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RESEARCH ARTICLE

THERMODYNAMICAL BEHAVIOUR OF THE VISCOUS GENERALIZED CHAPLYGIN GAS IN A FLAT UNIVERSE

Abhinath Barman[1], Purna Chandra Barman[2] and Sukalyan Mistry[1]

[1]Department of Mathematics, Alipurduar University, Alipurduar Court, Alipurduar, 736122, West Bengal, India
[2]Department of Physics, Raiganj University, College Para, Raiganj, 733134, West Bengal, India
[1*]Department of Physics, Alipurduar University, Alipurduar Court, Alipurduar, 736122, West Bengal, India

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**Corresponding author:*
Abhinath Barman

ABSTRACT

In this paper, we delve into the viscous generalized Chaplygin gas (VGCG) model, focusing on its thermodynamic stability. We utilize an equation of state represented as $p = -\frac{A}{\rho^{\alpha}}$, where A is a positive universal constant, incorporating bulk viscosity into our analysis. Our study reveals how the thermodynamic properties of the universe change as we vary the bulk viscosity parameter. By examining the thermal equation of state, we find that the equation of state parameter has a direct dependency on temperature. Additionally, we investigate how this exotic viscous fluid contributes to the accelerated expansion of the universe. A key finding is that for the fluid to meet standard stability conditions, the viscosity parameter must be strictly negative, ensuring that $\left(\frac{\partial p}{\partial V}\right)_S < 0$ and $\left(\frac{\partial p}{\partial V}\right)_T < 0$ are satisfied throughout cosmic evolution. Our analysis indicates that the fluid maintains stability during adiabatic processes. We also demonstrate that this model upholds the condition of positive heat capacity at constant volume (c_V) and adheres to the third law of thermodynamics.

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INTRODUCTION

Various astronomical observationssuch as high-redshift Type Ia supernovae (SNIa) [1–5], cosmic microwave background radiation (CMB) [6], and matter power spectra [7], strongly imply that a "dark sector", which is thought to be the cause of the universe's accelerated expansion, accounts for nearly 96 percent of the universe's current total energy content. In the context of dark energy, the nature of this acceleration has been thoroughly examined [1–3, 8–10]. Dark matter and dark energy are generally considered to be the two components that make up this dark region. In the context of Einstein's General Relativity, a variety of cosmological models have been created to investigate the characteristics of this sector. Einstein's cosmological constant (Λ) is among the best models for explaining dark energy, which is created by the negative pressure of the universe [11–15]. On sufficiently massive cosmic scales, the cosmological principle is considered to hold, suggesting that the world is homogeneous and isotropic. Assuming this, the matter- energy content of the cosmos is frequently represented as a perfect fluid, an idealized material with homogeneous characteristics and no viscosity. The cosmological constant (Λ), which emerges naturally from the framework of general relativity and quantum field theory, is among the most widely accepted hypotheses for dark energy. Alternative methods, such dynamical dark energy models, contend that dark energy is not static but rather changes over time [16–24]. Despite offering a more comprehensive foundation, these models have drawbacks and unsolved problems. The Chaplygin gas has been suggested as an unusual fluid model that could provide a cohesive explanation of dark energy and dark matter in response to these constraints. As a dark energy candidate, we are interested in the Chaplygin gas (CG) [25–32]. To maintain consistency with observational data, the initial model has been extended to the generalized Chaplygin gas (GCG) [33–37]. Models like the modified Chaplygin gas (MCG) [38] and the modified cosmic Chaplygin gas (MCCG) [39] are the result of additional changes. Under some conditions, a feasible dark energy model should be able to approximate the cosmological constant model, even though it is still consistent with data [40]. The issue of late-time cosmic acceleration is effectively handled by the majority of models based on Chaplygin gas. They frequently fall short in addressing the initial singularity problem, though. Consequently, only a small number of models try to concurrently tackle the initial singularity and the late-time acceleration difficulties. The equation of state (EoS) of the Generalized Chaplygin Gas (GCG) can be written as,

$$p = -\frac{A}{\rho^{\alpha}} \tag{1}$$

where $1 > \alpha > 0$, and ρ and p represent the fluid's density and pressure, respectively. Furthermore, it has been demonstrated that bulk viscosity is crucial to cosmology [41– 43]. The Chaplygin gas, which was first proposed in [44] and subsequently confirmed by additional research in [45–50], might have offered an early hint of the significance of viscosity. The viscous modified cosmic Chaplygin gas model

(VMCCG), and the viscous modified Chaplygin gas (VMCG) model, which both take time-dependent energy density into account, are specifically covered in Refs. [43, 48]. Furthermore, the viscous Chaplygin gas model has been explored within the framework of a non-flat FriedmannRobertsonWalker (FRW) universe in Ref. [46].

Numerous investigations have been performed on the viscous generalized Chaplygin gas (VGCG) hypothesis as a potential explanation for the universe's observed accelerated expansion. This concept provides a cohesive framework for comprehending dark matter and dark energy. In this paper, we investigate the joint impacts of the Chaplygin gas and bulk viscosity in a flat Friedmann-Robertson-Walker (FRW) universe. The Chaplygin gas component further alters the standard Friedmann equations, which are modified by the addition of bulk viscosity, $\xi = \xi_0 \rho^{1/2}$ is the model for the bulk viscous coefficient, as explained in Ref. [51]. The equation of state in Ref. [52] uses the parameter $= \frac{1}{2}$, and we employ the same value for our research. Under this assumption, we examine the VGCG model's thermodynamic stability and discover that the stability requirements are conditionally met. Following the methodology for the generalized in Santos *et.al.* [53], and modified Chaplygin gas models [54], based on the thermodynamic criteria outlined in Ref. [55], we examine whether the conditions $\left(\frac{\partial p}{\partial V}\right)_S < 0$, $\left(\frac{\partial p}{\partial V}\right)_T < 0$ and $c_V > 0$ hold. These conditions are essential for ensuring instantaneous thermodynamic stability. In accordance with these references, [53, 54, 56] and [57, 58], we analyze various cosmological parameters within the VGCG model, including the effective pressure, effective equation of state parameter, effective deceleration parameter, and the adiabatic speed of sound. Additionally, Ref. [59] investigates the interaction between VGCG and $f(R, T)$ gravity in the FRW frame-work, where the modified Friedmann equations incorporate time-dependent energy density and pressure contributions from both the Chaplygin gas and dark energy.

