Available online at http://www.journalcra.com

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

International Journal of Current Research
Vol. 6, Issue, 07, pp.7458-7465, July, 2014

ISSN: 0975-833X

RESEARCH ARTICLE

A NOTE ON FUZZY CRITICAL PATH ANALYSIS IN PROJECT NETWORKS

Stephen Dinagar, D. and *Abirami, D.

Department of Mathematics, TBML College, Porayar, India

ARTICLE INFO

Article History:
Received 09th April, 2014
Received in revised form
30th May, 2014
Accepted 17th June, 2014
Published online 20th July, 2014

Key words:

Interval valued fuzzy numbers, Fuzzy arithmetic, Project network, Critical path.

ABSTRACT

In this paper, two different approaches such as ranking value of a fuzzy number approach and a distinct approach to critical path analysis in project networks are presented in a fuzzy environment. In project network, the activity times are uncertain and are represented by interval valued fuzzy numbers (IVFNS) in the work. The corresponding fuzzy arithmetics and fuzzy ranking values of a fuzzy numbers are utilized to determine the fuzzy critical path without converting the fuzzy activity time to classical number. Relevant numerical examples are also included to justify the proposed notions.

INTRODUCTION

A project defines a combination of interrelated activities that must be executed in a certain order before the entire task can be completed. The activities are interrelated in a logic sequence, an activity in a project is usually viewed as a job requiring time and resources for its completion. In general a project is a one-time effort that in the same sequence of activities may not be repeated in future. A fundamental approach to solve these problems is applying fuzzy sets. Introducing the fuzzy set by zadeh in 1965 opened promising new horizons to different scientific area such as project scheduling fuzzy theory, several methods have been developed during the last three decades. The first method was proposed by Chanas and Kamburowski (1961). They presented the project completion time in the form of a fuzzy set in the time space. Gazdik (1983) developed fuzzy network of an priori unknown project to estimate the activity duration and fuzzy algebraic operator to calculates the duration of the project and its critical path. The fuzzy networking was proposed by Nasution (1994) and Lorterapong and Moselhi (1996). Following on this, Mc cahon (1993) Chang *et al.* (1995) and Lin and Yao (2003) presented three methodologies to calculate fuzzy completion project time. Other resources such as Kuchta (2001) and Oliveros and Rabinson (2005) using fuzzy numbers presented other methods to obtain fuzzy critical paths, critical activities and activity delay. Previous work on network scheduling using fuzzy theory provides methods for scheduling projects. Chen and Huang (2007), applied fuzzy method for measuring criticality in project network. Ravishankar *et al.* (2010) proposed an analytical method for finding critical path in a fuzzy project network. In this paper, a new approach of ranking value of a fuzzy numbers and a distinct approach to fuzzy critical analysis are introduced. It is also assumed that the uncertain parameters are represented by IVFNS. An algorithm to tackle the problem in fuzzy project decision analysis is proposed. Finally an illustrative numerical example is given to demonstrate the validity of the proposed methods.

The rest of this paper is organized as follows

In section 2, the basic concepts of fuzzy numbers, ranking and other related results are presented.
In section 3, two different approaches for analyzing the fuzzy critical path are discussed with relevant numerical examples.

2. PRELIMINARIES

In this section, some important definitions and results which are useful to this work are presented.

Corresponding author: Abirami, D.
Department of Mathematics, TBML College, Porayar, India.

2.1 Definition

A Fuzzy set $\tilde{a}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics.

$\tilde{a}$ is convex, i.e., $\tilde{a}$ (λ X_1 + (1-λ) X_2) = Minimum { $\tilde{a}$ (X_1), $\tilde{a}$ (X_2)}, for all X_1,X_2 ∈ R and λ∈ (0,1)
$\tilde{a}$ is normal i.e., there exists an X_0 ∈ R such that $\tilde{a}$ (X_0) = 1
$\tilde{a}$ is piecewise continuous.

2.2 Definition

An IVFN $\tilde{A}$ on R is given by

$\tilde{A}$ = { x,(μ_A^L(x), μ_A^U(x)), x∈R } and μ_A^L(x) ≤ μ_A^U(x) for all x∈R.

