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RESEARCH ARTICLE

DYNAMICAL ANALYSIS OF JOSEPHSON JUNCTION OSCILLATOR WITH QUADRATIC DAMPING AND TOPOLOGICAL NONTRIVIAL BARRIER

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ABSTRACT

The dynamical analysis and microcontroller implementation of Josephson junction (JJ) oscillator with quadratic damping and topological nontrivial barrier (JJOQDTNB) is investigated in this paper. JJOQDTNB is described by nonlinear resistive capacitive inductive shunted JJ (NRCLSJJ) with 4π-periodic superconducting current. Two steady-states are obtained from the rate equations describing JJOQDTNB. According to the Routh-Hurwitz criteria, one steady-state is stable and the other steady-state is unstable. JJOQDTNB exhibits periodic oscillations, period doubling route to chaotic oscillations and coexistence between limit cycle and chaotic oscillations. The numerical simulations and microcontroller implementation results of JJOQDTNB show a good qualitative agreement.

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INTRODUCTION

Recent progress in condensed matter physics and superconductivity have led to significant shift in the development of quantum circuits with various dimensions. The potential for researchers to read out, control the energy states, and experiment with superconductor circuits has posed a significant challenge for a considerable number of years. A notable and noteworthy phenomenon arising from superconductivity is the quantum tunneling [1]. The phenomenon whereby electrons tunnel across a potential barrier comprising two superconductor layers without dissipation of energy and power bias is known as the Josephson Effect. In the contemporary era of technological headway, this field has witnessed a paradigm shift towards the integration of quantum circuits, which have emerged as a pivotal component of modern technological applications. The scope of these applications is manifold and encompasses; voltage standards, highly sensitive detectors, the design of parametric amplifiers, superconductor electronic devices such as terahertz pulse generators [2–5], phase qubits [6], and superconducting quantum interference devices [7].

In the literature, there are two main models employed for describing the JJ; resistive capacitive shunted JJ [8–10], and the resistive capacitive inductive shunted JJ [11–13]. In superconducting quantum circuits, regular damping and nonlinear damping occurs under temperatures and voltages conditions [14]. Superconductors properties has prompted rigorous research investigation in the physics of Majorana bound states, which employ the search for topological superconductor and the existence of pairing gap in the bulk and gapless surfaces [15–17]**.** The theoretical exploration of Majorana bound states and topological quantum computing occurred between 2000 and 2010 and over the subsequent decade (2010–2020), a transitioned in the concepts gradually trend down to practical implementations. The periodic behavior of the current-phase relation in JJ associated to Majorana particles is an interesting aspect of topological superconductor [18-20]**.** The characterization of Shapiro steps in the presence of a $4\pi -$ periodic Josephson current component unveiled by Park *et al*. [21]. Optimal conditions for observing fractional Josephson effect in topological JJs probed by Yeongmin and Yong-Joo [22]. Domínguez *et al.* studied JJ dynamics in the presence of 2π- and 4π-periodic supercurrents [23]. The motivation of this paper is the investigation of dynamical behaviors of JJOQDTNB described by NRCLSJJ with 4π-periodic superconducting current. This paper is organized as follow. The theoretical analysis of JJOQDTNB and its microcontroller implementation are studied in Section 2 and Section 3 unveils the conclusion.

Analysis of JJOQDTNB and its microcontroller implementation: The rate equations of Josephson junction oscillator with quadratic damping and topological nontrivial barrier is given by **[24-26]**:

$$\frac{dv}{dt} = \frac{1}{\beta_C}\left[i_e - i_l - kv^2 - \sin\left(\frac{\varphi}{2}\right)\right], \quad (1.a)$$

$$\frac{di_l}{dt} = \frac{1}{\beta_L}\left(v - i_l\right), \quad (1.b)$$

$$\frac{d\varphi}{dt} = v, \quad (1.c)$$

where v the voltage of resistive capacitive inductive shunted JJ (RCLSJJ), i_e the external bias current source, i_l the current developed across the inductor, $\varphi = \varphi_2 - \varphi_1$ the phase difference across the two superconductors, k the constant of nonlinear resistor and β_L, β_C the positive parameters of System (1). System (1) has two steady states: $S_1\left(0,\ 0,\ 2\,\mathrm{Arcsin}\left(i_e\right)\right)$ and $S_2\left(0,\ 0,\ 2\left(\pi - \mathrm{Arcsin}\left(i_e\right)\right)\right)$ for $i_e \le 1$ and no steady states for $i_e > 1$. The characteristic equation is given by:

$$\lambda^3 + \left(\frac{1}{\beta_L}\right)\lambda^2 \pm \frac{1}{2\beta_C\beta_L}\left(\beta_L\sqrt{1-i_e^2} \pm 2\right)\lambda \pm \frac{1}{2\beta_C\beta_L}\sqrt{1-i_e^2} = 0\,. \quad (2)$$

According to the Routh-Hurwitz stability criteria, the steady state $S_1\left(0,\ 0,\ 2\,\mathrm{Arcsin}\left(i_e\right)\right)$ is stable and the steady state $S_2\left(0,\ 0,\ 2\left(\pi - \mathrm{Arcsin}\left(i_e\right)\right)\right)$ is unstable.