Cosmological models based on viscous modified Chaplygin gas (VMCG) and related frameworks have been the subject of several investigations. The VMCG model was studied in both classical and loop quantum cosmology (LQC) in Ref. [60], emphasizing its cosmological implications. Using current background data, Ref. [61] performed an observational constraints analysis on the equation of state (EoS) parameters of the viscous generalized Chaplygin gas model, achieving appropriate values in agreement with observations. The viscous Modified Cosmic Chaplygin Gas (MCCG) model was examined in a flat Friedmann-Robertson-Walker (FRW) universe characterized by a cosmological constant in [62]. Using the square of the speed of sound, the study examined how the cosmological constant and viscosity affected the evolution of the universe and compared it to the Cardassian universe model to assess the model's stability. With a thorough examination of dynamical stability, Ref. [63] concentrated on a VMCG model in both classical and LQC situations. In addition to solutions to the coincidence problem and the time of cosmic acceleration, the study offered encouraging predictions for the deceleration and EoS parameters. Furthermore, in a (2+1)-dimensional FRW space-time a modified Chaplygin gas model with bulk viscosity was examined in Ref. [64]. In order, to determine important physical quantities including energy density, the Hubble parameter, and the deceleration parameter, several variants of the bulk viscosity coefficient were examined. The speed of sound was once more used to evaluate the model's stability. Lastly, explained a generalized Chaplygin gas in terms of bulk viscosity without the necessity of a cosmological constant in Ref. [65]. The work found that applying bulk viscosity to normal FRW cosmology naturally connects the dark matter and the dark energy.

Basic equations of VGCG model

We are aware that the Friedmann-Robertson-Walker measure can be expressed as follows:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \tag{2}$$

where, $d\Omega^2 = d\theta^2 + sin^2\theta d\varphi^2$, and $a(t)$ is the expansion rate of the universe. In addition to $k = 0, +1$, and -1 denoting flat, closed, and open universes, respectively, the dimensionless coordinates r, θ, and φ are also known as comoving coordinates. Since we are studying in flat spacetime in this instance, ($k = 0$), the Einstein equation can be expressed as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G T_{\mu\nu}}{c^4} \tag{3}$$

where $c = 1$, $8\pi G = 1$, and $\Lambda = 0$ are the parameters. We discovered that the universe's viscous generalized Chaplygin gas (VGCG) functions as dark energy with EoS (1). With bulk viscous fluid, we obtain the following statement using equations (2) and (3) (47, 49) as

$$T_{\mu\nu} = (\rho + p_{eff})u_\mu u_\nu - p_{\mu\nu} g_{\mu\nu} \tag{4}$$

where u^μ and ρ stand for the four-velocity vector and energy-density vector, respec- tively. $g_{\mu\nu}$ is the metric tensor and $T_{\mu\nu}$ is the energy-momentum tensor of the universe, and $u^\mu u_\nu = 1$ is the normalization condition. Therefore, the total effective pressure of the viscous fluid can be written as

$$p_{eff} = p + \Pi = -\frac{A}{\rho^\alpha} - \sqrt{3}\xi_0 \rho \tag{5}$$

where the bulk viscosity coefficient is $\xi = \xi_0 \rho^{1/2}$, and the viscous pressure is $\Pi = -3\xi H$.

According to the definition, the energy-density is

$$\rho_{vgcv} = \frac{U}{V} \tag{6}$$

where the fluid's volume is denoted by V and its internal energy by U. We have the thermodynamics relation [66]

$$\left(\frac{\partial U}{\partial V}\right) = -p_{eff} \tag{7}$$

From equations (1), (5), (6) and (7), we get the relation

$$\left(\frac{\partial U}{\partial V}\right) = A\left(\frac{U}{V}\right) - \left(-\sqrt{3}\xi_0\right)\frac{U}{V} \quad (8)$$

The expression for internal energy U obtained by integrating the above equation is

$$U = \left(\frac{A}{1-\sqrt{3\xi_0}} V^{\alpha+1} + \frac{b}{V^{(-\sqrt{3}\xi_0)(\alpha+1)}}\right)^{\frac{1}{\alpha+1}} \quad (9)$$

where b is an arbitrary integration constant.

The above expression can also be written as

$$U = \left(\frac{A}{1-\sqrt{3\xi_0}}\right)^{\frac{1}{\alpha+1}} V\left(1+\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right)^{\frac{1}{\alpha+1}} \quad (10)$$

where,

$$\epsilon = \left[\frac{b(1-\sqrt{3}\,\xi_0)}{A}\right]^{\frac{1}{(1-\sqrt{3})(\alpha+1)}} \quad (11)$$

Consequently, the energy-density can alternatively be expressed using the parameter ξ_0 as

$$\rho_{vgcg} = \left(\frac{A}{1-\sqrt{3\xi_0}}\right)^{\frac{1}{\alpha+1}}\left(1+\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right)^{\frac{1}{\alpha+1}} \quad (12)$$

In the ensuing subsections, this expression aids in the computation of the values of several cosmological parameters.

Effective Pressure: We calculate the VGCG model's effective pressure using the viscous parameter ξ_0 and volume V. Equations (5) and (12) are used to obtain

$$p_{eff} = \rho_{vgcg}\left[\left(-\sqrt{3}\xi_0\right) - \frac{1-\sqrt{3\xi_0}}{\left[1+\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]}\right] \quad (13)$$

It can also be stated as

$$p_{eff} = -\frac{\left(\frac{A}{1-\sqrt{3\xi_0}}\right)^{\frac{1}{\alpha+1}}\left[1+\sqrt{3}\xi_0\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]}{\left[1+\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]^{\frac{\alpha}{\alpha+1}}} \quad (14)$$

Effective pressure is expressed by the equation above. A negative total effective pressure results from the bulk viscous pressure phrase becoming more dominant and negative than the thermodynamic pressure. Dark energy is characterized by this "negative pressure" or tension, which propels the universe's observable faster expansion. Given the circumstances, we now obtain relations:

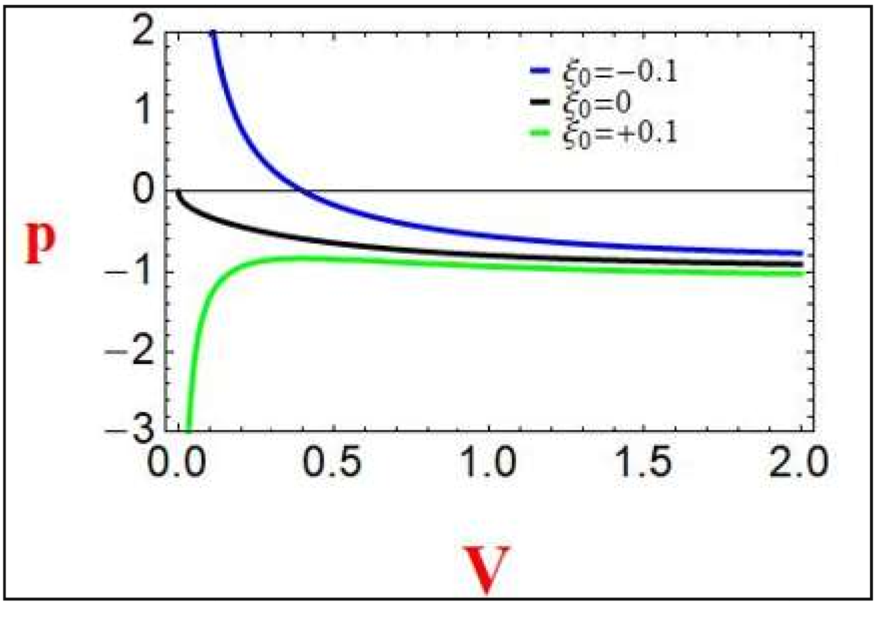

Fig. 1 The plot of p_{eff} with V for various values of ξ_0 with $A = 1$, $\alpha = 0.5$ and $b = 1$

For $\xi_0 = 0$, $A \neq 0$ and $\alpha \neq 0$, the equation (14) reduces to as,

$$p_{eff} = -\frac{(A)^{\frac{1}{\alpha+1}}}{\left[1+\left(\frac{\epsilon}{V}\right)^{(\alpha+1)}\right]^{\frac{\alpha}{\alpha+1}}} \quad (15)$$

This is comparable to the generalized Chaplygin gas model [53], as anticipated. The thermodynamic pressure is the same as the effective pressure. Both density and pressure typically fall as volume increases in an expanding universe.