Denote $\tilde{A} = (\tilde{A}^L, \tilde{A}^U)$, where $\tilde{A}^L = (a_1^L, a_2^L, a_3^L, a_4^L)$ and $\tilde{A}^U = (a_1^U, a_2^U, a_3^U, a_4^U)$ are the trapezoidal fuzzy numbers.

It is also noted that $a_1^U \le a_1^L$, $a_2^U \le a_2^L$, $a_3^L \le a_3^U$, $a_4^L \le a_4^U$

Example : 2.1

Let $\tilde{A}$ = ((2,4,5,7), (1,3,6,8))

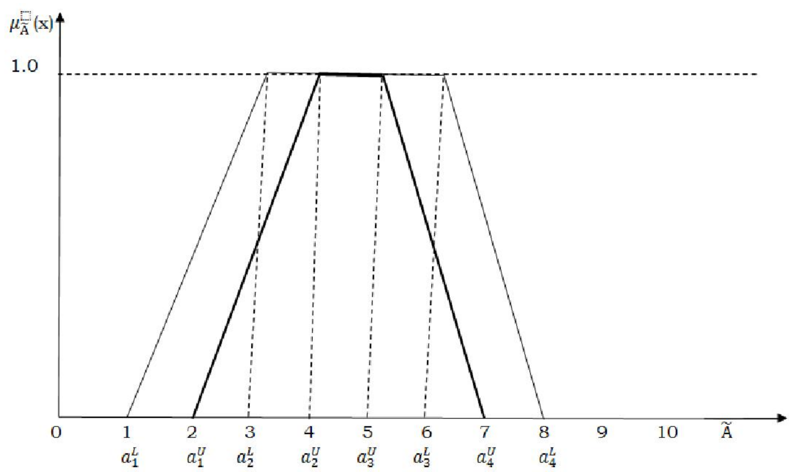

Fig 2.2.1 – IVFN Ã

$\tilde{A} = (a^L, a^U) = ((a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U))$
Here $a_1^U \le a_1^L$, $a_2^U \le a_2^L$, $a_3^U \ge a_3^L$, $a_4^U \ge a_4^L$

2.3: Arithmetic operations on IVFNS

Let $\tilde{A} = ((a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U))$ and $\tilde{B} = ((b_1^L, b_2^L, b_3^L, b_4^L), (b_1^U, b_2^U, b_3^U, b_4^U))$ then we define

(i)Addition:

$\tilde{A} \oplus \tilde{B} = ((a_1^L+b_1^L, a_2^L+b_2^L, a_3^L+b_3^L, a_4^L+b_4^L),(a_1^U+b_1^U, a_2^U+b_2^U, a_3^U+b_3^U, a_4^U+b_4^U))$

(ii)Subtraction:

$\tilde{A} \ominus \tilde{B} = ((a_1^L-b_4^U, a_2^L-b_3^L, a_3^L-b_2^L, a_4^L-b_1^L),(a_1^U-b_4^U, a_2^U-b_3^U, a_3^U-b_2^U, a_4^U-b_1^U))$

2.4 Definition

An efficient approach for comparing the fuzzy number is by the use of ranking function R: F(R) → R, where F(R) is a set of fuzzy numbers defined on a set of real numbers, which maps each fuzzy number into a real number where a natural order exists.

For $\widetilde{A} = (\tilde{a}^L, \tilde{a}^U) \in F(R)$, Then the ranking function $R : F(R) \rightarrow R$ defined as:

$R(\widetilde{A}) = (a_1^L + a_2^L + a_3^L + a_4^L + a_1^U + a_2^U + a_3^U + a_4^U)/8$

2.5 Remark:

Let $\widetilde{A} = ((a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U))$ and $\widetilde{B} = ((b_1^L, b_2^L, b_3^L, b_4^L), (b_1^U, b_2^U, b_3^U, b_4^U))$

(i) We have the following comparison:

$\widetilde{A} > \widetilde{B}$ iff $R(\widetilde{A}) > R(\widetilde{B})$
$\widetilde{A} < \widetilde{B}$ iff $R(\widetilde{A}) < R(\widetilde{B})$
$\widetilde{A} = \widetilde{B}$ iff $R(\widetilde{A}) = R(\widetilde{B})$

(ii) An IVFN is said to be positive if $R(\widetilde{A}) > 0$.That is $\widetilde{A} > \tilde{0}$ if $R(\widetilde{A}) > 0$.
Also $\widetilde{A} = \tilde{0}$ iff $R(\widetilde{A}) = 0$ and $\widetilde{A} < \tilde{0}$ if $R(\widetilde{A}) < 0$

(iii) If $\widetilde{A}$ and $\widetilde{B}$ are said to be equivalent, if $R(\widetilde{A}) = R(\widetilde{B})$.

