Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 7, Issue, 06, pp. 17327-17341, June 2015

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

DERIVATION OF MODIFIED REYNOLDS EQUATION FOR HYDROMAGNETIC SQUEEZE FILMS AND ITS APPLICATIONS TO ROUGH POROUS RECTANGULAR PLATES WITH LUBRICANT ADDITIVES

*,[1]Syeda Tasneem Fathima, [2]Naduvinamani, N. B. and [3]ShivaKumar, H. M. and [4]Hanumagowda, B. N.

[1]Department of Mathematic, Reva Institute of Technology and Management, Bangalore-560064, India
[2]Department of Mathematic, Gulbarga University, Gulbarga-585106, India
[3]Department of Mathematic, East West Institute of Technology, Bangalore-560091, India
[4]Department of Mathematics, East Point College of Engineering & Technology, Bangalore - 560049, India

ARTICLE INFO

Article History:
Received 26th March, 2015
Received in revised form
12th April, 2015
Accepted 29th May, 2015
Published online 30th June, 2015

Key words:

Squeeze films, Lubricant additives, Porous, Magnetic field, Rough surface.

ABSTRACT

Derivation of modified Reynolds equation is obtained for the hydromagnetic squeeze films with lubricant additives on the basis of hydromagnetic flow model and Stokes Microcontinuum fluid model. An example of porous rough rectangular plates is used to guide the Stochastic modified Reynolds equation with lubricant additives and MHD effects. The equations are solved analytically using the stochastic method developed by Christensen. It is found that, there exists a critical value of the aspect ratio above which the roughness effects are more pronounced due to size effect of the lubricant additives in the presence of transverse magnetic field. Further, this critical value is dependent on the microstructure size of the additives and magnetic parameter. The coarser the microstructure, higher is the critical value of the aspect ratio in the presence of magnetic field. It is found that the squeeze film characteristics are more pronounced for rough porous rectangular plates with increasing values of magnetic parameter.

Citation: Syeda Tasneem Fathima, Naduvinamani, N. B. and ShivaKumar, H. M. and Hanumagowda, B. N. 2015. "Derivation of modified reynolds equation for hydromagnetic squeeze films and its applications to rough porous rectangular plates with lubricant additives", *International Journal of Current Research*, 7, (6), 17327-17341.

INTRODUCTION

In recent years, surface roughness effects has attracted the attention of many researchers in the field of tribology. In the study of surface roughness effects on the lubrication characteristics of various bearing surfaces, in reality it is found that bearing surfaces are rough to some extent. Generally it is observed that, the roughness asperity height is of the same order as the mean separation between the two lubricated contacts. The concept of surface roughness has greater importance in the study of porous bearings, since the roughness is inherent in the process used for their manufacture. A stochastic model was investigated by Christensen (1969) for the first time, to study the rough surfaces in hydrodynamic lubrication of solid bearings. Christensen (1969) stochastic theory is widely used by many investigators for the study of concept of surface roughness in various bearing systems. Prakash and Tiwari (1982) studied the surface roughness effect on the characteristics of porous bearings, Gururajan and Prakash (1999) studied the surface roughness effects in an infinitely long porous journal bearing, Naduvinamani *et al*. (2004) studied the effect of surface roughness on the characteristics of squeeze film lubrication between isotropic porous rectangular plates with lubricant additives. The concept of Squeeze film phenomenon occurs, when two porous surfaces approach each other only part of the fluid between them is squeezed out, the remainder flows out through the porous media and thus the bearing decreases and film thickness attained in a specific time also decreases. The squeeze film behaviour when one surface was porous and backed by a solid impermeable wall was analyzed and discussed by Wu (1972) and Prakash and Vij (1973). The classical theory of fluids called Stokes Microcontinuum (1966) theory which allows polar effects such as the presence of couple stresses and body couples, essentially describes the peculiar behavior of Newtonian fluids containing microstructures which accounts for particle size effects. Couple stresses appear most particularly in thin film problems. Many authors have used this Sokes couple-stress model to study the various problems of hydrodynamic lubrication, for example, journal bearings (2004, 2007), porous circular disks (2008), slider bearings (2011).

Corresponding author: Syeda Tasneem Fathima,
Department of Mathematic, Reva Institute of Technology and Management, Bangalore-560064, India.

When under high temperature liquid metals are used as lubricants, the load carrying capacity of the bearing decreases because of their highly conducting properties and low viscosity (1957). The electromagnetic fields have been applied (1973) to improve upon these characteristics. Naduvinamani *et al.* (2010) have studied the magnetic effects in porous rectangular plates and investigated that bearing characteristics such as pressure distribution, load capacity, and squeezing time is significant for increasing values of Hartman number. Many other authors have studied the MHD effects (2012), (2012a) on the squeeze film bearings.

The aim of this paper is to derive a modified Stochastic Reynolds equation with lubricant additives effect with an application to rough porous rectangular plates. The modified Stochastic Reynolds equation governing the squeeze film pressure is derived on the basis of Stochastic roughness by Christensen and Stoke's microcontinuum theory to account for the lubricants blended with various additives. The results of squeeze film characteristics such as the squeeze film pressure, load carrying capacity and squeeze film time are presented for various values of magnetic parameter, lubricant additives parameter and roughness parameter.

Mathematical formulation

Schematic diagram of the squeeze film geometry under consideration is shown in Fig.1. A squeezing flow of non Newtonian fluid blended with lubricant additives between two rough porous rectangular plates is considered in the present paper. The upper rectangular plate having surface roughness is approaching towards lower isotropic porous rectangular plates.

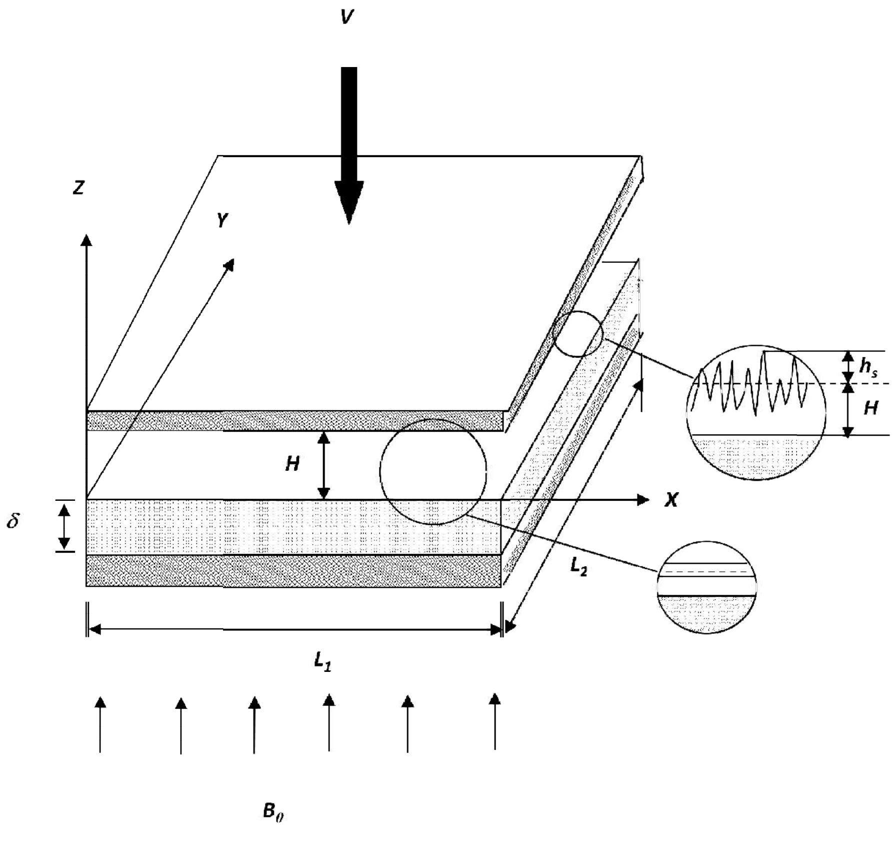

Fig.1. Physical configuration of the porous rough rectangular plates

In addition to the usual assumptions of hydrodynamic and hydromagnetic lubrication, we assume that the fluid is incompressible and body forces and couples are absent. Under these assumptions, the equations of the motion derived by the stokes (1966) takes the form

$$\mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \sigma B_0^2 u = \frac{\partial p}{\partial x} \quad \text{……………(1)}$$

$$\mu \frac{\partial^2 v}{\partial z^2} - \eta \frac{\partial^4 v}{\partial z^4} - \sigma B_0^2 v = \frac{\partial p}{\partial y} \quad \text{……………(2)}$$

$$\frac{\partial p}{\partial z} = 0 \quad \text{…………...(3)}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad \text{..............(4)}$$

where u, v and w are the fluid velocity components along x, y, and z directions respectively in the film region and p is the pressure in the film region.