For $\xi_0 = 0$, the results are reduced to the Chaplygin gas model [66] as $A = 0$ and $\alpha = 1$

$$p_{eff} = -\frac{(A)^{\frac{1}{2}}}{\left[1+\left(\frac{\epsilon}{V}\right)^{(2)}\right]^{\frac{1}{2}}} \quad (16)$$

As seen in Fig.1, we plot effective pressure against volume for a range of viscous param- eter ξ_0 values. For "dust" (non-relativistic), the thermodynamic pressure is practically negligible. As a result, decelerated expansion is associated with an effective pressure of zero. Depending on the parameter's value, the effective pressure may be either positive or negative. Pressure is always negative for $\xi_0 = 0$ and $\xi_0 > 0$, indicating that the gas of Chaplygin type. Additionally, it demonstrates that when $\xi_0 < 0$, the effective pressure is negative for large volumes and positive for small volumes. In a viscous cosmological fluid, the volume and pressure have a complicated connection that is influenced by the expansion rate (H). Include a negative pressure component from bulk viscosity that dissipates and is proportional to the rate of expansion (H). As the universe's volume grows, it can turn negative and serve as a source of repulsive gravity, accelerating expansion.

The critical volume (V_c) for the VGCG model is zero pressure condition, of $p_{eff} = 0$, is obtained

$$V_c = \epsilon\left(-\sqrt{3}\xi_0\right)^{\frac{1}{(1-\sqrt{3}\xi_0)(\alpha+1)}} \quad (17)$$

Alternatively,

$$V_c = \left(\frac{\left[b(1-\sqrt{3}\xi_0)(-\sqrt{3}\xi_0)\right]}{A}\right)^{\frac{1}{(1-\sqrt{3}\xi_0)(\alpha+1)}} \quad (18)$$

A viable decelerated universe is indicated by a positive effective pressure (p_{eff}) when the magnitude of the critical volume is greater than volume V, i.e., $V_c > V$. $p_{eff} = 0$ for $V = V_c$. Effective pressure turns negative when $V > V_c$, which further suggests that the cosmos is accelerating. In the analysis, we obtain a new scale of V_c when a dust-dominated universe enters the acceleration phase. We discover that ϵ and V_c are of the same order of magnitude. The decelerated universe is represented by $V << \epsilon$, whereas the very vast volume with a conceivable accelerated universe is represented by $V >> \epsilon$.

EoS Parameter of VGCG model: The EoS for the bulk viscous is determined in this section and is defined as

$$\omega_{eff} = \frac{p_{eff}}{\rho_{eff}} \quad (19)$$

Equations (12) and (14) can also be used to write the aforementioned expression as

$$\omega_{eff} = \left(-\sqrt{3}\xi_0\right) - \frac{1-\sqrt{3}\xi_0}{\left[1+\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]} \quad (20)$$

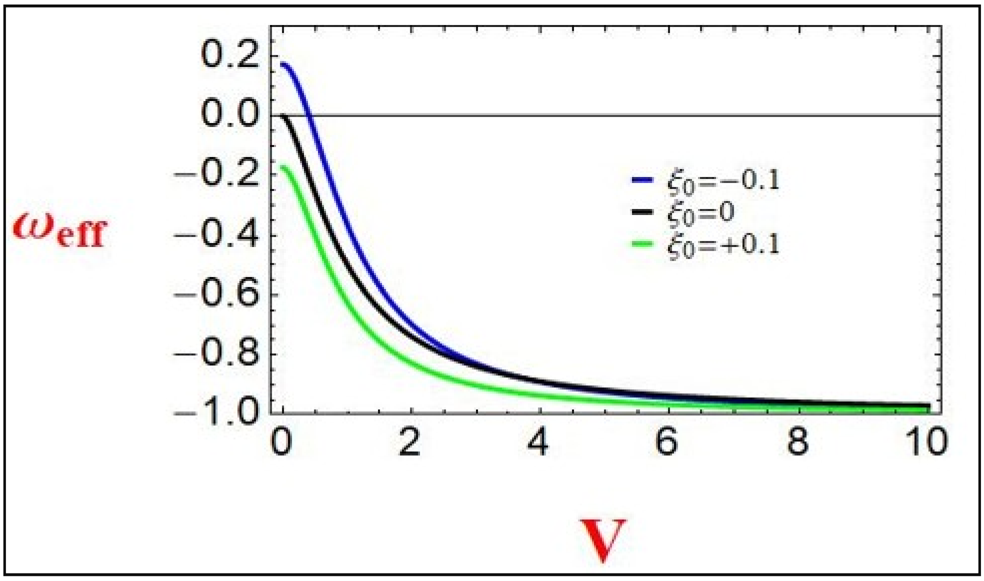

Fig. 2 Plot of ω_{eff} with V for various values of ξ_0 with $A = 1$, $\alpha = 0.5$ and $b = 1$.

In a viscous cosmology, the characteristics of the cosmic fluid and its bulk viscosity shape the dynamic relationship between the effective EoS parameter and volume. Instead of being constant, the effective EoS parameter changes when the cosmic scale factor (a) changes. The impact of bulk viscosity on the EoS is minimal during early eras when the cosmos is dense. The bulk viscosity may become dynamically significant as the universe gets denser and expands, pushing the effective EoS in the direction of negative values ($\omega_{eff} < -1/3$), which accelerates expansion.

The effective pressure is $p_{eff} = (-\sqrt{3}\xi_0)\rho_{vgcg}$ and $\omega_{eff} = (-\sqrt{3}\xi_0)$ when volume is small, $V \ll \epsilon, i.e., \frac{\epsilon}{V} > 1$. It is observed that the viscous parameter ξ_0 determines the effective pressure. A phantom-like universe is produced if ξ_0 is positive $(i.e., \xi_0 > \frac{1}{\sqrt{3}})$, and a quiescence phenomenon is indicated if $\omega_{eff} < -1$ is negative (i.e., $\xi_0 < 0$), in which case $\omega_{eff} > -1$. In this instance as well, if $\xi_0 = 0$, we obtain $\omega_{eff} \approx 0$, which reveals that $p_{eff} = 0$, indicating a cosmos dominated by dust. The EoS parameter, $\omega_{eff} \approx -1$ is also obtained when V >> ϵ, that is, when $\frac{\epsilon}{V} < 1$, this indicates the ΛCDM model. As seen in Fig.2, we plot effective EoS versus volume V for a different value of ξ_0 values, indicating that the universe is accelerating at a huge volume.