2.6 Definition

An IVFN $\widetilde{A}$ is called as zero fuzzy number if $\widetilde{A} = ((0,0,0,0), (0,0,0,0))$ and denoted by $\tilde{0}$

2.7 Definition

An IVFN $\widetilde{A}$ is called as zero – equivalent fuzzy number if $R(\widetilde{A}) = 0$ and denoted by $\hat{0}$.

3. FUZZY CRITICAL PATH ANALYSIS

In this section, two different approaches to analysis the fuzzy critical path are discussed.

3.1 Ranking value of fuzzy number Approach

Let $\widetilde{A}_i = ((a_i^L, b_i^L, c_i^L, d_i^L), (a_i^U, b_i^U, c_i^U, d_i^U))$, i = 1,2….n, be fuzzy numbers with membership functions $\mu_{\widetilde{A}_i}$ respectively.

Let x_1 = Infimum D, x_2 = Supremum D, and $D_i = \{ x | \mu_{\widetilde{A}_i}(x) \geq 0\}$, i = 1,2,3…..n, $\mu_{\widetilde{A}^L}(x)>0$ $\mu_{\widetilde{A}^U}(x)>0$ Then ranking value of fuzzy number $\widetilde{A}_i$, $R(\widetilde{A}_i)$, is defined as

$$R(\widetilde{A}_i) = \frac{\beta}{2}((d_i^L+d_i^U) - x_1 / x_2 - x_1 - (c_i^L+c_i^U) + (d_i^L+d_i^U)) + \left(\frac{1-\beta}{2}\right)(2-((x_2 - (a_i^L+a_i^U))/(x_2 - x_1 + (b_i^L+b_i^U) - (a_i^L+a_i^U)) \quad (3.1)$$

The value of β can be referred to as decision makers risk index. If β < 0.5, it implies that the decision maker is a risk averter. If β = 0.5, it implies that the risk attitude of decision maker is natural. If β > 0.5, it implies that the decision maker is risk lover. For a fuzzy critical path (FCP) analysis problem, using the IVFNS such as $F\widetilde{E}T_{ij} = ((a_{ij}^L, b_{ij}^L, c_{ij}^L, d_{ij}^L), (a_{ij}^U, b_{ij}^U, c_{ij}^U, d_{ij}^U))$ to denote the fuzzy activity time of activity $\widetilde{A}_{ij}$, the decision maker's risk attitude index β can be obtained by

$$\beta = \left(\sum_i \sum_j \frac{(b_{ij}^L+b_{ij}^U)-(a_{ij}^L+a_{ij}^U)}{(((b_{ij}^L+b_{ij}^U)-(a_{ij}^L+a_{ij}^U))+((d_{ij}^L+d_{ij}^U)-(c_{ij}^L+c_{ij}^U)))} \right) \Big/ t, \quad \widetilde{A}_{ij} \in ACT \quad (3.2)$$

Where ACT and t denote the set of all activities and the number of activities in a project network, respectively.

We can write x_1 as x_1 = Minimum $\{(a_1^L+a_1^U), (a_2^L+a_2^U)…. (a_n^L+a_n^U)\}$ and x_2 as x_2 = Maximum $\{(d_1^L+d_1^U), (d_2^L+d_2^U)…. (d_n^L+d_n^U)\}$

Taking β value calculated by the equation (3.2) and by using equation (3.1),we can easily calculate the ranking values of the n IVFNS. Then based on the ranking rules described above, the ranking of the n IVFNS can be effectively determined.