The flow of non-Newtonian fluid blended with lubricant additives in the porous matrix is governed by the modified form of the Darcy law, which account for the polar effects;

$$\vec{q}^{\,*}=-\frac{k}{\mu(1-\beta)}\nabla p^{*} \qquad \text{..............(5)}$$

Where $\vec{q}^{\,*}=(u^*,v^*,w^*)$, $\beta=(\eta/\mu)/K$, μ the isotropic viscosity of the fluid, k is the permeability of the porous matrix. The ratio (η/μ) is of dimension length square and this length may be identified as the chain length of the polar additives in a non-polar lubricant. Due to continuity of the lubricant flow, the pressure P^* in the porous matrix satisfies the Laplace equation

$$\frac{\partial^2 p^*}{\partial x^2}+\frac{\partial^2 p^*}{\partial y^2}+c^2\frac{\partial^2 p^*}{\partial z^2}=0$$

where $c=\sqrt{\dfrac{1-\beta+\dfrac{k\sigma B_0^2}{\mu M}}{1-\beta}}$ ....(6)

The relevant boundary conditions for the velocity components are
At the upper solid rough rectangular plate $(z=H)$

$u=v=0$ (no slip), $w=\dfrac{dH}{dt}=V$ (Squeezing velocity) ....(7a)

$\dfrac{\partial^2 u}{\partial z^2}=\dfrac{\partial^2 v}{\partial z^2}=0$ (vanishing of couple stresses) ....(7b)

At the lower porous rectangular plate (z=0)

$\dfrac{\alpha}{\sqrt{k}}(u-u^*)=\dfrac{\partial u}{\partial z}$ (B-J Slip condition) .... (8a)

$\dfrac{\alpha}{\sqrt{k}}(v-v^*)=\dfrac{\partial v}{\partial z}$ (B-J Slip Condition) ....(8b)

$w=w^*$ (Continuity of vertical component of velocity) ....(8c)

$\dfrac{\partial^2 u}{\partial z^2}=\dfrac{\partial^2 v}{\partial z^2}=0$ (vanishing of couple stresses) .... (8d)

The Modified Darcy's equations are

$$u^*=\frac{-k}{\mu\left(1-\beta+\dfrac{k\sigma B_0^2}{\mu M}\right)}\frac{\partial p^*}{\partial x} \quad v^*=\frac{-k}{\mu\left(1-\beta+\dfrac{k\sigma B_0^2}{\mu M}\right)}\frac{\partial p^*}{\partial y} \quad w^*=\frac{-k}{\mu(1-\beta)}\frac{\partial p^*}{\partial z}$$

Solving equation (1) and (2) using the boundary conditions of (7) and (8) we get

$$u=\frac{1}{\sigma B_0^2}\frac{\partial p}{\partial x}\frac{1}{(A^2-B^2)}\left[f_{11}\left\{\left(\frac{B^2 Sinh\frac{A(z-H)}{l}}{Sin\frac{AH}{l}}\right)-\left(\frac{A^2 Sinh\frac{B(z-H)}{l}}{Sin\frac{BH}{l}}\right)\right\}-f_{12}\right]$$

$$v=\frac{1}{\sigma B_0^2}\frac{\partial p}{\partial y}\frac{1}{(A^2-B^2)}\left[f_{11}\left\{\left(\frac{B^2 Sinh\frac{A(z-H)}{l}}{Sin\frac{AH}{l}}\right)-\left(\frac{A^2 Sinh\frac{B(z-H)}{l}}{Sin\frac{BH}{l}}\right)\right\}-f_{12}\right]$$

where $l=(\eta/\mu)^{1/2}$ is the couple stress parameter

$$A=\left\{\frac{1+\left(1-4l^2\sigma B_0^2/\mu\right)^{1/2}}{2}\right\}^{1/2},\quad B=\left\{\frac{1-\left(1-4l^2\sigma B_0^2/\mu\right)^{1/2}}{2}\right\}^{1/2}.$$

$$f_{11}=\left\{\frac{\left\{1-\xi_1 Co\sec h\frac{AH}{l}+\xi_2 Co\sec h\frac{BH}{l}\right\}-\frac{k\sigma B_0^2}{\mu\xi_3}}{\left(1-\xi_1 Coth\frac{AH}{l}+\xi_2 Coth\frac{BH}{l}\right)}\right\}$$

$$f_{12}=\left(B^2 Sinh\frac{Az}{l}Co\sec h\frac{AH}{l}-A^2 Sinh\frac{Bz}{l}Co\sec h\frac{BH}{l}+(A^2-B^2)\right)$$

$$\xi_1=\frac{\sigma^* AB^2}{(A^2-B^2)l}\qquad \xi_2=\frac{\sigma^* BA^2}{(A^2-B^2)l}\qquad \xi_3=1-\beta+\frac{k\sigma B_0^2}{\mu M}$$

The boundary conditions(7b)and (8d) indicate vanishing of couplestresses on the bearing surfaces at $z=H$ and $z=0$ respectively, and the boundary conditions (8a)and (8b) are the Beavers and Joseph (1967) slip boundary conditions where α is the non-dimensional slip constant, where z depends on the characteristics of the porous medium.

Integrating the mass equation (4) with respect to z using u and v from equation (1) and (2), and using independence of the pressure with respect to z, and the corresponding boundary conditions in (7a)-(8d),the modified Reynolds equation is obtained in the form

$$\frac{\partial}{\partial x}\left\{g(H,\sigma^*,l,B_0)\frac{\partial p}{\partial x}\right\}+\frac{\partial}{\partial y}\left\{g(H,\sigma^*,l,B_0)\frac{\partial p}{\partial y}\right\}=\sigma B_0^2\left\{\frac{dH}{dt}+\frac{k}{\mu(1-\beta)}\frac{\partial p^*}{\partial z}\Big|_{z=0}\right\} \quad \text{………... (9)}$$

Where $k=\sigma^{*2}\alpha^2$ $\quad g(H,\sigma^*,l,B_0)=\frac{l}{(A^2-B^2)}\left\{g_{11}g_{12}+\frac{H}{l}(A^2-B^2)\right\}$

$$g_{11}=\left\{\frac{\left(2-\xi_1 Coth\frac{AH}{2l}+\xi_2 Coth\frac{BH}{2l}\right)-\frac{\sigma^{*2}\alpha^2\sigma B_0^2}{\xi_3\mu}}{1-\xi_1 Coth\frac{AH}{l}+\xi_2 Coth\frac{BH}{l}}\right\}$$

$$g_{12}=\left\{\frac{B^2}{A}tanh\frac{AH}{2l}-\frac{A^2}{B}\tanh\frac{BH}{2l}\right\}$$

The film thickness is considered to be made up of two parts

$$H=h(t)+h_s(x,y,\xi) \quad \text{………. (10)}$$

Where the nominal smooth part of the film thickness is represented by h, the random part h_s resulting from the surface roughness asperities measured from the nominal level and ξ is an index describing the definite roughness arrangements.

Modified Stochastic Reynolds equation

In the study of roughness phenomenon two types of surface roughness are of interest, namely (i) the one-dimensional longitudinal roughness where roughness pattern are in the form of long narrow ridges and valleys running in the x-direction, and (ii) the one-dimensional transverse roughness where, the roughness striations are in the form of long narrow ridges and valleys running in the y-direction. In the present study, the analysis is restricted to only one- dimensional longitudinal roughness, because, both the roughness patterns can be made identical, by just a rotation of co-ordinate axes.