When the volume is very small, $\omega_{eff} > 0$ for $\xi_0 < 0$ in the early universe. ω_{eff} tends to zero when V rises to V_c. So, we also find $V = V_c = \left(\frac{[b(1-\sqrt{3}\xi_0)(-\sqrt{3}\xi_0)]}{A}\right)^{\frac{1}{(1-\sqrt{3}\xi_0)(\alpha+1)}} = \epsilon(-\sqrt{3}\xi_0)^{\frac{1}{(1-\sqrt{3}\xi_0)(\alpha+1)}}$ at $\omega_{eff} = 0$. Once more, V rises and ω_{eff} falls.

Deceleration Parameter: Based on the specification, the effective deceleration parameter that corresponds to the bulk viscosity of the VGCG model can be expressed as

$$q_{eff} = \frac{1}{2} + \frac{3}{2}\left(\frac{p_{eff}}{\rho_{vgcg}}\right) \tag{21}$$

The equation can be written using the viscous parameter ξ_0 using equation (20) as

$$q_{eff} = \frac{1}{2} + \frac{3}{2}\left[(-\sqrt{3}\xi_0) - \frac{1-\sqrt{3}\xi_0}{\left[1+\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]}\right] \tag{22}$$

The deceleration parameter in a cosmology where a viscous fluid predominates dynamically changes with the universe's volume, which is represented by the scale factor. As a result of cosmic acceleration driven by viscosity, the deceleration parameter changes from a positive (decelerating) value in the early universe to a negative (accelerating) value in the later universe. In order, to explain the observable expansion history, this dynamic behavior offers an alternative to the cosmological constant (Λ). Radiation and matter densities were quite high in the early cosmos. In a universe dominated by matter, where viscosity is zero, ($q = 0.5$). Because of the large energy density and initial insignificant negative pressure associated with bulk viscosity, q is positive, signifying a decelerating phase. Energy density falls as the universe gets bigger and its volume rises. However, the negative viscous pressure may start to dominate. As the cosmic volume expands, the negative bulk viscous pressure becomes more significant. In order, to move from a decelerating to an accelerating phase, the deceleration parameter falls, exceeds the threshold ($q = 0$) (which corresponds to a constant expansion rate), and becomes negative ($q < 0$).

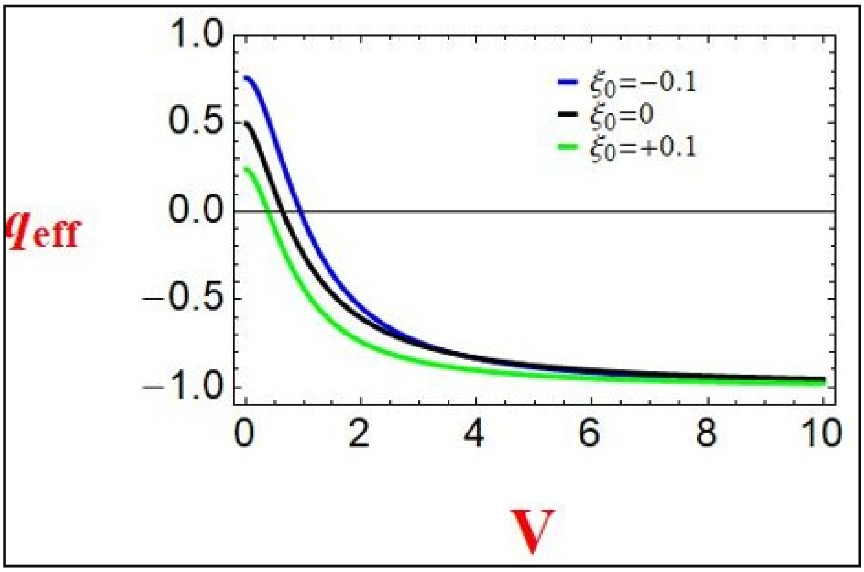

Fig. 3. Plot of qeff with V for various values of ξ0 with A = 1, α = 0.5 and b = 1.

For Small volume (i.e., early universe), $V \ll \epsilon$, i.e., $\frac{\epsilon}{V} = \infty$, then we get $q_{eff} = \frac{1}{2}(1 - \sqrt{3}\xi_0)$. The effective deceleration parameter is positive for $\xi_0 \leq 0$ when volume is minimal, suggesting that the universe may be decelerating. The universe is accelerated when $V \gg \epsilon$, i.e., $\frac{\epsilon}{V} = 0$. This is indicated by the value $q_{eff} \approx -1$. In this instance, the *flip* volume (V_f) will occur when the effective deceleration parameter is zero. In many viscous models, the deceleration parameter gets closer to $q_{eff} = -1$ as the scale factor gets closer to infinity. This is equivalent to a de Sitter universe, which behaves like a cosmological constant and experiences exponential growth driven by a constant vacuum energy density.

Consequently, the flip volume expression can be stated as

$$V_{eff} = \frac{\epsilon}{\left[\frac{2}{3(-\sqrt{3}\xi_0)+1}\right]^{\frac{1}{(1-\sqrt{3}\xi_0)(\alpha+1)}}} = \left(\frac{\left[b\left(1-\sqrt{3}\xi_0\right)(1-3\sqrt{3}\xi_0)\right]}{2A}\right)^{\frac{1}{(1-\sqrt{3}\xi_0)(\alpha+1)}} \quad (23)$$

According to Fig.3, the cosmos accelerates as volumes rise after q_{eff} initially goes to zero. The equation above demonstrates that the value of V_f must be genuine when $\xi_0 < 0$; otherwise, there will not be any *flip*. Accordingly, the universe accelerates when $V > V_f$ and decelerates when $V < V_f$. Therefore, we determine that two scales of volume are V_c and V_f, respectively, from the zero-pressure condition, and that a potential deceleration-acceleration transition takes place at the effective deceleration parameter.