Figure 1 presents the largest Lyapunov exponents (LLE) of system (1) in the parameter space $(k,\ i_e)$.

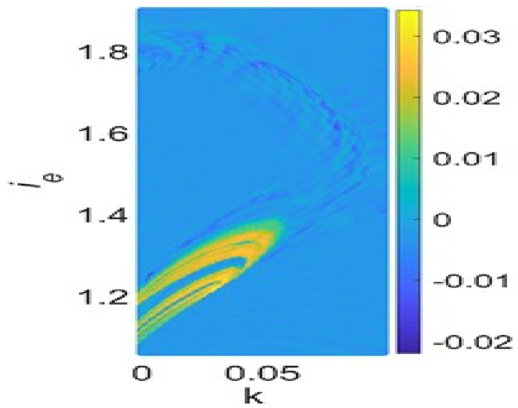


Figure 1. LLE of system (1) in the plane $(k,\ i_e)$ for $\beta_L = 4.21$ and $\beta_C = 1.58$. The initial conditions are $\left(v(0),\ i_l(0),\ \varphi(0)\right) = (0.0,\ 0.0,\ 1.0)$.

In Figure 1, chaotic areas have LLE>0 shown by the yellow, green and light blue colours. The periodic areas have LLE<0 shown by blue colour. The bifurcation diagram of v and LLE as function of i_e are depicted in Fig. 2.

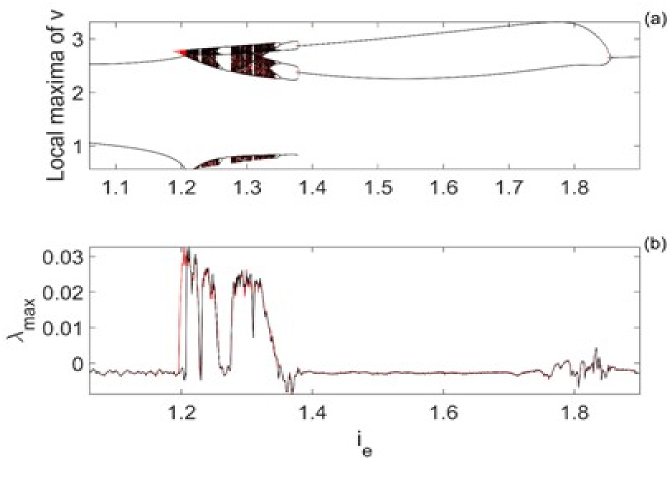
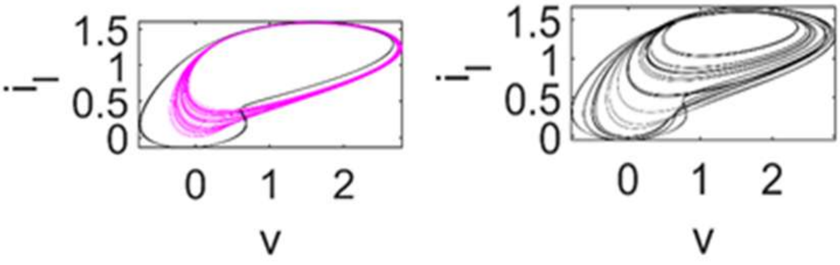


Figure 2. Local maxima of v (a) and associated LLE (b)versus i_e for $\beta_C = 1.58$, $\beta_L = 4.21$, and $k = 0.03$

Figure 2 (a) reveals period-2 oscillation to coexistence at $i_e = 1.2$ between chaotic zone and period doubling, preceded by interception of monostable complex zones with periodic zones to reverse period doubling, a long range of period-2 oscillation and finally to limit circle. The dynamical behaviors found in Fig. 2 (a) are confirmed by the LLE of Fig. 2(b). The dynamical behaviour found in Fig. 2 are depicted in Fig. 3.



(a) (b)

Figure 3. Phase portraits of system (1) in the plane $(v,\ i_l)$ with $\beta_C = 1.58$, $k = 0.03$, $\beta_L = 4.21$ and for given value of i_e : (a) $i_e = 1.2$ and $i_e = 1.3$. The initial conditions $(v(0),\ i_l(0),\ \varphi(0)) = (4.5,\ 0.0,\ 1.0)$, and $(v(0),\ i_l(0),\ \varphi(0)) = (0.0,\ 0.0,\ 1.0)$, align with plots in cyan and black, respectively.