The film thickness for the longitudinal one-dimensional roughness, assumes the form

$$H = h(t) + h_s(y,\xi) \qquad \text{..........(11)}$$

Taking the expected value on both side of equation (12) and assuming in accordance with the Christensen stochastic approach for rough surfaces, the pressure gradient in the direction of roughness and flux perpendicular to it are stochastic variables with zero or negligible variance. The variables with zero variance are the pressure gradient in x and the velocity along y. Then the equation for longitudinal roughness takes the form.

$$\frac{\partial^2 E(p)}{\partial x^2} + R\frac{\partial^2 E(p)}{\partial y^2} = \frac{\sigma B_0^2}{S}\left\{\frac{\partial H}{\partial t} + \frac{k}{\mu(1-\beta)}\frac{\partial p^*}{\partial z}\Big|_{z=0}\right\} \qquad \text{..................(12)}$$

Where

$$E\left(g(H,\sigma^*,l,B_0)\right) = \frac{35}{32c^7}\int_{-c}^{c} g(H,\sigma^*,l,B_0)(c^2-h_s^2)^3\,dh_s \quad E\left(1/g(H,\sigma^*,l,B_0)\right) = \frac{35}{32c^7}\int_{-c}^{c}\frac{(c^2-h_s^2)^3}{g(H,\sigma^*,l,B_0)}dh_s$$

$$S = E\left(g(H,\sigma^*,l,B_0)\right)$$

$$R = \left[E\left(g(H,\sigma^*,l,B_0)\right).E\left(1/g(H,\sigma^*,l,B_0)\right)\right]^{-1} \qquad \text{..................(13)}$$

$E(*)$ denotes the expectancy operator defined by

$$E(*) = \int_{-\infty}^{\infty}(*)f(h_s)dh_s \qquad \text{..................(14)}$$

Where $f(h_s)$ is the probability distribution function of the random variable h_s. In most of the engineering application, the rough surfaces are of Gaussian type and hence, the following polynomial function is chosen to approximate the Gaussian distribution; Following Christensen (1969), We assume that

$$f(h_s) = \begin{cases}\frac{35}{32c^7}(c^2-h_s^2)^3. & -c < h_s < c\\ 0 & elsewhere\end{cases} \qquad \text{...............(15)}$$

Where $\bar{\sigma} = \frac{c}{3}$ is the standard deviation.

The relevant boundary conditions for pressure field are

(i) for the fluid region

$$E\left(p(x,y)\right) = 0 \quad \text{at} \quad x = 0, L_1 \quad \text{and} \quad y = 0, L_2 \qquad \text{..............(16)}$$

(ii) for the porous region

$p^*(x,y,z)=0$ at $x=0,L_1$ and $y=0,L_2$ ……………(17)

$$\left.\frac{\partial p^*}{\partial z}\right|_{z=-\delta}=0 \quad \text{(solid baking)} \qquad ……………(18)$$

(iii)at the interface

$E\left(p(x,y)\right)=p^*(x,y,0)$ (Pressure continuity) ……………(19)

The solution of (6) subject to the boundary conditions (17) and (18) is

$$p^*\left(x,y,z\right)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_{mn}\cosh\left[\gamma_{mn}\left(z+\delta\right)\right]\times\sin\left(\alpha_m x\right)\sin\left(\beta_n y\right), \qquad …………….(20)$$

Where

$$\alpha_m=\frac{m\pi}{L_1},\quad \beta_n=\frac{n\pi}{L_2},\quad \gamma_{mn}=\left(\alpha_m^2+\beta_n^2\right)^{1/2}$$

The solution of equation (12) subject to the boundary conditions (16) and(19) is obtained in the form

$$E\left(p(x,y)\right)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}b_{mn}Sin(\alpha_m x)Sin(\beta_n y) \qquad …………(21)$$

$$E\left(p(x,y)\right)=$$

$$=-\frac{16\mu M_0^2\,(dH/dt)}{h_0^2}\sum_{m=1,3,5,....}^{\infty}\sum_{n=1,3,5,.....}^{\infty}\frac{Sin(m\pi x/L_1)Sin(n\pi y/L_2)}{mn\pi^2\left\{\left(\frac{m^2\pi^2}{L_1^2}+R\frac{n^2\pi^2}{L_2^2}\right)S+\frac{M_0^2}{h_0^2}\frac{k\gamma}{c\left(1-\beta\right)}\tan h\left(\frac{\gamma\delta}{c}\right)\right\}}$$

where $\gamma_{mn}=\sqrt{\frac{m^2\pi^2}{L_1^2}+\frac{n^2\pi^2}{L_2^2}}$

$$P^*=-\frac{E(p(x,y))h_0^3}{\mu L_1^2\left(dH/dt\right)}$$

$$=\frac{16M_0^2}{\pi^2}\sum_{m=1,3,5,....}^{\infty}\sum_{n=1,3,5,....}^{\infty}\frac{Sin(m\pi x^*)Sin(\lambda n\pi y^*)}{mn\left\{\pi^2\left(m^2+R^*\lambda^2 n^2\right)S^*+\frac{M_0^2\sqrt{\lambda}}{(1-\beta)}\frac{\psi\gamma_{mn}^*}{\delta^*}\tan h\left(\frac{\gamma_{mn}^*\delta^*}{\sqrt{\lambda}}\right)\right\}}$$

Where $k=\sigma^2\alpha^2$ $\quad H^*=\frac{h_1}{h_0}$ $\quad l^*=\frac{2l}{h_0}$ $\quad s=\frac{\sigma^*}{h_0}$ $\quad \psi=\frac{k\delta}{h_0^3}$ $\quad M_0=B_0h_0(\sigma/\mu)^{1/2}$

$$\delta^*=\frac{\delta}{\sqrt{L_1L_2}},\ \lambda=\frac{L_1}{L_2},\ \gamma_{mn}^*=\gamma_{mn}L_1$$

$$g^*(H^*,s,l^*,M_0)=\frac{l^*}{2(A^{*2}-B^{*2})}\left\{g_{11}^*g_{12}^*+\frac{2(A^{*2}-B^{*2})}{l^*}(H^*+h_s^*)\right\}$$

$$g_{11}^*=\left\{\frac{\left(2-\xi_1^*Coth\frac{A^*(H^*+h_s^*)}{l^*}+\xi_2^*Coth\frac{B^*(H^*+h_s^*)}{l^*}\right)-\frac{s^2\alpha^2M_0^2}{\xi_3^*}}{1-\xi_1^*Coth\frac{2A^*(H^*+h_s^*)}{l^*}+\xi_2^*Coth\frac{2B^*(H^*+h_s^*)}{l^*}}\right\}$$

$$g_{12}^* = \left\{ \frac{B^2}{A^*} tanh \frac{A^*(H^* + h_s^*)}{l^*} - \frac{A^2}{B^*} \tanh \frac{B^*(H^* + h_s^*)}{l^*} \right\}$$

$$\xi_1^* = \frac{2sA^* B^{*2}}{(A^{*2} - B^{*2})l^*} \qquad \xi_2^* = \frac{2sA^{*2} B^*}{(A^{*2} - B^{*2})l^*} \qquad \xi_3^* = 1 - \beta + \frac{s^2 \alpha^2 M_0^2}{M}$$

$$S^* = E\left(g^*(H^*, s, l^*, M_0)\right)$$

$$R^* = \left[E\left(g^*(H^*, s, l^*, M_0)\right).E\left(1/g^*(H^*, s, l^*, M_0)\right)\right]^{-1}$$

The load carrying capacity of the squeeze film is obtained by integrating the pressure filed over the surface of the top plate

$$E(w) = \int_0^{L_2} \int_0^{L_1} E\left(p(x,y)\right) dxdy \qquad \text{……………….. (22)}$$

$$E(w) =$$

$$= -\frac{64\sigma B_0^2 \left(dH/dt\right) L_1^3 L_2}{\pi^4} \sum_{m=1,3,5,....}^{\infty} \sum_{n=1,3,5,....}^{\infty} \frac{1}{m^2 n^2 \left\{ \pi^2 \left(m^2 + Rn^2\lambda^2\right) S + \frac{\sigma B_0^2 k L_1^2}{\mu(1-\beta)} \gamma_{mn} \tan h\left(\gamma_{mn}\delta\right) \right\}}$$