We now obtain the relation from equations (18) and (23) as

$$\frac{V_f}{V_c} = \left[\frac{1-3\sqrt{3}\xi_0}{2(-\sqrt{3}\xi_0)}\right]^{\frac{1}{(1-\sqrt{3}\xi_0)(\alpha+1)}} \quad (24)$$

Sound Speed of VGCG model: Here, we looked at the VGCG model's stability condition. The definition of sound speed in a viscous fluid is

$$v_s^2 = \left(\frac{dp_{eff}}{d\rho_{eff}}\right)_S = \frac{\left(\alpha-\sqrt{3}\xi_0\left[1+\alpha+\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]\right)}{\left[1+\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]} \quad (25)$$

The velocity of sound in the cosmic fluid is a dynamically changing quantity that is governed by the bulk viscosity and thermodynamic parameters of the fluid rather than being a constant in viscous cosmology. The relation between sound speed and volume is driven by the expansion of the universe, as opposed to static fluids where density and elastic characteristics control sound speed. Sound speed is comparatively stable in the early cosmos because adiabatic processes dominate the fluid's characteristics. In the late universe, bulk viscosity increases with increasing volume, causing the sound speed to dynamically vary and possibly turn negative. This is a prerequisite for the fluid to function as a source of rapid expansion.

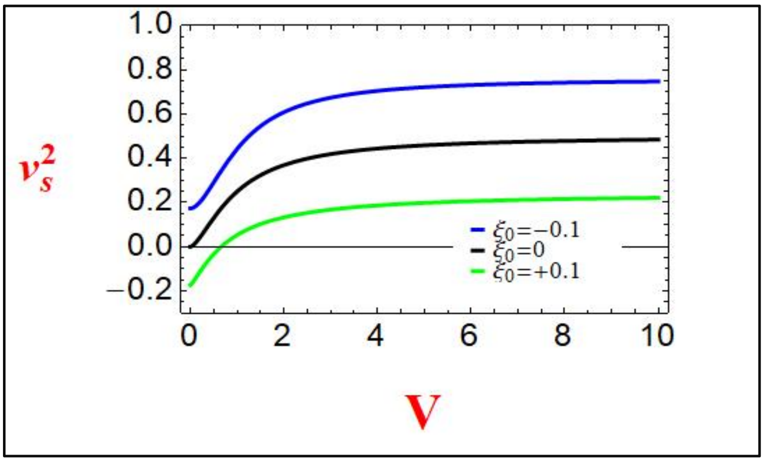

Fig. 4. The variation of square speed of sound v_s^2 and volume V for various values of ξ_0 with A = 1, α = 0.5 and b = 1.

We also know that $0 < v_s^2 < 1$ must be the range of the speed of sound. This range was the focus of our investigation. It lowers to $v_s^2 = -\sqrt{3}\xi_0$ when volume is very tiny (V << ϵ), i.e., at early universe. When (V >> ϵ), is large, equation (25) yields

$$v_s^2 = \alpha + \left(-\sqrt{3}\xi_0\right)(\alpha+1) \quad (26)$$

In the equation above, ξ_0 and α are necessary. The phantom type universe is the result of the thermodynamic stability condition for values of $\alpha = \frac{1}{2}$ and $\xi_0 = \frac{1}{\sqrt{3}}$, as we shall demonstrate later [67, 68]. Furthermore, for $\alpha > 0$, the foregoing calculation yields an imaginary speed of sound, resulting in a perturbative cosmology. We discovered in ref. [69] that for holographic DE, the sound speed is always negative and non-negative for tachyon matter and generic Chaplygin gas. As seen in Fig.4, the properties of the squared speed of sound change depending on the value of the parameter ξ_0 for $\alpha = \frac{1}{2}$. It is evident that $\alpha = \frac{1}{2}$ yields thermodynamically stable results throughout the range of $-\frac{1}{3\sqrt{3}} < \xi_0 < \frac{1}{3\sqrt{3}}$. Panigrahi & Chatterjee [58] also looked at the variable MCG model, which allows for both positive and negative sound speed values. With bulk viscosity playing a major role in its evolution, the effective sound speed is an essential measure for assessing the stability and behavior of cosmic disturbances.

Thermodynamics Stability of VGCG model: We examine a fluid's thermodynamic stability conditions throughout the universe's evolution. The stability requirements of thermodynamics are known to be **[55]**: In both adiabatic and isothermal expansion, the pressure decreases as $\left(\frac{\partial p}{\partial V}\right)_S < 0$, $\left(\frac{\partial p}{\partial V}\right)_T < 0$ and $c_V > 0$.

Differentiating equation (14) w.r.t. volume, we obtained as

$$\left(\frac{\partial p_{eff}}{\partial V}\right)_S = -\frac{\left(\frac{A}{1-\sqrt{3}\xi_0}\right)^{\frac{1}{\alpha+1}}\left(\frac{1-\sqrt{3}\xi_0}{V}\right)\left[\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]}{\left[1+\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]^{\frac{2\alpha+1}{\alpha+1}}}\left[\alpha-\sqrt{3}\xi_0\left[\alpha+1+\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]\right] \quad (27)$$

The above expression can be expressed as

$$\left(\frac{\partial p_{eff}}{\partial V}\right)_S = -\frac{p_{eff}\left(\frac{1-\sqrt{3}\xi_0}{V}\right)}{\left[1+\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]^{\frac{2\alpha+1}{\alpha+1}}}\left[\alpha-\sqrt{3}\xi_0\left[\alpha+1+\left(\frac{\epsilon}{V}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]\right] \quad (28)$$

If we put $\xi_0 = 0$ and $\alpha \neq 0$, we find that equation (28) becomes a similar work of *Santos* [**53**], we get

$$\left(\frac{\partial p_{eff}}{\partial V}\right)_S = -\frac{\alpha p_{eff}}{V}\left[1+\left(\frac{\epsilon}{V}\right)^{(\alpha+1)}\right]^{-1}\left(\frac{\epsilon}{V}\right)^{(\alpha+1)} \quad (29)$$

Which gives $\left(\frac{\partial p_{eff}}{\partial V}\right)_S < 0$ for $\alpha > 0$. The above expression is similar like to the Generalized Chaplygin Gas (GCG).
From equation (28), when $\left(\frac{\partial p_{eff}}{\partial V}\right)_S$ $vs\ V$ for various values of ξ_0 as shown in Fig.5, which indicates that the negative adiabatic condition of the fluid leads to stable. So, we have seen that when $\xi_0 \leq 0$, our model is stable during the universe's evolution.

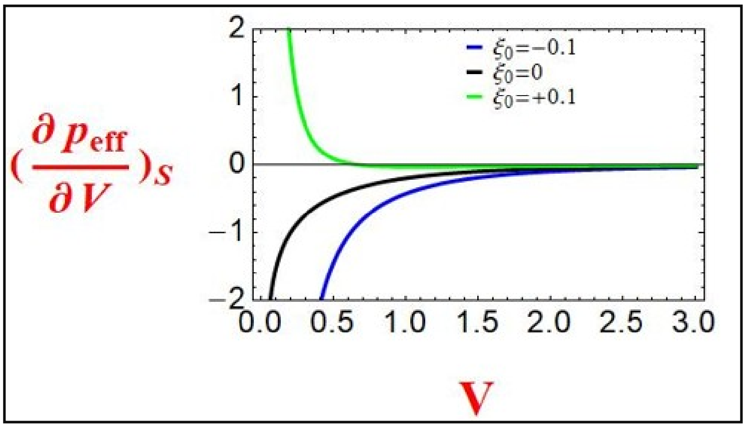

Fig. 5 The variation of effective pressure gradient $\left(\frac{\partial p_{eff}}{\partial V}\right)_V$ and volume V for various values of ξ_0 with $A = 1$, $b = 1$, and $\alpha = 0.5$. Here, the graphs $\xi_0 = -0.1$ (blue line), $\xi_0 = 0$ (black line) and $\xi_0 = 0.3$ (green line); clearly shows that for throughout the evolution period.

