order rotatable design in three dimensions [11]. The goal was to optimize and enhance the production efficiency of *rose coco* beans by applying Response Surface Methodology (RSM) through an existing A-optimal and D-efficient experimental design. The research utilized a response surface methodology (RSM) framework, specifically adopting one of the six known second-order rotatable designs in three dimensions. The design, based on previous work by Mutiso [9] and Koech [7], was selected. Mutiso calculated a calculus optimum value of 1.1072569 for the free parameter in one of the existing six specific three-dimensional second-order rotatable designs. Koech further evaluated the relative efficiencies of these designs and concluded that the twenty four points configuration was both the most D-efficient and A-optimal. Based on these findings, this study focused on the twenty four points design to practically examine its effectiveness in optimizing *rose coco* bean yield under controlled greenhouse conditions.

The primary objective was to achieve optimal and efficient production of *Rose Coco* beans through the application of an existing A-optimal and D-efficient second-order rotatable design. This design involved the use of three independent fertilizer treatment factors—nitrogen (N), phosphorus (P), and potassium (K)—and aimed at estimating linear and interaction effects on bean yield within the specified design space. The regression model expressed *Rose Coco* bean yield as a continuous function of the application rates of nitrogen, phosphorus, and potassium. Importantly, the study demonstrated that deviations from the optimum fertilizer levels—whether higher or lower—could lead to a decline in yield. By implementing this RSM approach, the research sought to determine the most effective combination and rate of fertilizer application to maximize yield, especially in contexts where resources are limited. The broader goal of this research was to apply RSM techniques in practical agricultural settings to optimize resource use and maximize yield. By identifying the most effective treatment combinations, this study contributes to improving existing fertilizer application strategies—particularly under resource constraints common in small-scale farming. Ultimately, the results aim to support more efficient and sustainable *Rose Coco* bean production by determining the best possible fertilizer application levels.

METHODOLOGY

The response surface methodology (RSM) is a collection of mathematical and statistical techniques useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response [8]. In most RSM applications, the precise functional relationship between the response and the independent variables is initially unknown. Therefore, the first step in applying RSM involves constructing an appropriate approximation of this relationship, typically through a polynomial model

To ensure efficient and accurate approximation, it is essential to employ a suitable experimental design for data collection. A commonly used representation in RSM is the linear model: $y = x\beta + \epsilon$, where y is a vector of observed responses, x is the design matrix (comprising combinations of coded variable levels), β is a vector of unknown model coefficients and ϵ is the vector of errors. The design matrix was a set of combinations of the values of the coded variables, which specifies the settings of the design parameters to be performed during experimentation.

The parameters β are typically estimated using the least squares method: $\beta = (x'x)^{-1}x'y$

In this study, we focus on statistical modeling to develop an appropriate response surface model that describes the relationship between the response variable y and three independent variables: x_1 (nitrogen), x_2 (phosphorus), x_3 (potassium). The second-order (quadratic) model is deemed necessary to capture potential curvature in the response surface. For three variables, the general form of the second-order model is expressed as:

$$\hat{y} = \beta_0 + \sum_i^k \beta_i x_i + \sum_i^k \beta_{ii} x_i^2 + \sum_{i < j=2}^k \sum_j^k \beta_{ij} x_i x_j \quad (1)$$

where $\hat{y}$ is the measured *rose coco* response, β_0 is the intercept term, β_i are the linear coefficients, β_{ij} is the logarithmic coefficient, β_{ii} are the quadratic coefficients and x_{iu} denotes the coded level of the i^{th} factor ($i=1,2,3$) in the u^{th} run ($u=1,2,\dots,24$) of the experiment. The parameters of the model $\beta_0, \beta_i, \beta_{ii}, \beta_{ij}$ are estimated by least squares estimation to provide $\hat{\beta}_0, \hat{\beta}_i, \hat{\beta}_{ii}, \hat{\beta}_{ij}$. It is said to be SORD if the variance of the estimate

of the response $\hat{y}_u$ is only a function of the distance $\left(d^2 = \sum_{i=1}^k x_i^2\right)$ of the point $(x_1, x_2, \dots, x_k)$ from the origin (centre) of the design [6]. This

model would likely be useful as an approximation to the true response surface in a relatively small region. The low and high factor settings are coded as negative and positive, the midpoint coded as 0.