The non-dimensional mean instantaneous load carrying capacity of the squeeze film is given by

$$W^* = -\frac{E(w) h_0^3}{\mu L_1^2 L_2^2 \left(dH/dt\right)} \qquad dt = -\frac{\mu L_1^2 L_2^2}{E(w) h_0^3} dH$$

$$= \frac{64 M_0^2 \delta^* \sqrt{\lambda}}{\pi^4} \sum_{m=1,3,5,....}^{\infty} \sum_{n=1,3,5,....}^{\infty} \frac{1}{m^2 n^2 \left\{ \frac{\delta^*}{\sqrt{\lambda}} \pi^2 \left(m^2 + R^* n^2 \lambda^2\right) S^* + \frac{M_0^2 \psi \gamma_{mn}^*}{(1-\beta)} \tan h\left(\frac{\gamma_{mn}^* \delta^*}{\sqrt{\lambda}} \right) \right\}}$$

$$T^* = -\frac{h_0^2 E(w) t}{\mu L_1^2 L_2^2} = \frac{64 M_0^2 \delta^* \sqrt{\lambda}}{\pi^4} \sum_{m=1,3,5,...}^{\infty} \sum_{n=1,3,5,......}^{\infty} I_{mn}$$

$$I_{mn} = \int_{h_1^*}^{1} \frac{1}{m^2 n^2 \left\{ \frac{\delta^*}{\sqrt{\lambda}} \pi^2 \left(m^2 + R^* n^2 \lambda^2\right) S^* + \frac{M_0^2 \psi \gamma_{mn}^*}{(1-\beta)} \tan h\left(\frac{\gamma_{mn}^* \delta^*}{\sqrt{\lambda}} \right) \right\}} dH^*$$

RESULTS AND DISCUSSION

In order to study squeeze film characteristics on the porous rough rectangular plates with non-Newtonian fluid blended with lubricant additives, the numerical computations are performed for various non-dimensional parameters, viz., lubricant additive parameter l^*, magnetic parameter M_0, roughness parameter and the slip parameter s . Figure 2 depicts the non-dimensional load carrying capacity W^* with $\log_{10}(\lambda)$ for various values of magnetic parameter M_0, it is observed that effect of magnetic field is to increase the load carrying capacity as compared to non-magnetic case $(M_0 \to 0)$. The variation of non-dimensional load carrying capacity W^* with $\log_{10}(\lambda)$ for various values of lubricant additive parameter l^* is shown in Figure 3 it is clear that the effect of lubricant additives is to increase the load carrying capacity for rough plates in the presence of magnetic field. The variation of W^* with $\log_{10}(\lambda)$ for different values of s for two values of c is depicted in Figure 4. It is observed that the effect of the slip parameter is to decrease the load capacity.

The effect of transverse magnetic field on the squeeze film characteristics is evaluated by the relative percentage difference. The increase in the non-dimensional load carrying capacity R_{W^*} and non-dimensional squeeze film time R_{W^*} and R_{T^*} are defined by $R_{W^*} = \left\{ \left(W^*_{magnetic} - W^*_{non-magnetic} \right) / W^*_{non-magnetic} \right\} \times 100$, $R_{T^*} = \left\{ \left(T^*_{magnetic} - T^*_{non-magnetic} \right) / T^*_{non-magentic} \right\} \times 100$.

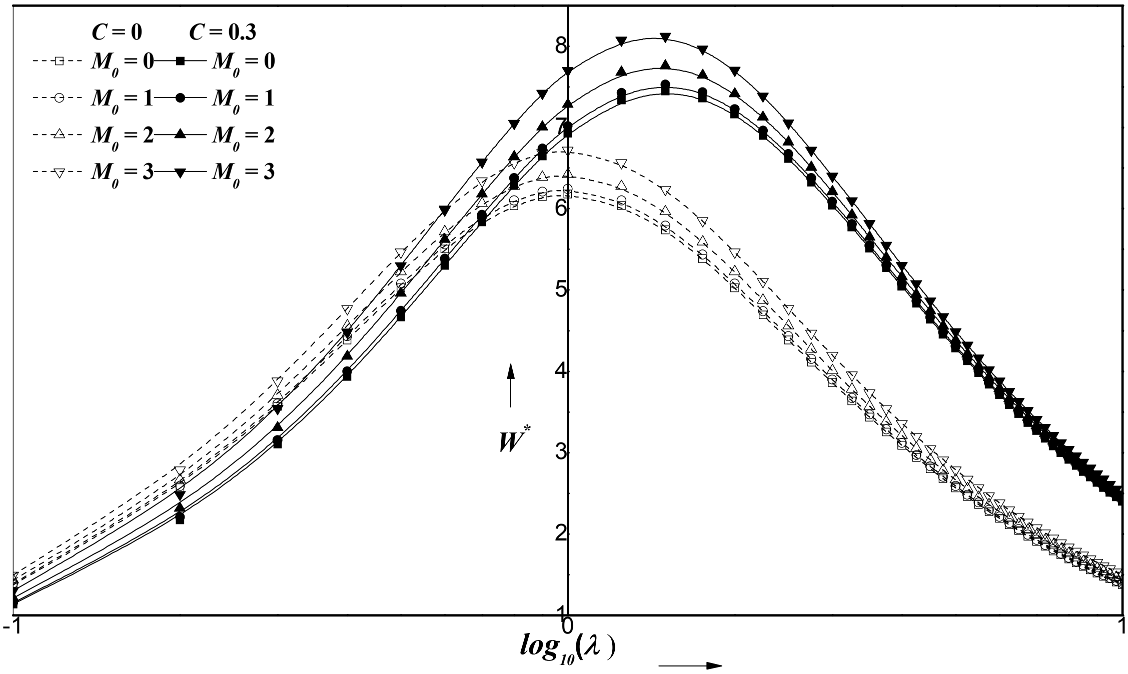

Fig.2 Variation of nondimensional Load carrying capacity W^* with $\log_{10}(\lambda)$ for different values of C and M_0 with $l^* = 0.3$, $H^* = 0.4$, $\psi = 0.001$, $s = 0.25$, $\alpha = 0.1$, $\beta = 0.2$, $\delta^* = 0.02$, $M' = 0.6$.

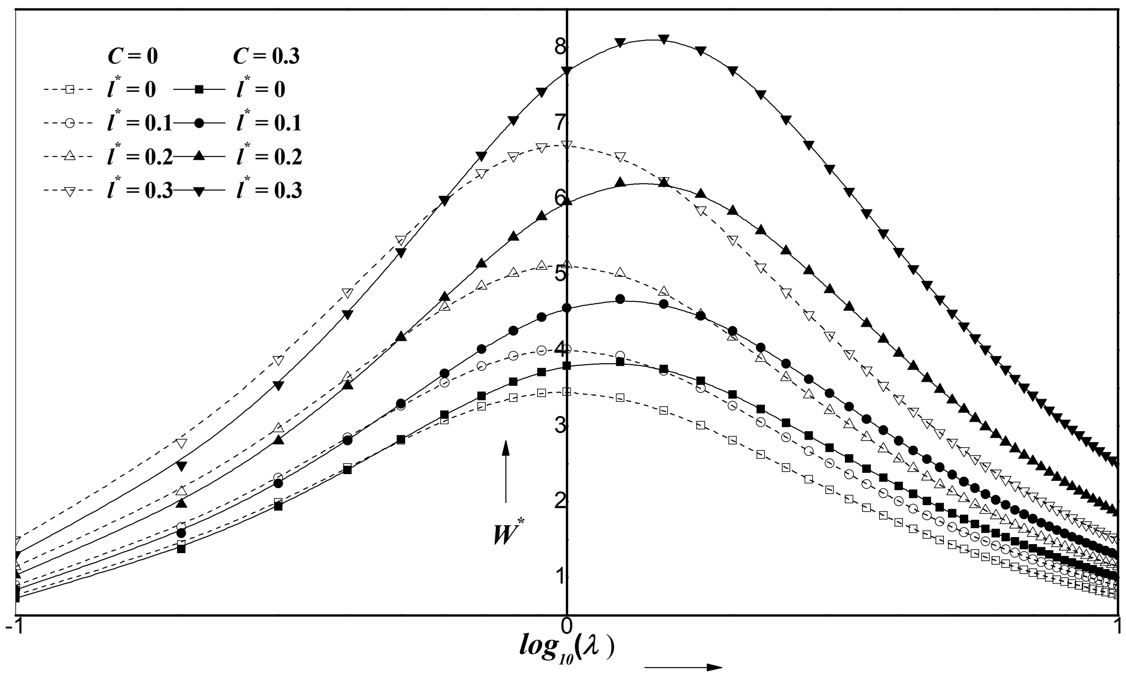

Fig.3 Variation of nondimensional Load carrying capacity W^* with $\log_{10}(\lambda)$ for different values of C and l^* with $M_0 = 3$, $H^* = 0.4$, $\psi = 0.001$, $s = 0.25$, $\alpha = 0.1$, $\beta = 0.2$, $\delta^* = 0.02$, $M' = 0.6$.