3.1.1. Notations

The notations that will be used in the presented methods are as follows

N : The set of all nodes in a project network.
A_{ij} : The activity between nodes i and j
$F\widetilde{E}T_{ij}$: The fuzzy activity time of A_{ij}.
$F\widetilde{E}S_j$: The earliest fuzzy time of node j.
$F\widetilde{L}F_j$: The latest fuzzy time of node j.
$F\widetilde{T}S_{ij}$: The total slack fuzzy time of A_{ij}.
S(j) : The set of all successor activities of node j.
NS(j) : The set of all nodes connected to all successor activities of node j. i.e., $NS(j) = \{ k|A_{jk} \in S(j), k \in N \}$
F(j) : The set of all predecessor activities of node j.
NP(j) : The set of all nodes connected to all predecessor activities of node j. i.e., $NP(j) = \{ i/ A_{ij} \in N \}$
P_i : The i^{th} path.
P : The set of all paths in a project network.
$F\widetilde{C}PM(P_K)$: The fuzzy completion time of path P_k in a project network.

3.1.2. SOME IMPORTANT PROPERTIES

In this section, some properties that will be used in this method for analyzing the FCP are presented. Set the initial node to be zero for starting.

i.e., $F\widetilde{E}S_1 = ((0,0,0,0),(0,0,0,0))$

Then the following properties are true :

Property 3.1

$F\widetilde{E}S_j$ = Maximum $\{ F\widetilde{E}S_i \oplus F\widetilde{E}T_{ij} | i \in NP(j), j \neq 1, j \in N \}$

Property 3.2

$F\widetilde{L}F_j$ = Minimum $\{ F\widetilde{L}F_k \ominus F\widetilde{E}T_{jk} | k \in NS(j), j \neq n, j \in N \}$

Property 3.3

$F\widetilde{T}S_{ij} = F\widetilde{L}F_j \ominus (F\widetilde{E}S_j \oplus F\widetilde{E}T_{ij}),\ 1 \leq i < j \leq n;\ i.j \in N$

Property 3.4

$$F\widetilde{C}PM(P_k) = \sum F\widetilde{T}S_{ij}\ ,\quad P_k \in p. \quad \begin{matrix} 1 \leq i < j \leq n \\ i, j \in P_k \end{matrix}$$

3.1.3 Definition

If there exists a path P_c in a project network such that $F\widetilde{C}PM(P_c)$ = Minimum $\{ F\widetilde{C}PM(P_i)|P_i \in P\}$ then the path P_c is a fuzzy critical path (FCP).

3.1.4. Algorithm for finding the fuzzy critical path

The FCP of a Project network can be obtained by using the following steps :

1. Identify activities in a project.
2. Establish precedence relationship of all activities.
3. Estimate the fuzzy activity time with respect to each activity.
4. Construct the project network.
5. Let $F\widetilde{E}S_1 = ((0,0,0,0), (0,0,0,0))$ and calculate
   $F\widetilde{E}S_j$, j = 2,3,….n by using property (3.1)
6. Let $F\widetilde{L}F_n = F\widetilde{E}S_n$ and calculate $F\widetilde{L}F_j$, j = n-1, n-2,… 2.1, by using property 3.2
7. Calculate $F\widetilde{T}S_{ij}$ with respect to each activity in a project network by using property 3.3

8. Find all the possible paths and calculate $F\tilde{C}PM(P_k)$ by using property 3.4
9. Find the FCP by using 3.1.3 Definition
10. Find the grade of membership that the project can be completed at scheduled time.

3.1.5. Illustration

Suppose there is a project network, as shown in Fig 3.1 with the set of node N = {1,2,3,4}, the fuzzy activity time for each activity as shown in Table 3.1 All of the duration are in hours. Find the FCP for the given network.