Experiment Layout of Twenty Four Points Calculus Optimum Value Design: *Rose coco* bean plants were cultivated in a greenhouse setting using a spacing of 75 cm $\times$ 30 cm. Inorganic fertilizers—nitrogen (N), phosphorus (P), and potassium (K)—were applied only once at planting, with no additional supplementation through top dressing or foliar spray thereafter. The experiment followed a twenty-four-point second-order rotatable design to evaluate the effects of varying levels of N, P, and K on crop performance. The greenhouse experiment, designated as GP2G (Group 2 Greenhouse Experiment), included three replications: GP2GA, GP2GB, and GP2GC. Each replication consisted of twenty-four experimental units (plants), making up the full design layout. The center point of the design consisted of a fertilizer combination of 20 grams of nitrogen, 30 grams of phosphorus, and 40 grams of potassium, which was applied uniformly across all center-point plots

Each replication received direct application of N, P, and K fertilizers in accordance with the design matrix. Prior to planting, the greenhouse soil surface was cleared of organic debris, followed by land preparation involving jembe ploughing and harrowing until a fine tilth was achieved.

Certified, viable, and uniform *rose coco* bean seeds were sown in February 2016 as a pure stand in the greenhouse. Prior to planting, seeds were treated with Aldrin at a rate of 5 g per kg of seed to control soil-borne pests, particularly bean fly (*Melanargromyza phaseoli*). Additionally, Furadan (5% carbofuran) was applied along the planting rows at sowing to manage cutworm infestations (*Agrotis ipsilon*). Drip irrigation was provided to all plots using drip line pipes. Fertilizer treatments were applied prior to sowing, and two seeds were planted per experimental unit. After one week, seedlings were thinned to one plant per unit to ensure uniformity. Weeding was performed twice: the first at two weeks after emergence, and the second four weeks later. The experimental design followed the twenty-four-point rotatable design principles as described by We consider a set of twenty four point's rotatable designs as highlighted by Draper [4], Mutiso [9] and Koske [5] is given as:

$$D_1 = \left[\frac{1}{2} G(f, f, 0) + \frac{1}{4} G(c_1, 0, 0) + \frac{1}{4} G(c_2, 0, 0) \right] \quad (2)$$

A practical greenhouse example: A second-order rotatable design (3) was implemented in a greenhouse located in Saroiyot, Kesses, Uasin Gishu County, Kenya, to evaluate the effects of three inorganic fertilizer components—nitrogen (N), phosphorus (P), and potassium (K)—on the yield of *rose coco* beans. The design consisted of 24 experimental points arranged according to a second-order rotatable configuration involving three factors. The actual quantities of fertilizer applied per planting hole were as follows: Nitrogen (N) $x_{1u}\Psi_1=20$ grams/hole; Phosphorus (P) $x_{2u}\Psi_2=30$ grams/hole; and Potassium (K) $x_{3u}\Psi_3=40$ grams/hole. The response variable of interest was the average yield of *rose coco* beans, measured in milligrams (mg) per plant.

According to Koske [5], calculations for the design parameters yielded the following values:

The calculations done by Koske [5] showed that $f=1.1072569$, $c_1=0.7829487$, $c_2=1.2735263$, hence from (2) the design D_1 yields an optimum design as:

$$D_2 = [\frac{1}{2} G(1.1072569, 1.1072569, 0) + \frac{1}{4} G(0.7829487, 0, 0) + \frac{1}{4} G(1.2735263, 0, 0)] \quad (3)$$