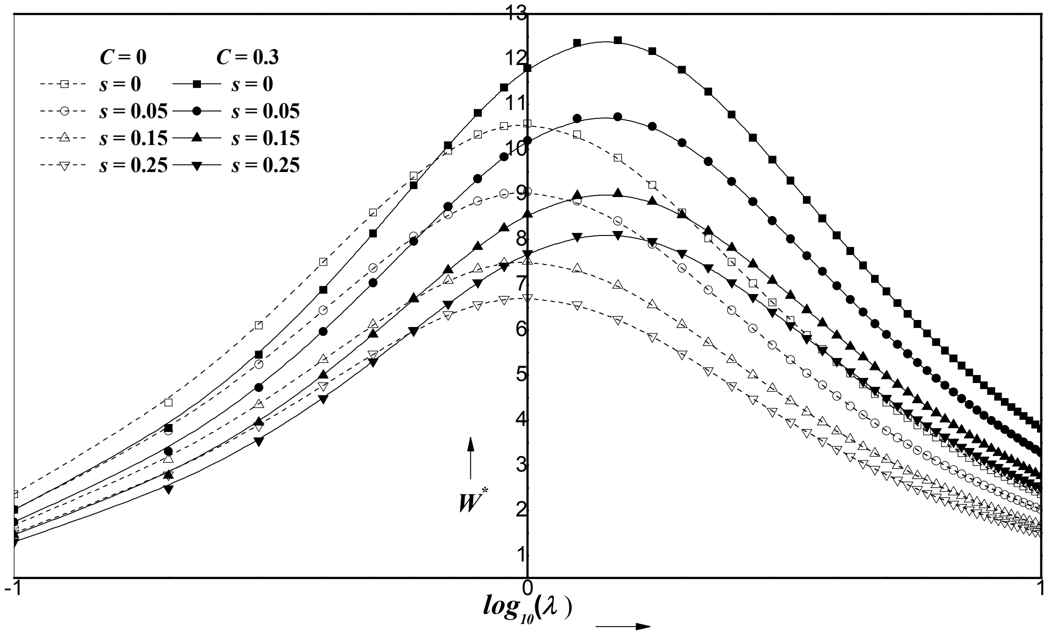

Fig.4 Variation of nondimensional Load carrying capacity W^* with $\log_{10}(\lambda)$ for different values of C and s with $M_0 = 3$, $l^* = 0.3$, $H^* = 0.4$, $\psi = 0.001$, $\alpha = 0.1$, $\beta = 0.2$, $\delta^* = 0.02$, $M' = 0.6$.

The variation of R_{W^*} with λ for different values of c and ψ is shown in Figure 5. This figure clearly indicates that, there exist a critical values of aspect ratio $\lambda_c = 0.6$ which is independent of ψ and at which the roughness effects are not predominant. The effect of surface roughness is to increase R_{W^*} for $\lambda > \lambda_c$ and is to decrease R_{W^*} for $\lambda < \lambda_c$. Similar trend is observed for different values of . Figure 6 shows the variation of R_{W^*} with λ for different values of l^* and c. It is interesting to note that the critical value of the aspect ratio λ_c is a function of lubricant additive parameter l^* and roughness parameter c in the presence of magnetic field. When $\lambda > \lambda_c$ the effect of c and l^* is to increase the load carrying capacity and when $\lambda < \lambda_c$ the effect of c and l^* is to reduce the load carrying capacity.

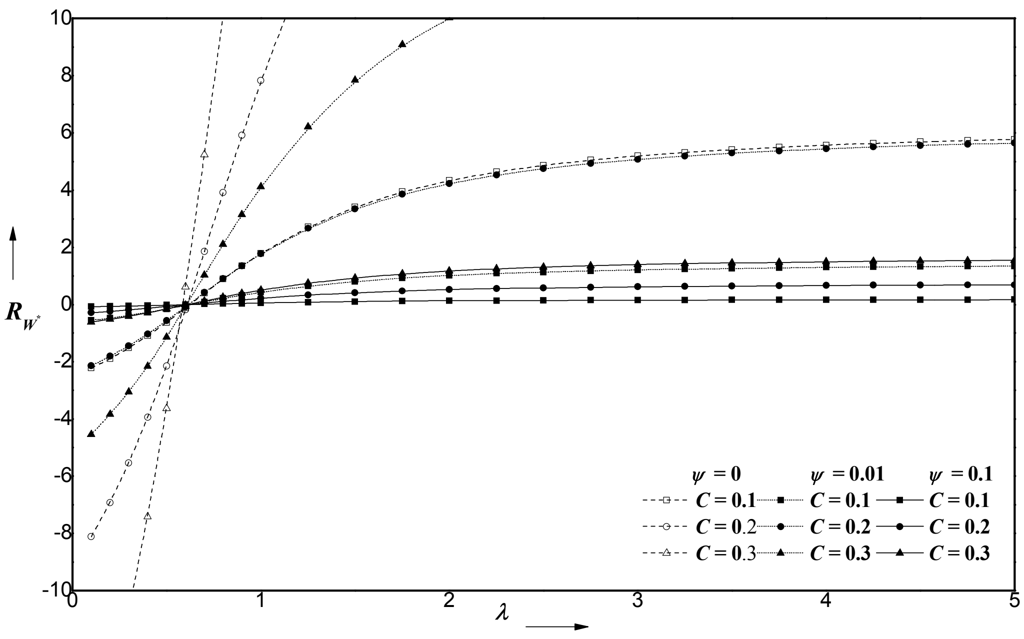

Fig.5. Variation of nondimensional Load carrying capacity R_{W^*} with λ for different values of ψ and C with $M_0 = 3$, $l^* = 0.3$, $H^* = 0.4$, $\alpha = 0.1$, $\beta = 0.2$, $\delta^* = 0.02$, $M' = 0.6$.

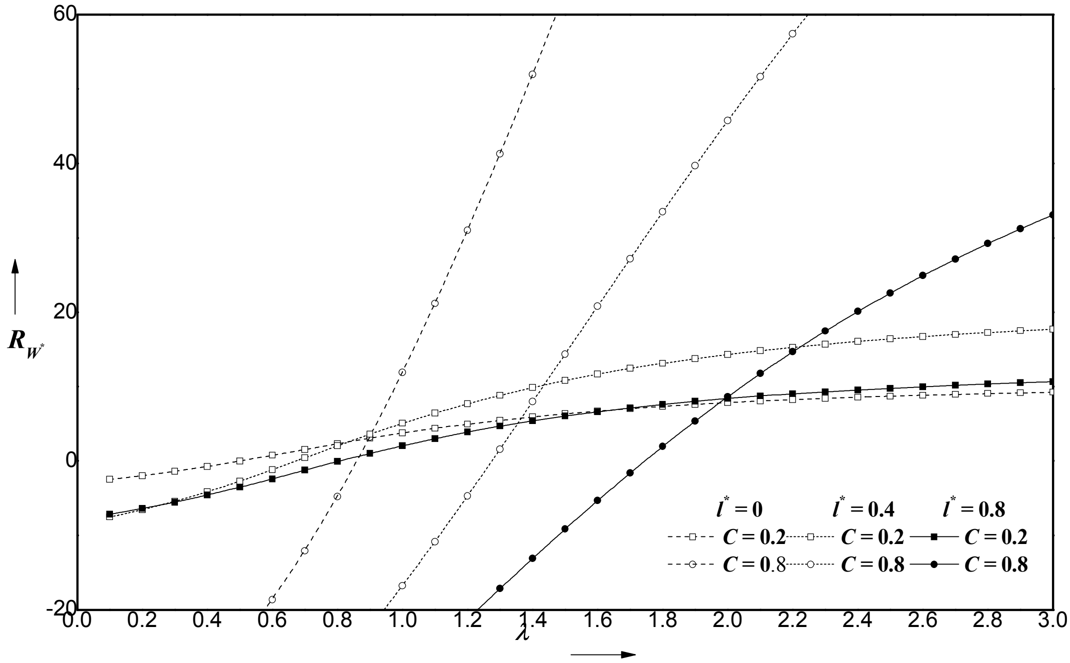

Fig.6 Variation of nondimensional Load carrying capacity R_{W^*} with λ for different values of l^* and C with $M_0 = 3$, $H^* = 0.4$, $\psi = 0.001$, $\alpha = 0.1$, $\beta = 0.2$, $\delta^* = 0.02$, $M = 0.6$.