Table 3.1 Fuzzy activity time for each activity

Activity A_{ij}	Fuzzy activity times (Hours) $F\widetilde{ET}_{ij}$
A_{12}	Approximately 5 Hours ((4,5,5,6), (3,5,5,7))
A_{13}	Approximately 10 Hours ((8,10,10,15), (5,10,10,15))
A_{23}	Approximately between 3 and 4 Hours ((2,3.3,3.7,4), (1,3,4,5))
A_{24}	Approximately between 4 and 5 Hours ((3,4.4,4.6,5), (2,4,5,6))
A_{34}	Approximately between 8 and 10 Hours ((7,8.2,9.8,10), (6,8,10,11))

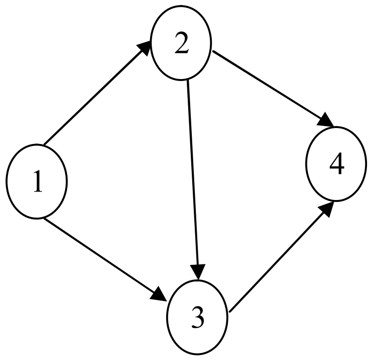

Fig 3.1. A Project network

By using equation (3.2) $\beta = 0.6$

The FCP of the given network shown in Fig 3.1, can be obtained by using the following steps :

Step 1

Set $F\widetilde{ES}_1$ = ((0,0,0,0), (0,0,0,0)) and calculate $F\widetilde{ES}_j$, j = 2,3,4 by Using property 3.1

$F\widetilde{ES}_2 = F\widetilde{ES}_1 \oplus F\widetilde{ET}_{12}$ = ((0,0,0,0), (0,0,0,0)) ⊕ ((4,5,5,6), (3,5,5,7)) = ((4,5,5,6), (3,5,5,7))

$F\widetilde{ES}_3$ = Maximum { $F\widetilde{ES}_1 \oplus F\widetilde{ET}_{13}$, $F\widetilde{ES}_2 \oplus F\widetilde{ET}_{23}$} = Maximum{((8,10,10,12), (5,10,10,15)), ((6,8.3,8.7,10), (4,8,9,12))}

Taking $\beta = 0.6$ the ranking value of ((8,10,10,12), (5,10,10,15)) and ((6,8.3,8.7,10), (4,8,9,12)) can be obtained :

x_1 = Minimum { 14, 9 }
x_2 = Maximum { 22, 27 }
$x_1 = 9$ $x_2 = 27$

Then, R ((8,10,10,12), (5,10,10,15)) = 0.50400
R ((6,8.3,8.7,10), (4,8,9,12)) = 0.43496
Since R ((8,10,10,12), (5,10,10,15)) > R ((6,8.3,8.7,10), (4,8,9,12))
$F\widetilde{ES}_3$ = ((8,10,10,12), (5,10,10,15))
$F\widetilde{ES}_4$ = Maximum { $F\widetilde{ES}_2 \oplus F\widetilde{ET}_{24}$, $F\widetilde{ES}_3 \oplus F\widetilde{ET}_{34}$}
= Maximum { ((7,9.4,9.6,11), (5,9,10,13)), ((15,18.2,19.8,22),(11,18,20,26)) } by using equation (3.1) and taking $\beta = 0.6$
$F\widetilde{ES}_4$ = ((15,18.2,19.8,22), (11,18,20,26))

Step 2

Set $F\tilde{L}F_4$= ((15,18.2,19.8,22),(11,18,20,26)) and calculate $F\tilde{L}F_j$, j = 3,2,1by property 3.2

$F\tilde{L}F_3$= $F\tilde{L}F_4 \ominus F\tilde{E}T_{34}$=((15,18.2,19.8,22),(11,18,20,26)) ⊖((7,8.2,9.8,10),(6,8,10,11))

$F\tilde{L}F_3$= ((5,8.4,11.6,12), (0,8,12,20))

$F\tilde{L}F_2$= Minimum { $F\tilde{L}F_4 \ominus F\tilde{E}T_{24}$, $F\tilde{L}F_3 \ominus F\tilde{E}T_{23}$}= Minimum{((10,13.6,15.4,19),(5,13,16,24)),((1,4.7,8.3,13), (-5,4,9,19))}

Taking β = 0.6

$F\tilde{L}F_2$ = ((1,4.7,8.3,10), (-5,4,9,19))

$F\tilde{L}F_1$ = Minimum { $F\tilde{L}F_3 \ominus F\tilde{E}T_{13}$, $F\tilde{L}F_2 \ominus F\tilde{E}T_{12}$} = ((-7,-1.6,1.6,7), (-15,-2,2,15))