Thermal EOS of VGCG model: Using the thermodynamics relation, we also verified that the specific heat was positive at constant volume. The specific heat can be expressed in terms of temperature and entropy as

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V = V\left(\frac{\partial \rho_{vgcg}}{\partial T}\right)_V \quad (30)$$

Where T and S are represents temperature and entropy, respectively. The equation is utilized to determine the fluid's temperature,

$T = \left(\frac{\partial U}{\partial S}\right)_V$ **[70],** it can be expressed

$$T = \left(\frac{\partial U}{\partial b}\right)_V\left(\frac{\partial b}{\partial S}\right)_V \quad (31)$$

Using equation (**9**), the expression temperature can be written as

$$T = \frac{V^{1-(1-\sqrt{3}\xi_0)(\alpha+1)}}{\alpha+1}\left[\frac{A}{(1-\sqrt{3}\xi_0)}+\frac{b}{V^{(1-\sqrt{3}\xi_0)(\alpha+1)}}\right]^{-\frac{\alpha}{\alpha+1}}\left(\frac{\partial b}{\partial S}\right)_V \quad (32)$$

Since b is a universal constant, we find that -$\left(\frac{\partial b}{\partial S}\right)_V = 0$. This implies that, regardless of the pressure and volume, the temperature is zero. However, in the case of Chaplygin gas, the temperature does change with expansion. Therefore, we must consider $\frac{db}{dS} \neq 0$. Lacking specific information on how b depends on S, we assume that $\frac{db}{dS} > 0$ [57, 58], which allows us to derive positive temperatures that decrease through adiabatic expansion.

Equation (9) can be obtained by dimensional analysis

$$[b] = [U]^{\alpha+1}\left[V^{(-\sqrt{3}\xi_0)(\alpha+1)}\right] \tag{33}$$

Since we are aware that [U]=[T][S], the equation above can be expressed as

$$[b] = [T]^{\alpha+1}[S]^{\alpha+1}\left[V^{(-\sqrt{3}\xi_0)(\alpha+1)}\right] \tag{34}$$

It is challenging to solve for b from equation (34). Consequently, the expression for b in a trial case can be expressed as τ and υ

$$b = \left(\tau\upsilon^{-\sqrt{3}\xi_0}\right)^{\alpha+1} S^{\alpha+1} \tag{35}$$

In this case, υ represents the volume dimension and τ (the constant) represents merely the temperature. Presently, we get

$$\left(\frac{db}{dS}\right)_V = (\alpha+1)\left(\tau\upsilon^{-\sqrt{3}\xi_0}\right)^{\alpha+1} S^{\alpha} \tag{36}$$

Using equations (35) and (36), the equation (32) becomes

$$T = V^{1-(1-\sqrt{3}\xi_0)(\alpha+1)}\left(\tau\upsilon^{-\sqrt{3}\xi_0}\right)^{\alpha+1} S^{\alpha}\rho^{-\alpha} \tag{37}$$

The above expression can be streamlined to

$$T = V^{(\sqrt{3}\xi_0)}\left(\tau\upsilon^{-\sqrt{3}\xi_0}\right)\left[1+\left(\frac{V}{\epsilon}\right)^{(1-\sqrt{3}\xi_0)(\alpha+1)}\right]^{-\frac{\alpha}{\alpha+1}} \tag{38}$$

Putting the value of b in equation (37), the entropy is given as

$$S = \left(\frac{A}{1-\sqrt{3}\xi_0}\right)^{\frac{1}{\alpha+1}} \frac{\left(\frac{V^{1-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)}{\left[\left(\frac{\tau\upsilon^{-\sqrt{3}\xi_0}}{TV^{1-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}-1\right]^{\frac{1}{\alpha+1}}} \tag{39}$$

Which means that is may also be written as

$$S = \left(\frac{A}{1-\sqrt{3}\xi_0}\right)^{\frac{1}{\alpha+1}}\left(\frac{V}{T}\right) \frac{\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}}{\left[1-\left(\frac{TV^{1-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}\right]^{\frac{1}{\alpha+1}}} \tag{40}$$

From equation (40), we see that the condition $0 < TV^{-\sqrt{3}\xi_0} < \tau\upsilon^{-\sqrt{3}\xi_0}$ must hold for positive and finite entropy. This condition is confirmed as it meets the constraints $\tau > T > 0$, and $\upsilon < V < \infty$, where υ represents the lowest volume and τ represents the highest temperature. If $T \to 0$, then equation (40) yields $S = 0$, indicating that the thermodynamics' third law is satisfied.
Substituting equations (38) and (40) into (30), it follows that

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V = \frac{1}{\alpha}\left(\frac{A}{1-\sqrt{3}\xi_0}\right)^{\frac{1}{\alpha+1}}\left(\frac{V}{T}\right) \frac{\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}}{\left[1-\left(\frac{TV^{1-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}\right]^{\frac{\alpha+2}{\alpha+1}}} \tag{41}$$

In terms of entropy S, the above equation can also be written as

$$C_V = \frac{1}{\alpha}\frac{S}{\left[1-\left(\frac{TV^{1-\sqrt{3}\xi_0}}{\tau\upsilon^{-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}\right]} \tag{42}$$

When $0 < TV^{-\sqrt{3}\xi_0} < \tau\upsilon^{-\sqrt{3}\xi_0}$ i.e., $\tau > T > 0$ and $\upsilon < V < \infty$, and $\alpha > 0$, then C_V is positive and $C_V > 0$, always satisfied. We found that in this model, the specific heat capacity disappears at zero temperature, or $C_V \to 0$, for $T \to 0$, verifies the third law of thermodynamics' validity. Thus, given the identical circumstance, we also saw that the thermal heat capacity C_V and the entropy S have positive values. Thus, for values of $\xi_0 < 0$ and $\alpha = \frac{1}{2}$ during the evolution, the VGCG model is thermodynamically stable. The plot of specific heat vs volume for a given value of $\alpha = \frac{1}{2}$ and different values of ξ_0 is displayed in Fig.6. In equation (42), if we enter $\xi_0 = 0$, we obtain an expression that is comparable to C_V, which was discovered by Santos *et. al.* (53) and correlates to GCG.