Coexistence between limit cycle and chaos is shown in Fig. 3 (a) and monostable chaos is displayed in Fig. 3 (b).

The microcontroller implementation of system (1) is done to verify the dynamical behaviors found in system (1) via numerical simulations. Figure 4 presents the set up used for this purpose

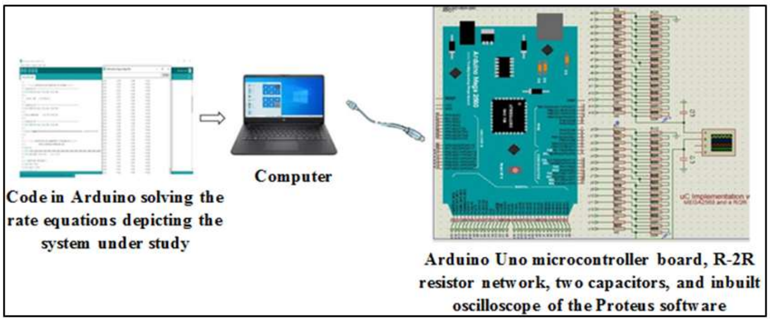
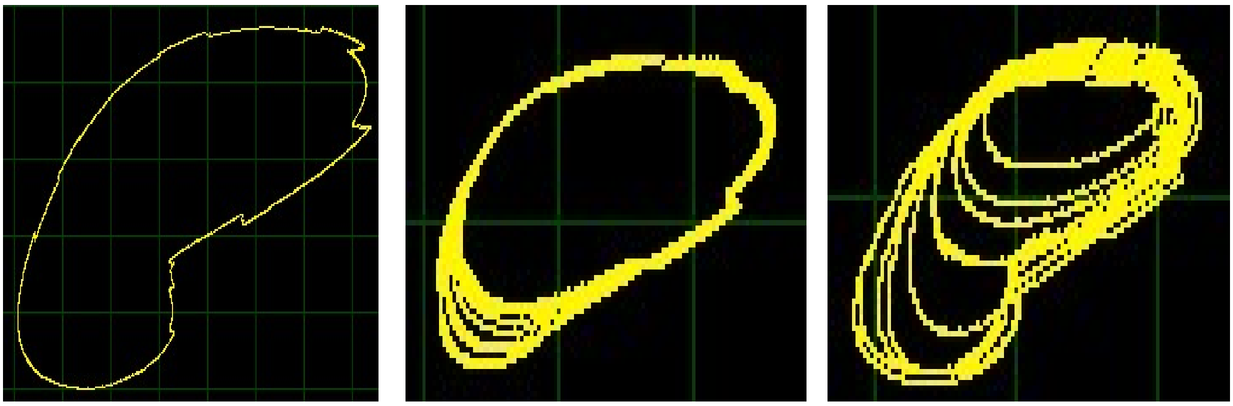


Figure 4. Microcontroller implementation representationofsystem (1)

Arduino Uno of In Figure 4 is embedded with a microcontroller chip which plays the role of the central processing unit that executes the instructions provided by the code. The microcontroller reads the inputs, processes the data and then controls the output. Once the code is compiled, it is then exported to the Proteus Software which is equally installed in the laptop. The latter has an inbuilt virtual oscilloscope that simulates various graphics based on specific initial conditions and parameter values of system (1). The dynamical behaviors found in system (1) are established in Fig. 5 from the microcontroller implementation set up of Fig. 4.



(a) (b) (c)

Figure 5. Phase portraits of System (1) in the plane $(v,\ i_l)$ obtained via microcontroller implementation of Fig. 4: (a) is obtained by using the initial conditions and parameters in Fig. 3 (a) (black lines), (b) is obtained by using the initial conditions and parameters in Fig. 3 (a) (cyanlines) and(c) is obtained by using the initial condition and parameters in Fig. 3 (b)

It is note that there is good qualitative correlation between Figs. 3 and 5.

CONCLUSION

This paper was devoted to the dynamical analysis and microcontroller implementation of Josephson junction oscillator with quadratic damping and topological nontrivial barrier (JJOQDTNB). Two steady-states were found in the rate equations describing JJOQDTNB have. The Routh-Hurwitz criteria was demonstrated that one steady-state was stable and the other steady-state was unstable. The numerical simulations were revealed that JJOQDTNB displayed periodic oscillations, period doubling route to chaotic oscillations and coexistence between limit cycle and chaotic oscillations. The microcontroller implementation of JJOQDTNB was used to confirm the results obtained during the numerical simulations.

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