Figure 7 shows the variation of T^* with $\log_{10}(\lambda)$ for different values of M_0 .It is observed that effect of magnetic parameter is to enhance the squeeze film time. Figure 8 shows the variation of T^* with $\log_{10}(\lambda)$ for different values of l^*. It is observed that effect of l^* is to increase load carrying capacity in the presence of M_0 and c .

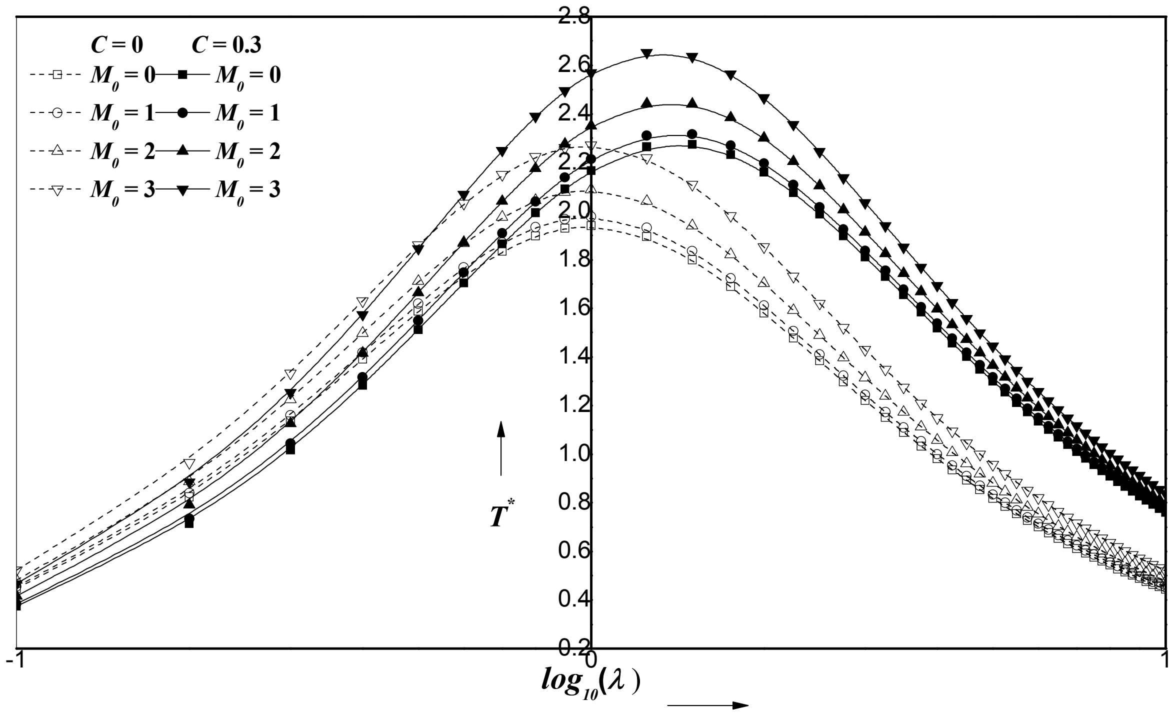

Fig.7 Variation of nondimensional squeeze film Time T^* with $\log_{10}(\lambda)$ for different values of C and M_0 with $l^* = 0.3$, $h_1^* = 0.3$, $\psi = 0.001$, $s = 0.25$, $\alpha = 0.1$, $\beta = 0.2$, $\delta^* = 0.02$, $M' = 0.6$.

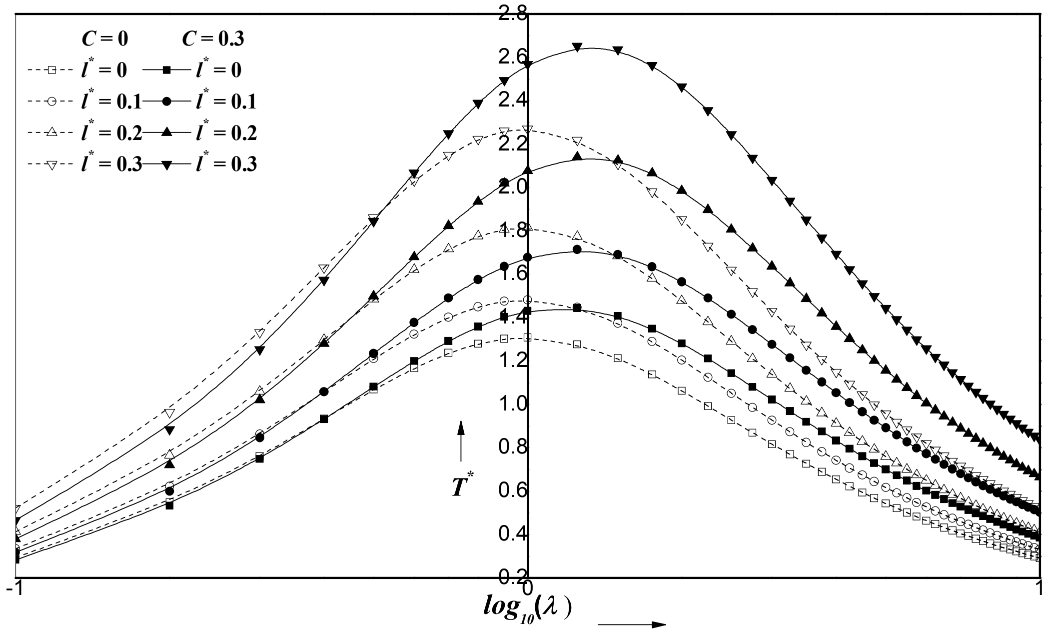

Fig.8 Variation of nondimensional squeeze film Time T^* with $\log_{10}(\lambda)$ for different values of C and l^* with $M_0 = 3$, $h_1^* = 0.3$, $\psi = 0.001$, $s = 0.25$, $\alpha = 0.1$, $\beta = 0.2$, $\delta^* = 0.02$, $M' = 0.6$.