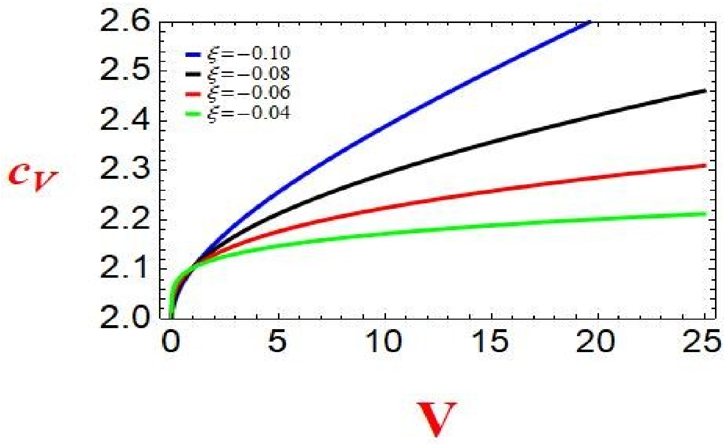

Fig. 6 Plot of C_V with V for various values of ξ0 with T = 1, τ = 2.73, v = 1 and α = 0.5.

Using equations (9), (35) and (40) to calculate the internal energy of this model, we

$$U = V\left(\frac{A}{1-\sqrt{3}\xi_0}\right)^{\frac{1}{\alpha+1}}\left[1-\frac{1}{1-\left(\frac{\tau v^{-\sqrt{3}\xi_0}}{TV^{1-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}}\right]^{\frac{1}{\alpha+1}} \quad (43)$$

For the stability of the VGCG model, we looked at the isothermal condition, or $\left(\frac{\partial p_{eff}}{\partial V}\right)_T < 0$. Applying $p = p\ (V, T)$ from thermodynamic relations and resolving equations (14) and (35), we obtain

$$p_{eff} = \left(\frac{A}{1-\sqrt{3}\xi_0}\right)^{\frac{1}{\alpha+1}} \frac{\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)\left[1-\sqrt{3}\xi_0-\left(\frac{\tau v^{-\sqrt{3}\xi_0}}{TV^{1-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}\right]}{\left[\left(\frac{\tau v^{-\sqrt{3}\xi_0}}{TV^{1-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}-1\right]^{\frac{1}{\alpha+1}}} \quad (44)$$

The above statement can also be expressed in terms of entropy, therefore we have

$$p_{eff} = -\left(\frac{TS}{V}\right)\left[1-\sqrt{3}\xi_0-\left(\frac{\tau v^{-\sqrt{3}\xi_0}}{TV^{1-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}\right] \quad (45)$$

Where density is expressed in temperature terms by

$$\rho_{vgcg} = \left(\frac{A}{1-\sqrt{3}\xi_0}\right)^{\frac{1}{\alpha+1}}\left[1-\frac{1}{1-\left(\frac{\tau v^{-\sqrt{3}\xi_0}}{TV^{1-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}}\right]^{\frac{1}{\alpha+1}} \quad (46)$$

The form of the relevant thermal EOS parameter of VGCG is

$$\omega_{eff} = \frac{\left[1-\sqrt{3}\xi_0-\left(\frac{\tau v^{-\sqrt{3}\xi_0}}{TV^{1-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}\right]}{\left(\frac{\tau v^{-\sqrt{3}\xi_0}}{TV^{1-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}} \quad (47)$$

Temperature (*T*) also affects the expression above. When the temperature is very high in the early stages of the universe, $T \to \tau$, the equation (47) indicates that the cosmos is dominated by dust, as $\omega_{eff} = 0$ and $p_{eff} = 0$. The ΛCDM model is shown by the equation (47), which yields $\omega_{eff} = -1$ when the universe is in its late stage, when the temperature is extremely low, i.e., $T \to 0$.

We will now analyze equation (44) to see if $\left(\frac{\partial p_{eff}}{\partial V}\right)_T \leq 0$. Consequently, we obtain

$$\left(\frac{\partial p_{eff}}{\partial V}\right)_T = -\frac{\left(\frac{\sqrt{3}\xi_0}{\alpha V}\right)\left(\frac{A}{1-\sqrt{3}\xi_0}\right)^{\frac{1}{\alpha+1}}\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}\left[\sqrt{3}\xi_0+\sqrt{3}\alpha\xi_0-\alpha+\alpha\left(1-\sqrt{3}\xi_0\right)\left(\frac{TV^{-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}\right]}{\left[1-\left(\frac{TV^{1-\sqrt{3}\xi_0}}{\tau v^{-\sqrt{3}\xi_0}}\right)^{1+\frac{1}{\alpha}}\right]^{\frac{\alpha+2}{\alpha+1}}} \quad (48)$$

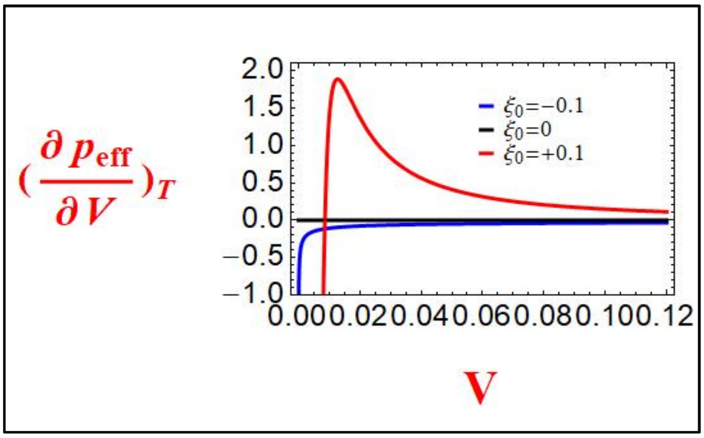

Fig. 7. The variation of effective pressure gradient $\left(\frac{\partial p_{eff}}{\partial V}\right)_T$ and volume V for various values of ξ_0 with $B = 1$, $\alpha = 0.5$, $T = 1$, $\tau = 2.73$ and $\nu = 1$

This clearly in Fig.7 indicates that the value of ξ_0 must be negative for $\left(\frac{\partial p_{eff}}{\partial V}\right)_T < 0$ throughout the evolution. Notably, when ξ_0 equals zero, $\left(\frac{\partial p_{eff}}{\partial V}\right)_T$ becomes zero as well. At this point, we take a moment to compare our findings with the earlier work of *Santos et al.* [53, 54]. They only calculated pressure as a function of temperature in their analysis of the generalized Chaplygin gas model, and they applied the same hypothesis to the MCG model, which led to $\left(\frac{\partial p_{eff}}{\partial V}\right)_T = 0$ in both scenarios. However, in our analysis, with $\xi_0 > 0$, $\left(\frac{\partial p_{eff}}{\partial V}\right)_T$ is consistently less than zero. This means that the isobaric curves in our viscous generalized Chaplygin gas model do not overlap with the isotherms within the thermodynamic state diagram. This distinction represents a meaningful advancement in our analysis. Thus, we conclude that $\left(\frac{\partial p_{eff}}{\partial V}\right)_V < 0$ and $\left(\frac{\partial p_{eff}}{\partial V}\right)_T < 0$ for negative values of ξ_0, which is satisfied the stability conditions of thermodynamics.