Step 3

Calculate $F\tilde{T}S_{ij}$ with respect to each activity by property 3.3

$F\tilde{T}S_{12} = F\tilde{L}F_2 \ominus (F\tilde{E}S_1 \oplus F\tilde{E}T_{12})$ = ((-5,-0.3,3.3,9), (-12,-1,4,16))

$F\tilde{T}S_{13} = F\tilde{L}F_3 \ominus (F\tilde{E}S_1 \oplus F\tilde{E}T_{13})$ = ((-7,-1.6,1.6,7), (-15,-2,2,15))

$F\tilde{T}S_{23} = F\tilde{L}F_3 \ominus (F\tilde{E}S_2 \oplus F\tilde{E}T_{23})$ = ((-5,-0.3,3.3,9), (-12,-1,4,16))

$F\tilde{T}S_{24} = F\tilde{L}F_4 \ominus (F\tilde{E}S_2 \oplus F\tilde{E}T_{24})$ = ((5,8.6,10.4,15), (-2,8,11,21))

$F\tilde{T}S_{34} = F\tilde{L}F_4 \ominus (F\tilde{E}S_3 \oplus F\tilde{E}T_{34})$ = ((-7,-1.6,1.6,7), (-15,-2,2,15))

Step 4

Find all the possible paths and calculate $F\tilde{C}PM$ (P_k) by using property 3.4

P = {(1,2,4), (1,2,3,4), (1,3,4)}

P_1 = (1,2,4) $F\tilde{C}PM(P_1)$ = $F\tilde{T}S_{12} \oplus F\tilde{T}S_{24}$ = ((0,8.3,13.7,24), (-14,7,15,37))

P_2 = (1,2,3,4) $F\tilde{C}PM(P_2)$ = $F\tilde{T}S_{12} \oplus F\tilde{T}S_{23} \oplus F\tilde{T}S_{34}$ = ((-17,-2.2,8.2,25),(-39,-4,10,47))

P_3 = (1,3,4) $F\tilde{C}PM$ (P_3) = $F\tilde{T}S_{13} \oplus F\tilde{T}S_{34}$ = ((-14,-3.2,3.2,14), (-30,-4,4,30))

Step 5

Find the FCP the taking β = 0.6.The ranking value of $F\tilde{C}PM$ (P_i), i = 1,2,3 can be obtained.

R ($F\tilde{C}PM$ (P_1)) = 0.50962

R ($F\tilde{C}PM(P_2)$) = 0.46723

R ($F\tilde{C}PM$ (P_3)) = 0.44126

Since R ($F\tilde{C}PM$ (P_3)) < R ($F\tilde{C}PM$ (P_2)) < R (($F\tilde{C}PM$ (P_3)) the FCP is P_3 and the project completion time is approximately between 18 and 20 hours. i.e., ((15,18.2,19.8,22), (11,18,20,26)

3.2 A Distinct approach to fuzzy critical path

A Fuzzy Project network is an acyclic digraph where the vertices represent events, and the directed edges represents the activities to be performed in a project.

Table 3.2. Fuzzy Critical Activity

	A_{12}	A_{13}	A_{23}	A_{24}	A_{34}
Duration	[(4,5,5,6),(3,5,5,7)]	[(8,10,10,12),(5,10,10,15)]	[(2,3.3,3.7,4),(1,3,4,5)]	[(3,4.4,4.6,5),(2,4,5,6)]	[(7,8.2,9.8,10),(6,8,10,11)]
Earliest Start	[(0,0,0,0),(0,0,0,0)]	[(0,0,0,0),(0,0,0,0)]	[(4,5,5,6),(3,5,5,7)]	[(4,5,5,6),(3,5,5,7)]	[(7,8.2,9.8,10),(6,8,10,11)]
Earliest Finish	[(4,5,5,6),(3,5,5,7)]	[(8,10,10,15),(5,10,10,15)]	[(6,8.3,8.7,10),(4,8,9,12)]	[(7,9.4,9.6,11),(5,9,10,13)]	[(15,18.2,19.8,22),(11,18,20,26)]
Latest Start	[(4,5,5,6),(3,5,5,7)]	[(-4,0,0,4),(-10,0,0,10)]	[(4,6.3,6.7,10),(0,6,7,14)]	[(10,13.6,15.4,19),(5,13,16,24)]	[(5,8.4,11.6,15),(0,8,12,20)]
Latest Finish	[(-2,0,0,2),(-4,0,0,4)]	[(8,10,10,12),(5,10,10,15)]	[(8,10,10,12),(5,10,10,15)]	[(15,18.2,19.8,22),(11,18,20,26)]	[(15,18.2,19.8,22),(11,18,20,26)]
Total Float	[(-2,1.3,1.7,6),(-7,1,2,11)]	[(-7,-1.6,1.6,7),(-15,-2,2,15)]	[(-5,-0.3,3.3,9),(-12,-1,5,16)]	[(6,8.6,10.4,15),(-2,8,11,21)]	[(-7,-1.6,1.6,7),(-15,-2,2,15)]