$$(X'X)^{-1} = \begin{bmatrix} 0.2735 & 0 & 0 & 0 & -0.1299 & -0.1299 & -0.1299 & 0 & 0 & 0 \\ 0 & 0.0700 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0700 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0700 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.1299 & 0 & 0 & 0 & 0.1282 & 0.0451 & 0.0451 & 0 & 0 & 0 \\ -0.1299 & 0 & 0 & 0 & 0.0451 & 0.1282 & 0.0451 & 0 & 0 & 0 \\ -0.1299 & 0 & 0 & 0 & 0.0451 & 0.0451 & 0.1282 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1663 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1663 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1663 \end{bmatrix} \quad (7)$$

$$(X'Y_{GP2G}) = \begin{bmatrix} 1097 \\ 20.6 \\ -10 \\ 39.3 \\ 620.8 \\ 692.1 \\ 718.7 \\ 36.8 \\ 17.2 \\ 6.1 \end{bmatrix} \quad \hat{B} = (X'X)^{-1} X'Y_{GP2G} = \begin{bmatrix} 36.1296 \\ 1.4399 \\ -0.6980 \\ 2.7503 \\ 0.6806 \\ 6.6025 \\ 8.8179 \\ 6.1174 \\ 2.8548 \\ 1.0196 \end{bmatrix} \quad (8)$$

$$Y_{GP2G} = 36.1296 + 1.4399X_1 - 0.6980X_2 + 2.7503X_3 + 0.6806X_1^2 + 6.6025X_2^2 + 8.8179X_3^2 \\ + 6.1174X_1X_2 + 2.8548X_1X_3 + 1.0196X_2X_3 \quad (9)$$

The $\hat{Y}_{GP2G}$ is the predicted response for *rose coco* beans in group 2. The regression coefficients suggest that quadratic effects of phosphorus ($p=0.0145$) and potassium ($p=0.0023$), the interaction effect N*P ($p=0.0397$) are significant factors on the *rose coco* bean, among these, K^2 is significant at the 1% significance level, while P^2 and N*P are significant at the 5% level, those other terms of the model showed no significant effect on the yield.

Table 1. 24-point's rotatable design of coded levels and natural levels with the yield of *rose coco* beans-GP2G at the ratio of 20:30:40 N: P: K fertilizers

(x _{1u})	x _{2u}	x _{3u})	Ψ _{1u}	Ψ _{2u}	Ψ _{3u}	Yield Y _i	Predicted yield- $\hat{Y}$
1.1072569	1.1072569	0	20.553628	30.332177	40	49	53.3803
-1.1072569	1.1072569	0	19.446372	30.332177	40	30	35.1916
1.1072569	-1.1072569	0	20.553628	29.667823	40	44	39.9260
-1.1072569	-1.1072569	0	19.446372	29.667823	40	55	51.7374
1.1072569	0	1.1072569	20.553628	30	41.107257	59	55.9146
-1.1072569	0	1.1072569	19.446372	30	41.107257	53	45.7258
1.1072569	0	-1.1072569	20.553628	30	38.892743	37	42.8239
-1.1072569	0	-1.1072569	19.446372	30	38.892743	45	46.6353
0	1.1072569	1.1072569	20	30.332177	41.107257	64	58.5578
0	-1.1072569	1.1072569	20	29.667823	41.107257	56	57.6034
0	1.1072569	-1.1072569	20	30.332177	38.892743	57	49.9671
0	-1.1072569	-1.1072569	20	29.667823	38.892743	54	54.0129
0.7829487	0	0	20.391474	30	40	34	37.6742
-0.7829487	0	0	19.608526	30	40	39	35.4194
0	0	0.7829487	20	30	40.782949	37	43.6884
0	0	-0.7829487	20	30	39.217051	42	39.3817
0	0.7829487	0	20	30.234885	40	47	39.6305
0	-0.7829487	0	20	29.765115	40	47	40.7235
1.2735263	0	0	20.636763	30	40	44	39.0672
-1.2735263	0	0	19.363237	30	40	30	35.3997
0	0	1.2735263	20	30	41.273526	46	53.9337
0	0	-1.2735263	20	30	38.726474	46	46.9285
0	1.2735263	0	20	30.382058	40	41	45.9491
0	-1.2735263	0	20	29.617942	40	41	47.7269