Pressure-Volume relation of VGCG model

Using the following extreme conditions, we have investigated the relationship between effective pressure and volume.
Now, for a small volume $V \ll \epsilon$, the effective pressure $p_{eff} = (-\sqrt{3}\xi_0)\rho_{vgcg}$, since $\epsilon/V = \infty$. Both pressure and energy density are extremely high in this instance. Equation (38) can now be used to express the temperature as

$$T = \frac{\tau v^{-\sqrt{3}\xi_0}}{V^{1-\sqrt{3}\xi_0}} \tag{49}$$

Early in the universe's existence, the transition from V to v (lowest volume) is associated with the transition from T to τ (maximum temperature). At this point, the temperature is significantly high.
By applying equation (49) and utilizing the relationship $\rho_{vgcg} = -\left(\frac{TS}{V}\right)$, we derive

$$\rho_{vgcg} = \frac{S\tau v^{-\sqrt{3}\xi_0}}{V^{1-\sqrt{3}\xi_0}} \tag{50}$$

Hence,

$$UV^{-\sqrt{3}\xi_0} = \rho_{vgcg}V^{1-\sqrt{3}\xi_0} = S\tau v^{-\sqrt{3}\xi_0} \tag{51}$$

At the early universe, we know that $p_{eff} = (-\sqrt{3}\xi_0)\rho_{vgcg}$, i.e., $p_{eff}V = (-\sqrt{3}\xi_0)U$, we have $p_{eff}V^{1-\sqrt{3}\xi_0} = S\tau v^{-\sqrt{3}\xi_0}$ (using equation (51)). In an adiabatic process, since the entropy is constant, the relation leads to $p_{eff}V^{1-\sqrt{3}\xi_0} = constant$. Thus, for small volume, such as, at high temperature, the VGCG is found to act as a fluid of $\gamma = \frac{C_p}{C_v} = (1 - \sqrt{3}\xi_0)$. The EoS can also be rewritten as $p_{eff} = (\gamma - 1)\rho_{vgcg}$. Since the pressure-volume relation at early universe $p_{eff}V^{\frac{4}{3}} = constant$ for $\xi_0 = -\frac{1}{3\sqrt{3}}$; $p_{eff}V^2 = constant$ for $\xi_0 = -\frac{1}{\sqrt{3}}$; $p_{eff}V^{\frac{2}{3}} = constant$ for $\xi_0 = \frac{1}{3\sqrt{3}}$ and $p_{eff}V = constant$ for $\xi_0 = 0$. When the volume is large (V >> ϵ), so we have $\frac{\epsilon}{V} = 0$. The entropy is significantly lower at low temperatures because of the low density at this point; thus, we obtain

$$p_{eff} = -\rho_{vgcg} \tag{52}$$

When, $\omega_{eff} = -1$, then from equation (12), we get

$$\rho_{vgcg} \approx \left(\frac{A}{1-\sqrt{3}\xi_0}\right)^{\frac{1}{\alpha+1}} \quad (53)$$

Now using equations (52) and (53), we get

$$p_{eff} = -\left(\frac{A}{1-\sqrt{3}\xi_0}\right)^{\frac{1}{\alpha+1}} \quad (54)$$

Dark energy is the state in which the effective pressure is negative and constant for adiabatic systems, which occur when the universe is late in its life cycle and at a lower temperature.

CONCLUSIONS

The thermodynamic stability and positivity of the thermal heat capacity (c_V) of the VGCG model against the backdrop of the FRW universe have been examined in this publication. Using physical characteristics like sound speed, deceleration parameter, effective pressure, and effective EoS, we also investigated the behavior of an expanding universe. The following is a summary of the aforementioned parameters:

- Plotting effective pressure with various viscous parameter ξ_0 values has revealed results. As illustrated in Fig.1, we also find that, depending on the value of ξ_0, the pressure can be both positive and negative. Additionally, we noticed that this model exhibits a constant negative pressure at large volumes while acting as a dust- dominated universe at small volumes. The pressure is always negative for $\xi_0 = 0$ and $\xi_0 > 0$, which denotes a Chaplygin-type gas.
- As seen in Fig.2, the EoS parameter for the VGCG model demonstrates that the viscous parameter ξ_0 determines the decelerated and accelerated phases of our universe. The critical volume was established at $\omega_{eff} = 0$, and its value rises with $\xi_0 > 0$.
- As seen in Fig.3, the universe accelerates when $V > V_f$, while it decelerates when $V < V_f$ for negative values of ξ_0. We determine the flip volume at acceleration when the flip happens. Additionally, we demonstrate how the universe accelerates at bigger volumes due to this flip volume, in contrast to its deceleration at lower sizes.
- Fig. 4 illustrates how sound speed changes with volume for a range of ξ_0 values. In the range of $-\frac{1}{3\sqrt{3}} < \xi_0 < \frac{1}{3\sqrt{3}}$, we examined the stability of the VGCG model using adiabatic sound speed and discovered that it was stable for the value of $\alpha = \frac{1}{2}$. We know that sound speed can be either positive or negative in the VMCG model [58], but in the GCCG model, there are no stable areas at the late universe [70].
- Finally, we examined the thermodynamic stability requirement in terms of specific heat, adiabatic states, and isothermal states. We demonstrate the adiabatically expansion of the VGCG model for any volume and the thermodynamic stability of the expansion at any pressure. Also, we verified the validity of the third law of thermodynamics. The specific heat at constant volume is always non-negative for the GCG and MCG models [53, 54], but it is always positive for the VGCG model when $0 < T < \tau$, $v < V < \infty$, and $\alpha > 0$. Additionally, we discovered that entropy is zero at temperature T = 0, confirming that the VGCG model complies with the third law of thermodynamics. The VGCG model is found to be thermodynamically stable in adiabatic conditions.
- As a stability condition in Fig.7, it is clear from equation (48) that $\left(\frac{\partial p_{eff}}{\partial V}\right)_T$ stays negative during the evolution when $\xi_0 < 0$. Nevertheless, $\left(\frac{\partial p_{eff}}{\partial V}\right)_T$ *is zero* when $\xi_0 = 0$. In order, to derive the expression for b, *Santos et al.* (53, 54) assumed $\left(\frac{\partial p_{eff}}{\partial V}\right)_T = 0$. This is comparable to the example they studied. In a particular pressure in the GCG model was only calculated as a function of temperature, and the same supposition was used for the MCG model, which is that $\left(\frac{\partial p_{eff}}{\partial V}\right)_T = 0$ in both scenarios. In contrast, $\left(\frac{\partial p_{eff}}{\partial V}\right)_T$ naturally becomes negative for $\xi_0 < 0$ in our case, suggesting that the isobaric curves of our viscous generalized Chaplygin gas (VGCG) model do not match its isotherms on the diagram of thermodynamic states.

This model based on theoretical findings, has been used to study the thermodynamic nature of the cosmos.

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