Fuzzy critical path is 1 → 3 → 4 The minimum fuzzy project duration is the length of the fuzzy critical path. The fuzzy project duration is in fuzzy hours.[(15,18.2,19.8,22),(11,18,20,26)]

3.2.1 Algorithm for Fuzzy Critical Activity

Let $F\tilde{E}S_i$ and $F\tilde{L}S_i$ be the earliest fuzzy event time, and the latest Fuzzy event time for event i, respectively Functions that define the earliest starting times, latest starting times and floats in terms of fuzzy activity durations are convex, normal whose membership functions are piecewise continues, hence the quantities such as earliest fuzzy event time $F\widetilde{E}S_i$, the latest fuzzy event time $F\widetilde{L}S_i$ and the floats $\widetilde{T}_i$ are also IVFNS for an event i respectively.

Step 1: Identify fuzzy activities in a fuzzy project
Step 2: Establish precedence relationship of all fuzzy activities,by applying fuzzy ranking function
Step 3: Construct the fuzzy project network with IVFNS as fuzzy activity times.
Step 4: Let $F\widetilde{E}S_1$ be the earliest Fuzzy event time and $F\widetilde{L}S_1$ be the latest fuzzy event time for the initial event $\widetilde{V}_1$ of the project network and assume that $F\widetilde{E}S_1 = F\widetilde{L}S_1=\widetilde{0}$ Compute the earliest fuzzy event time $F\widetilde{E}Sj$ of the event $\widetilde{V}_j$ by using the formula

$$F\widetilde{E}S_j = \max_{i\in N:i\rightarrow j}\{F\widetilde{E}Si + \widetilde{A}_{ij}\} \quad (3.3)$$

Step 5: Let $F\widetilde{E}S_n$ be the earliest fuzzy event time and $F\widetilde{L}S_n$ be the latest fuzzy event time for the terminal event $\widetilde{V}_n$ of the fuzzy project network and assume that $F\widetilde{E}S_n = F\widetilde{L}S_n$. Compute the latest fuzzy event time $F\widetilde{L}S_i$ by using the following equation

$$F\widetilde{L}S_i = \min_{i\in N}\{ F\widetilde{L}S_j - \widetilde{A}_{ij} \} \quad (3.4)$$

Step 6: Compute the total float $\widetilde{T}_{ij}$ of each fuzzy activity $\tilde{a}_{ij}$ by using the following equation $\widetilde{T}_{ij} = \{ F\widetilde{L}S_j - F\widetilde{E}S_i - \widetilde{A}_{ij} \}$ (3.5)

Hence we can obtain the earliest fuzzy event time, latest fuzzy event time, and the total float of every fuzzy activity by using equations (3.3),(3.4)and(3.5).

Step 7: If $\widetilde{T}_{ij} = \widetilde{0}$, then the activity $\widetilde{A}_{ij}$ is said to be a fuzzy critical activity. That is activities with zero equalent of total float are called, fuzzy critical activities, and are always found on one or more fuzzy critical paths.

Step 8: The length of the longest fuzzy critical path from the start of the fuzzy project to its finish is the minimum time required to complete the fuzzy project. This is fuzzy project duration.

3.2.2 Illustration

Let us consider the same illustration(3.1.5) and finding the fuzzy critical path by using the above algorithm(3.2.1).