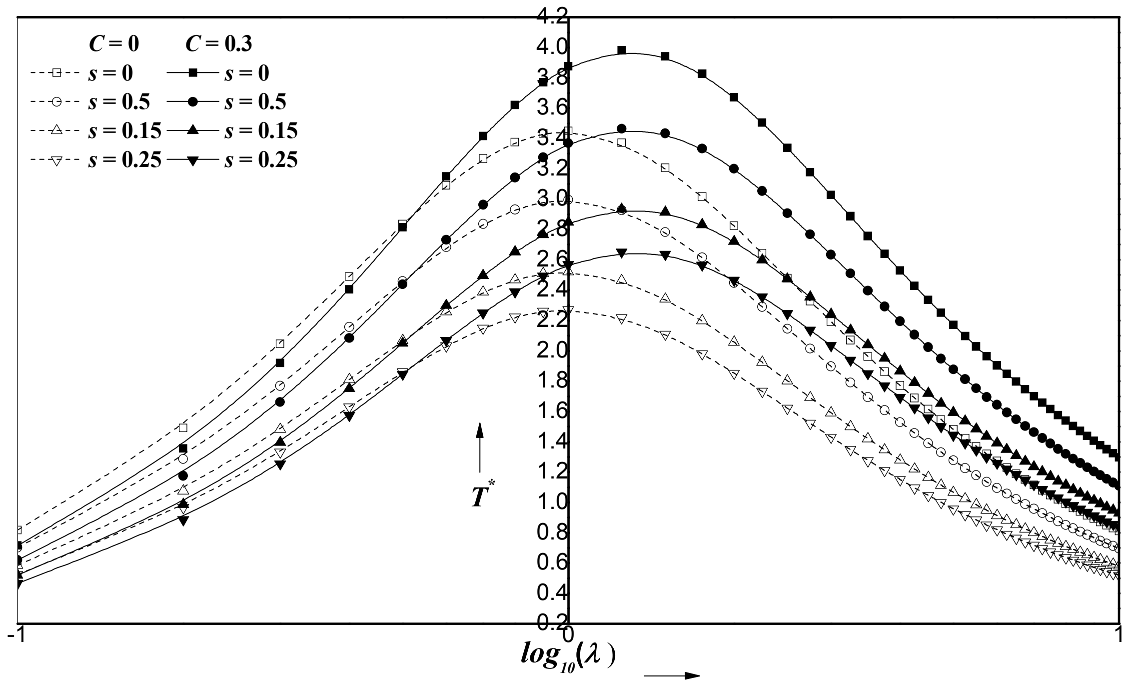

Fig.9 Variation of nondimensional Squeeze film Time T^* with $\log_{10}(\lambda)$ for different values of C and s with $M_0 = 3$, $l^* = 0.3$, $h_1^* = 0.3$, $\psi = 0.001$, $\alpha = 0.1$, $\beta = 0.2$, $\delta^* = 0.02$, $M' = 0.6$.

Figure 9 shows the variation of T^* with $\log_{10}(\lambda)$ for different values of s. It is observed that, the effect of the slip is to decrease squeeze film time T^* for all values of λ. Figure 10 shows the variation of squeeze film time T^* with h_1^* for different values of l^*. It is observed that the effect of lubricant additives and surface roughness is to delay the squeezing time T^* in the presence of transverse magnetic field. The variation of R_{W^*} and R_{T^*} with M_0 is shown in Table 1 for increasing values of c, it is observed that, there is an increase of 13.39%,and 12.56% in R_{W^*} and R_{T^*} respectively for $M_0 = 2,\ c = 0.3$. Variation of W^*

with T^* is shown in Table 2. It is observed that as $M_0 \to 0$ these results are in good agreement with Naduvinamani *et al.*(2004) [4]. There is an increase of 5.96% and 4.29% in W^* and T^* respectively for $l^* = 0.2,\ c = 0.3, M_0 = 3.$

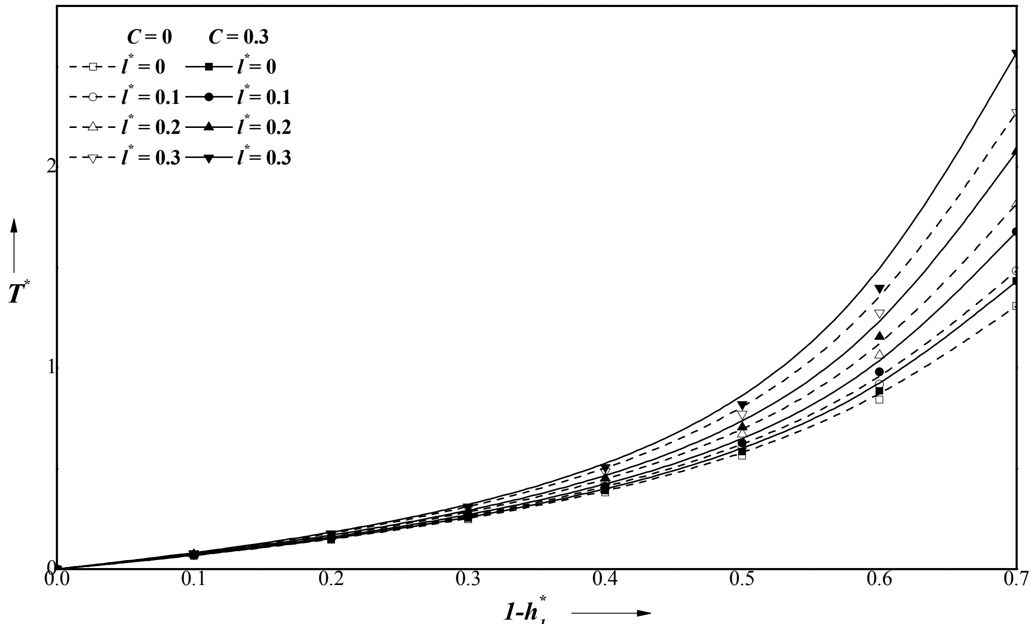

Fig.10 Variation of nondimensional squeeze film time T^* with h_1^* for different values of C and l^* with $M_0 = 3$, $\lambda = 1$, $\psi = 0.001$, $s = 0.25$, $\alpha = 0.1$, $\beta = 0.2$, $\delta^* = 0.02$, $M' = 0.6$.

Table 1. Variation of R_{W^*} and R_{T^*} with M_0 for different values C with $H^* = 0.4$, $l^* = 0.3$, $\lambda = 1$, $\psi = 0.001$, $s = 0.25$, $\alpha = 0.1$, $\beta = 0.2$, $\delta = 0.02$, $M' = 0.6$, $h_1^* = 0.3$

M_0	C	R_W^*	R_T^*
	0.1	1.275242	1.219254
0	0.2	5.25127	5.295814
	0.3	12.0504	11.72836
	0.1	1.28968	1.223381
1	0.2	5.346323	5.341793
	0.3	12.42947	11.9864
	0.1	1.32512	1.227717
2	0.2	5.583811	5.427522
	0.3	13.3952	12.56111
	0.1	1.364269	1.216378
3	0.2	5.854156	5.452577
	0.3	14.55408	13.06374

Table 2. Squeeze film characteristics W^*, T^* and comparison with non magnetic case by Naduvinamani et. al. (2004) with $H = 0.4$, $\lambda = 1$, $\psi = 0.001$, $s = 0.25$, $\alpha = 0.1$, $\beta = 0.2$, $\delta = 0.02$, $h_1^* = 0.3$

		Naduvinamani et al. (2004) analysis			Present analysis					
					$M_0 = 0$			$M_0 = 3$		
	C	$l^* = 0$	$l^* = 0.2$	$l^* = 0.4$	$l^* = 0$	$l^* = 0.2$	$l^* = 0.4$	$l^* = 0$	$l^* = 0.2$	$l^* = 0.4$
W^*	0	2.74279	4.5111	8.12101	2.74257	4.5111	8.12101	3.45397	5.13049	8.58242
	0.1	2.76629	4.56923	8.21366	2.76607	4.56923	8.21366	3.48396	5.19966	8.68795
	0.2	2.84275	4.76069	8.48353	2.84252	4.76069	8.48353	3.58344	5.43611	9.01651
	0.3	2.99914	5.15146	8.85789	2.9989	5.15146	8.85789	3.79515	5.96131	9.55822
T^*	0	1.92956	3.18557	5.78935	1.92941	3.18557	5.78935	2.43323	3.62768	6.12511
	0.1	1.94691	3.22926	5.86169	1.94675	3.22926	5.86169	2.45527	3.67943	6.20715
	0.2	2.00625	3.38415	6.09778	2.00609	3.38415	6.09778	2.53147	3.8678	6.48844
	0.3	2.13345	3.70724	6.47459	2.13328	3.70724	6.47459	2.7009	4.29589	7.00108

Table 3. Squeeze film characteristics W^*, T^* for increasing values of λ, ψ and its comparison with non magnetic case by Naduvinamani et al. (2004) $h^* = 0.4$, $C = 0.3$, $s = 0.25$, $\beta = 0.2$, $\alpha = 0.1$, $\delta = 0.02$, $h_1^* = 0.6$