The positive coefficient of P^2 (quadratic phosphorus) K^2 (quadratic potassium), N^*P enhance the yield since they are the largest coefficients in the model equation (4.62). The X_1 represent Nitrogen (N), X_2 represent Phosphorus (P), X_3 represents Potassium (K). Table 4.20 suggests that quadratic effects of phosphorus (P^2), potassium (K^2) and interaction N^*P were the determining significant factors on the *rose coco* bean yield as these had the more or less the same coefficients and those other terms of the model showed no significant effect on the yield. The P^2 , K^2 and N^*P showed the positive coefficient, meaning all the three (quadratic phosphorus, quadratic potassium and nitrogen and phosphorus combined) enhance the *rose coco* yield.

Table 2. The ANOVA results in the fertilizer concentration on *rose coco* beans-GP2G.

	Type I	Sum				
Regression	DF	of Squares	mean sq	R-Square	F Value	Pr > F
First Order	3	144.555104	48.185	0.0778	1.10	0.3815
Pure Quadratic	3	821.441245	273.814	0.4419	6.26	0.0065
Two-Factor Interaction	3	280.250000	93.417	0.1508	2.13	0.1417
Residuals	14	612.71	43.765			
Lack of fit	14	612.71	43.765			
Pure error	0	0.00				
Total Model	9	1246.246349	0.6704	3.16	0.0264	

Coefficient of variation (CV)=14.47, coefficient determination (R^2)=0.6704, Adjusted R-squared =0.4585, correlation coefficient (r)=0.8188, root MSE=6.615523, response mean=45.708333, PRESS=2091.1898012

Table 2 is the results of the averaged data of the three replicates in GP2G. In the table, the combined quadratic terms are significant at 1% with a p-value of 0.0065 also the total model was significant at 5% with a p value of 0.0264. We tested the first order model it was insignificant ($p=0.3815$). Linear and cross product terms were not significant, which corresponds to the fact that the quadratic model for GP2G was appropriate. Therefore, the analysis of variance (F-test) showed that the second order model fits well with the experimental data in GP2G. The R^2 of quadratic equation was 67.04% with mean response of 45.7083 grams of *rose coco* beans, implying that R^2 in GP2G of second order model explain 67.04% of the variation in the model than $R^2=7.83\%$ of the first order model. The model was adequate to express the actual relationship between the response and significant variables, with a coefficient of determination ($R^2=0.6704$), which indicated 67.04% of the variability in the response could be explained by the second-order polynomial predictive equation (4.62), meaning that the model was unable to explain 32.96% of the total variations. The value of R (0.8188) for equation (9) being close to 1 indicated a close agreement between the experimental results and the theoretical values predicted by the model equation.

CONCLUSIONS

This study successfully employed Response Surface Methodology (RSM) to model the relationship between nitrogen (N), phosphorus (P), and potassium (K) as fertilizer variables in the production of *rose coco* beans, using a twenty four points second-order rotatable design within a greenhouse environment. The average yield of *rose coco* beans was obtained from three replications for the GP2G group, and parameter coefficients were estimated accordingly. The Analysis of variance (ANOVA) for the response surface indicated that the model for the GP2G group had a coefficient of determination (R^2) of 0.6707, suggesting that 67.07% of the variability in *rose coco* yield could be explained by the selected fertilizer factors (N, P, and K). The coefficient of variation (CV) was found to be 14.47%, which are within acceptable limits. Generally, high values of CV indicate that experimental design developed was inadequate. In addition, quadratic was found to be significant at 5% for the GP2G. The group GP2Gtotal model was significant at 2.64% ($p = 0.0264$). The findings showed GP2G fitted the second order models well for the *rose coco* yield using the three fertilizer treatments, nitrogen (N), phosphorus (P) and potassium (K). Furthermore, the investigation also needs to be done if the certain amount of each component of the fertilizer affects the effectiveness of each other on the yield of *rose coco* beans.

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