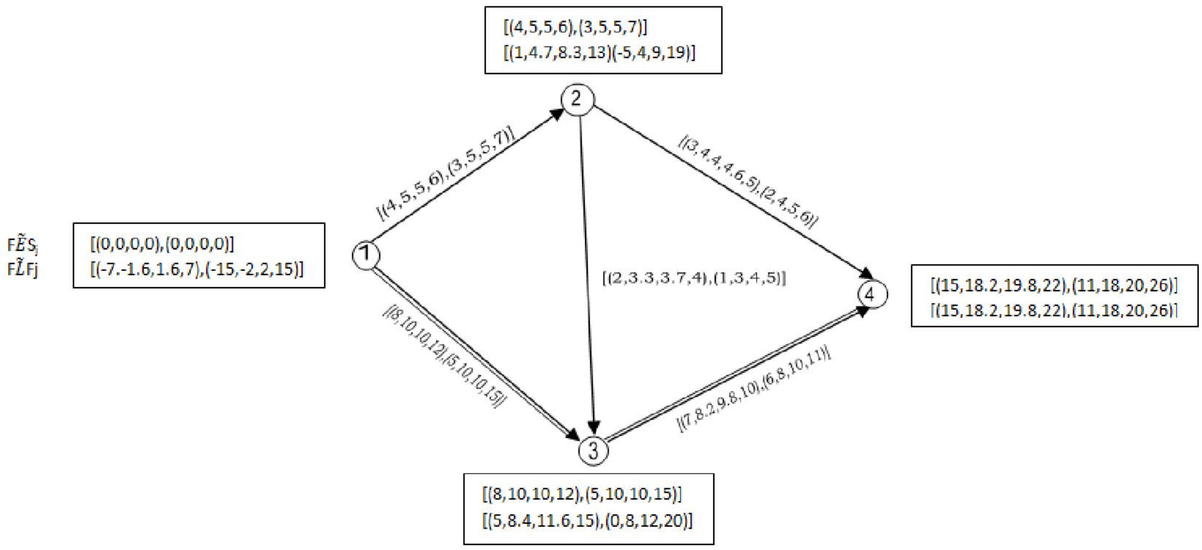

Fig 3.2
Fuzzy Critical Path

Conclusion

In this project evaluation analysis, two different approaches namely ranking value of a fuzzy numbers and a distinct approach to fuzzy critical analysis are introduced. In this work, it is important to note that a relevant numerical illustration was utilized to justify the both proposed notions. The project characteristics like earliest time latest time and total float time in terms of IVFNS are calculated without converting the fuzzy nature to classical nature.

REFERENCES

Chanas S., J. Kamurowsks, The Use of fuzzy variables in PERT, fuzzy sets and systems, 5(1)(1961) 11-19.

Chang S., Y. Tsujimur, T. Tazawa, An efficient approach for large scale project planning based on fuzzy Delphi method, Fuzzy Sets and Systems,76(1995) 277-288.

Chen C.T., S.F. Huang, Applying fuzzy method for measuring critically in project network, Information Sciences,177(2007) 2448-2458.

Gazdik J., Fuzzy-network-planning-FNET, IEEE Transaction Reliability, 32(2)(1983) 304-314.

Kuchta D., Use of fuzzy numbers in project risk(critically) assessment, International Journal of project management, 19(2001) 305-310.

Lin F.T., J.S.Yao, Fuzzy Critical Path method based on signed distance ranking and statistical confidence internal estimates, *Journal of Super computing*, 24(3)(2003) 305-325.

Lorterapong P., O.Moselhi, Project-network analysis Using Fuzzy Sets theory, *Journal of construction Engineering and Management,* 122(4)(1996) 308-318.

Mc Cahon C.S., Using PERT as an approximation of fuzzy project-network analysis, *IEEE transaction on engineering management*, 40(2)(1993) 146-153.

Nasution S.H., Fuzzy critical path method, IEEE Transaction on systems, MAN, AND cybernetics,41(1)(1994) 48-57.

Oliveros A., A. Robinson, Fuzzy logic approach for activity delay analysis scheduling updating, construction Engineering and Management 131(1)(2005) 42-51.

Ravishankar N., V.Sireesha, P. Phani Booshan Rao, An analytical method for finding critical path in a fuzzy project network, *International Journal Contemp Mathematical Sciences*,5(20)(2010) 953-962.

****