λ		Naduvinamani *et al.*		Present Analysis			
	ψ	$l^* = 0.2$		$l^* = 0.4$, $M_0 = 0$		$l^* = 0.4$, $M_0 = 2$	
		W^*	T^*	W^*	T^*	W^*	T^*
1	0	12.9701	0.787914	12.9701	0.787914	13.6321	0.911387
	0.001	8.85789	0.727451	8.85789	0.727451	9.1804	0.836571
	0.01	2.31337	0.462442	2.31337	0.462442	2.34411	0.517648
	0.1	0.276119	0.130265	0.276119	0.130265	0.277818	0.138281
	1	0.028158	0.0208458	0.028158	0.0208458	0.0283069	0.0215524
2	0	15.072	0.72252	15.072	0.72252	15.6259	0.819637
	0.001	9.06891	0.657048	9.06891	0.657048	9.28842	0.742031
	0.01	1.98907	0.399329	1.98907	0.399329	2.00785	0.442077
	0.1	0.226208	0.111475	0.226208	0.111475	0.227492	0.118385
	1	0.022937	0.0186685	0.022937	0.0186685	0.0230571	0.0193769
3	0	13.5181	0.579903	13.5181	0.579903	13.9052	0.652878
	0.001	7.61896	0.524711	7.61896	0.524711	7.76094	0.58858
	0.01	1.55754	0.316476	1.55754	0.316476	1.57013	0.349237
	0.1	0.174245	0.0912466	0.174245	0.0912466	0.175206	0.0971965
	1	0.0176351	0.0163392	0.0176351	0.0163392	0.017727	0.0170419

Variation of W^* with T^* is shown in Table 3 for increasing values of λ and ψ. As $\psi \to 1$ there is decrease in W^* and T^* values. There is an increase of 9.28% and 0.74% in W^* and T^* respectively for $l^* = 0.4,\ M_0 = 2, \lambda = 2, \psi = 0.001$

Conclusion

On the basis of Christensen's Stochastic Theory and Stokes microcontinuum theory to account for fluids blended with lubricant additives, the squeeze film lubrication between porous rough rectangular plates with the combined effect of MHD and lubricant additives is presented in this paper. The modified Stochastic Reynolds equation governing the squeeze film pressure, load carrying capacity and squeeze film time is derived. It is found that the effect of magnetic field is to increase the mean load carrying capacity and hence to lengthen the mean squeeze film time compared to the non-magnetic case. As $(M_0 \to 0)$, the squeeze film characteristics reduces to the non-magnetic case studied by Naduvinamani *et al*. (2004). It is also found that, in the presence of applied magnetic field the effect of lubricant additives and surface roughness is to increase the load carrying capacity and hence to lengthen the squeeze film time as compared to Newtonian and non-magnetic case.

Nomenclature

B_0	applied magnetic field
c	maximum asperity deviation from the normal film height
C	dimensionless roughness parameter c/h_0
E	expectancy operator defined by equation (14)
$E(w)$	mean load carrying capacity
h	film thickness after time Δt
h^*	dimensionless film thickness after time Δt
H	film thickness
h_0	Initial film thickness
H^*	dimensionless film thickness $\left(= H/h_0\right)$
k	permeability of porous facing
l	couple stress parameter $(\eta/\mu)^{1/2}$
l^*	dimensionless couple stress parameter $\left(2l/h_0\right)$
L_1	length of plate
L_2	width of plate

M	porosity
M_0	Hartmann number $\left\{= B_0 h_0 (\sigma/\mu)^{1/2}\right\}$
p	pressure in the film region
p^*	pressure in the porous region
P^*	dimensionless pressure $\left\{- p(x,y) h_0^3 / \mu L_1^2 \left(\partial H/\partial t\right)\right\}$
x, y, z	rectangular coordinates
x^*, y^*	dimensionless rectangular coordinates $x^* = x/L_1$, $y^* = y/L_1$
s	slip parameter σ^*/h_0
t	time of approach
Δt	time required for the film thickness to decrease to a value h
T^*	dimensionless time of approach $\left(- h_0^2 W^* t / \mu L_1^2 L_2^2\right)$
u, v, w	velocity components in film region
u^*, v^*, w^*	modified Darcy velocity components in x, y, z directions, respectively
V	squeezing velocity $\left(-\partial H/\partial t\right)$
W	load carrying capacity
W^*	dimensionless load carrying capacity $\left\{-W h_0^3 / \mu L_1^2 L_2^2 \left(\partial H/\partial t\right)\right\}$
α	dimensionless slip constant
β	ratio of microstructure size to pore size $\left\{= \left(\eta/\mu\right)/k\right\}$
δ	porous layer thickness
δ^*	$\left(= \delta/\sqrt{L_1 L_2}\right)$
λ	Aspect ratio $\left(L_1/L_2\right)$
η	material constant characterizing couple stress
μ	viscosity coefficient
σ	electrical conductivity
σ^*	parameter $\left(\sqrt{k}/\alpha\right)$
ψ	permeability parameter $\left(= k\delta/h_0^3\right)$

REFERENCES

Alyaqout, S.F. and Elsharkawy, A. A. 2011. "Optimal film shape for two-dimensional slider bearings lubricated with couple stress fluids." *Trib.International*, Volume 44, Issue 3, PP.336–342.

Beavers, G.S. and Joseph, D. D. 1967. "Boundary conditions at a Naturally permeable Wall." *J.fluid Mechanics*, Volume 30, Issue 1, PP.197-207.

Bujurke, N.M., D. P. Basti, D. P., R. B. and Kudenatti, R. B. 2008. "Surface roughness effects onsqueeze film behavior in porous circular disks with couple stress fluid." *Transport in Porous Media,* Volume 71, Issue 2,PP.185–197.

Chiang, H.L., Hsu, C. H. and Lin, J.R. 2004. "Lubrication performance of finite journal Bearings considering effects of couple stresses and surface roughness." *Tri. International*, Volume 37, Issue 4,PP.297–307.

Christensen, H. 1969. "Stochastic models for hydrodynamic lubrication of rough Surfaces". *Proc Inst. Mech. Eng*. (Part I), Volume 184, Issue 55, PP.1013-1026.

Gururajan, K. and Prakash, J. 1999. “Surface roughness effects in infinitely long porous journal bearings.” *J. Trib*., Volume 121, Issue 1 ,PP.139-147.

Kudenatti, R, B., Murulidhara, N. H. P. and Patil, H. P. 2012a. “Numerical solution of the MHD Reynolds equation for squeeze-film lubrication between porous and rough rectangular plates.” Hindawi Publishing Corporation, ISRN Tribology, Volume 2013, PP.1-10

Kudenatti, R.B., Basti, D. P. and Bujurke, N.M. 2012. Numerical solution of the MHD Reynolds equation for squeeze film lubrication between two parallel surfaces. *Applied Mathematics and Computation*, Volume 218, Issue 18, PP.9372–9382.

Morgan, V. T. and Cameron, A. 1957. “Mechanism of lubrication in porous metal bearings.” Pro.lub.wear, London, Volume 89,PP.151-157.

Naduvinamani, N. B., Fathima, S, T. and Jamal, S. 2010. “ Effect of roughness on hydromagnetic squeeze films between porous rectangular plates.” *Trib.International*, Volume 43, Issue 11, PP.2145–2151.

Naduvinamani, N. B., Syeda Tasneem Fathima, Hiremath, P.S. 2004. “On the squeeze film effect of lubricants with additives between rough porous rectangular plates” ZAMM. Z.Angew.Math, Mech, Volume 84, PP.825-834.

Naduvinamani, N.B. and Patil, S.B. 2007. “Combined effects of surface roughness and couple stresses on static and dynamic behavior of squeeze film lubrication of porous journal bearings.” *Tribology Materials, Surfaces and Interfaces*, Volume 1, Issue 3, PP.145–153.

Prakash, J. and Tiwari, K. 1982. An Analysis of the Squeeze Film between Porous Rectangular Plates Including the Surface Roughness Effects.” *J Mech. Eng. Sci*., Volume 24, Issue 1, PP.45-49.

Prakash, J., Vij, S. K. 1973. “Load capacity and time height relation for squeeze films between porous plates.” Wear, Volume 24, PP.309-315.

Sinha, P.C. and Gupta, J.L. 1973. “Hydromagnetic squeeze film between porous rectangular plates.” *J.lub.Tec, ASME*, Volume 95,PP.394-398.

Stokes, V.K. 1966. “ Couple stresses in fluids.”*Physics of Fluids*, Volume 9,PP.1709-1715.

Wu, H. 1972. “An analysis of squeeze films between porous rectangular plates.” *ASME J. Lub Tech*, Volume 94, PP.64-68.

****